Estimation of the Earth's Impulse Response: Deconvolution and Beyond

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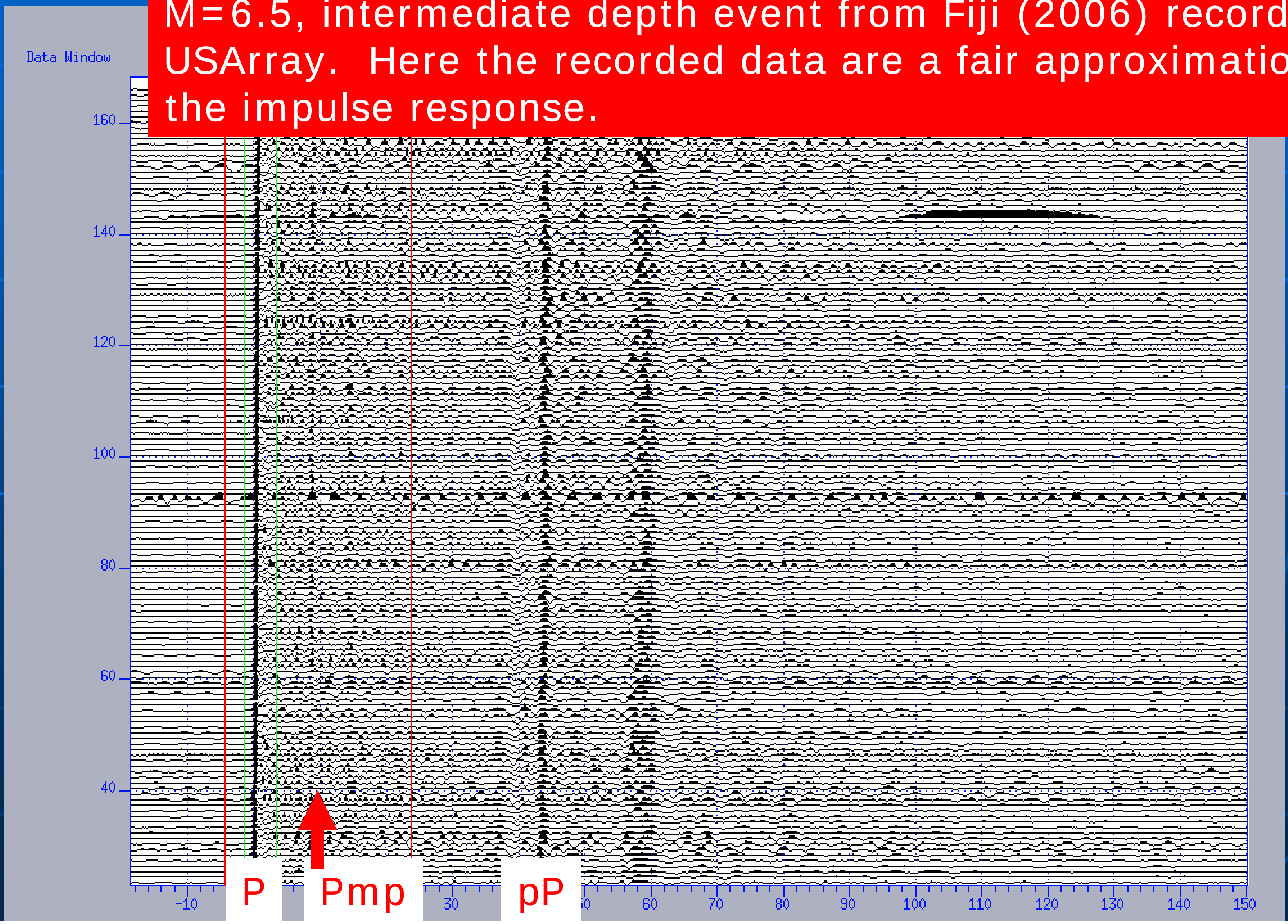
Outline

Guiding principle: This talk is mainly for students, post-docs, and those new to this area. For the experts it is mostly review.

- Fundamentals
  - Impulse response concept
  - Geometry
  - Scattering assumptions
- Conventional deconvolution
  - Single station methods [Tutorial 1]
    - Frequency domain methods
    - Time domain methods
  - Multichannel methods
    - Receiver array methods [Tutorial 2: f-k filtering]
    - Common receiver gather methods [Tutorial 3: pseudostation stacking]
- Discussion of some fundamental assumptions
- Beyond receiver functions
  - Baig and Bostock's blind deconvolution conjecture
  - Weglein, Fan et al.'s inverse scattering series [Tutorial 4]
  - Model-based approach
- Next steps

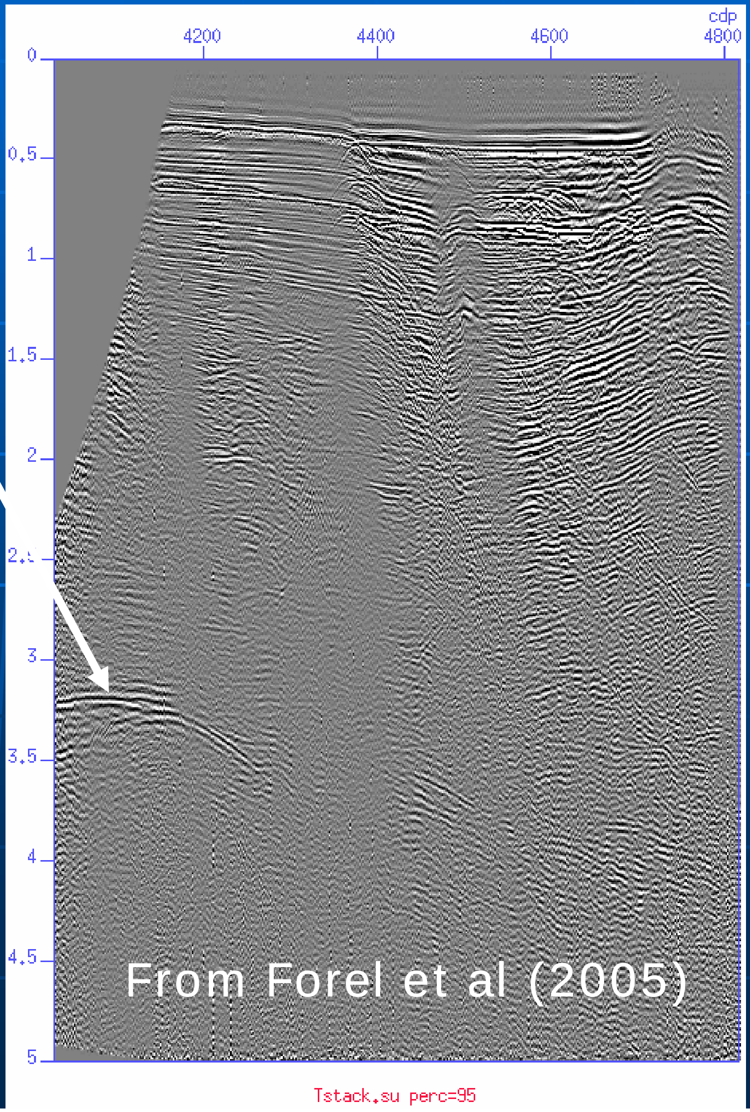
What is an impulse response?

M=6.5, intermediate depth event from Fiji (2006) recorded on USArray. Here the recorded data are a fair approximation to the impulse response.

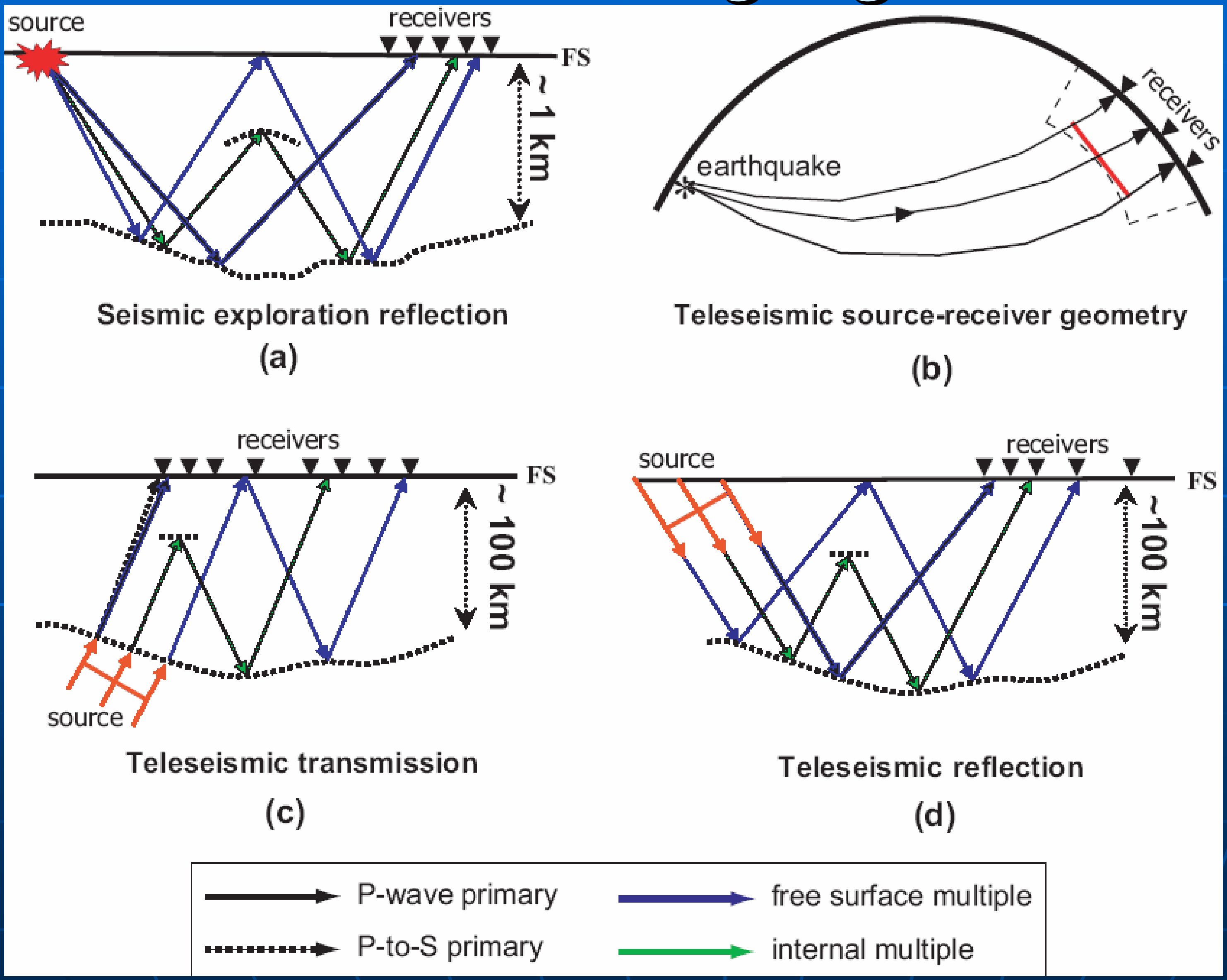


Why is the impulse response fundamental to wavefield imaging?

- Imaging
  - Manipulates full seismogram to construct image
  - If not impulse, image distorted by source wavelet
  - Most imaging assumes “primaries” only?
- How does this differ from seismic tomography?
- “Classic” inversion methods
  - Extract parameters as data (e.g. arrival times)
  - Invert for model using parametric data
  - Regularization always leads to smoothed models while imaging methods focus on discontinuities



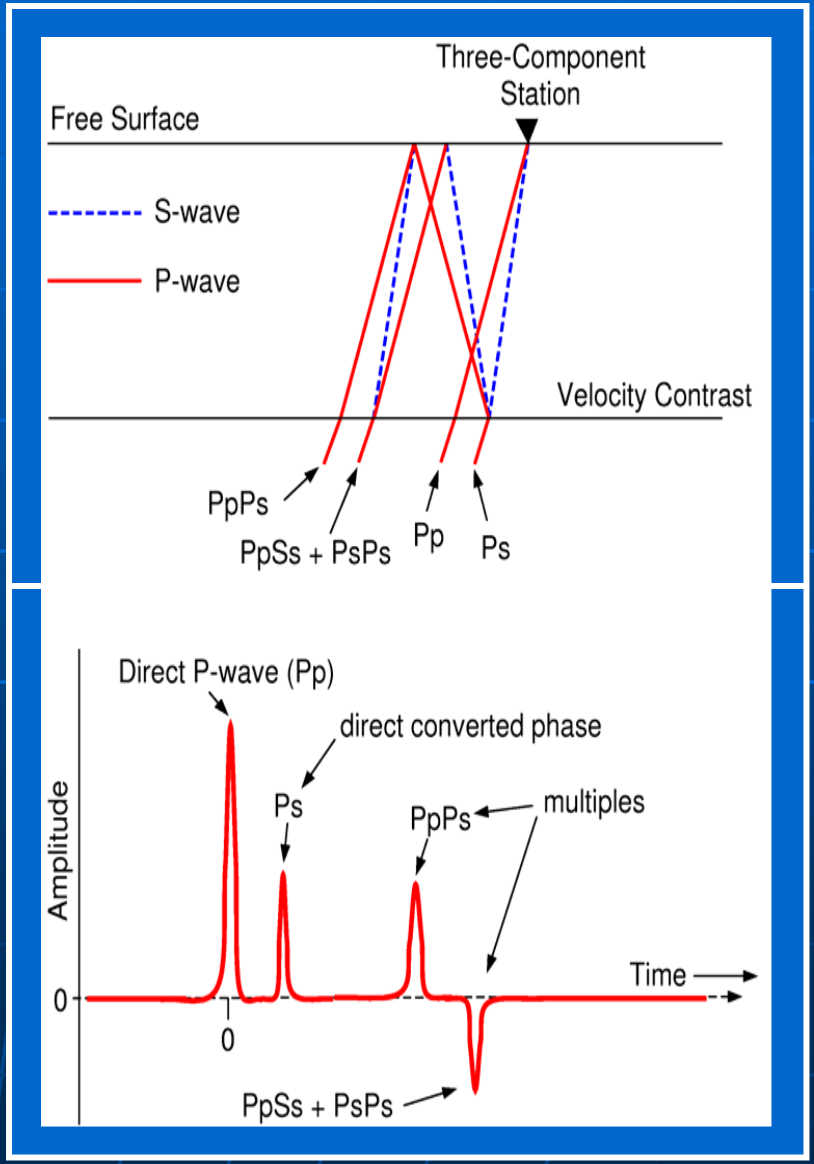
Teleseismic Imaging Geometry



Fan et al, 2006

Receiver function concept

- Focuses on P to S conversions
  - Rotation to L,R,T
  - Standard deconvolution method discards P
- Demonstrated success in many papers



Conventional Theory for Impulse Response and Receiver Functions

The standard starting point is to assume the data are related to the impulse response by the convolutional model

$$
\begin{aligned}
d_1 &= s \star i_1 \\
d_2 &= s \star i_2 \\
d_3 &= s \star i_3
\end{aligned}
$$

where s is the source wavelet and i_k is the impulse response for component k.

P-SV-SH wavefield separation

Matrix form:

$$\mathbf{d} = s \star \mathbf{i}$$

Data are ALWAYS transformed to some type of ray coordinates. That is,

$$\mathbf{d}' = \mathbf{F}\mathbf{d} = s \star \mathbf{F}\mathbf{i}$$

where $\mathbf{F}$ is a 3×3 transformation matrix.

Forms of matrix F

- Single rotation to R,T,Z
- Double rotation to R,T,L
- Free Surface Transformation operator (Kennett, 1991) – NOT an orthogonal transformation

A key point commonly misunderstood: this is NEVER a complete wavefield separation. Even in a 1d earth with homogenous layers separation is incomplete except at normal incidence.

The Receiver Function (subject of Tutorial 1)

Using the convolution theorem the impulse response equations can be written as

$$D'_k(\omega) = S(\omega) I'_k(\omega)$$

where ω is angular frequency. The receiver function, $R(\omega)$ in the frequency domain is then simply

$$R(\omega) = \frac{D'_S(\omega)}{D'_P(\omega)} = \frac{S(\omega) I'_S(\omega)}{S(\omega) I'_P(\omega)} = \frac{I'_S(\omega)}{I'_P(\omega)}$$

where the P and S subscripts are components of D'_k oriented to ray coordinates for P and S (SV or SH) defined by the transformation matrix $\mathbf{F}$.

Frequency-domain RF Estimate

The receiver function is then estimated as

$$r(t) = \mathcal{F}^{-1}\left[\frac{D'_S(\omega)}{\tilde{D}'_P(\omega)}\right]$$

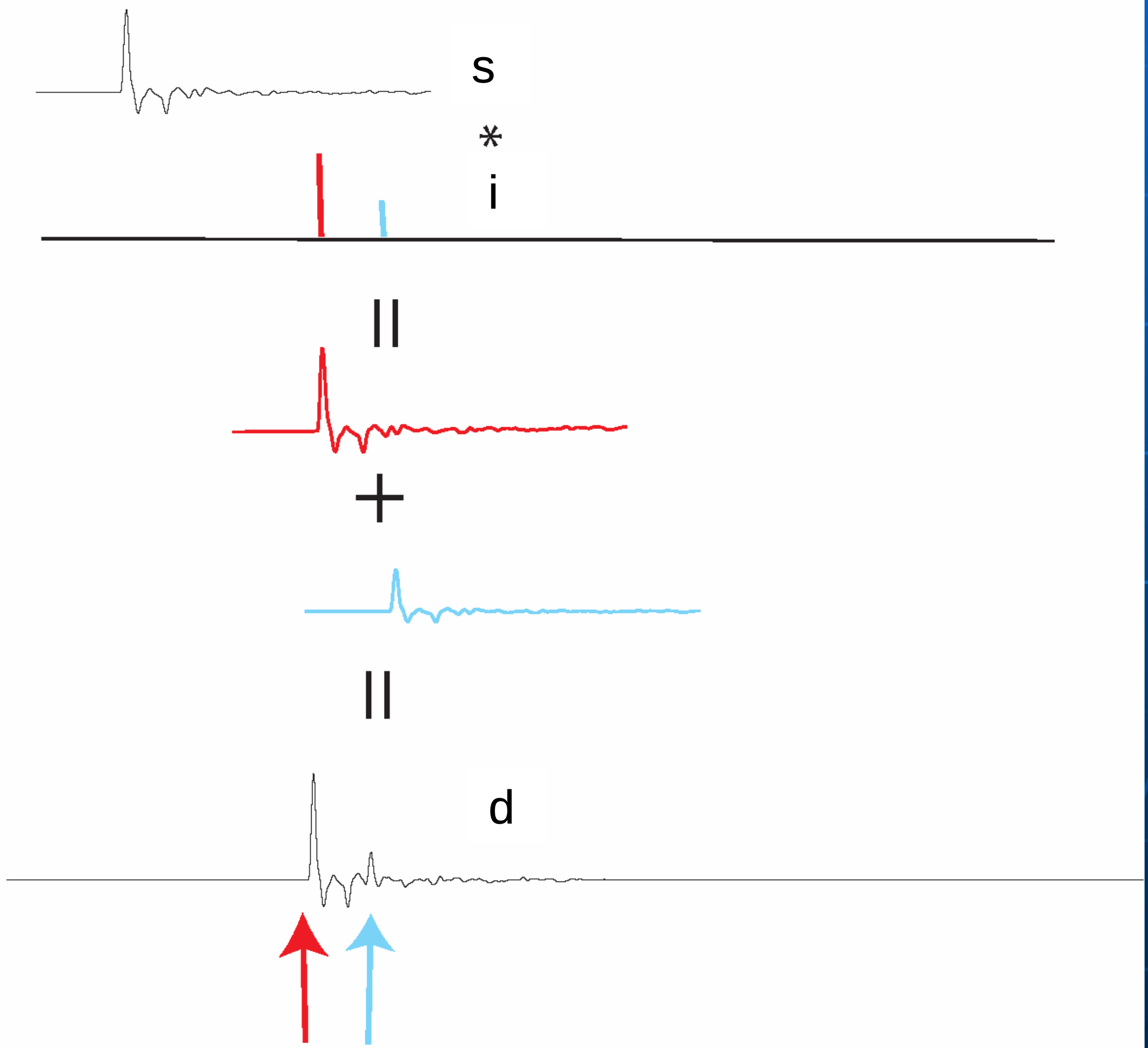
where $\mathcal{F}^{-1}$ is the inverse Fourier transform operator and $\tilde{D}'_P$ is D'_P modified for stability.

- Issues
  - Divide by zero or nearly so
  - Small values magnify “noise” (????)
- Methods
  - Water level
  - Other regularizations
  - Multitaper algorithm (robust spectral estimator)

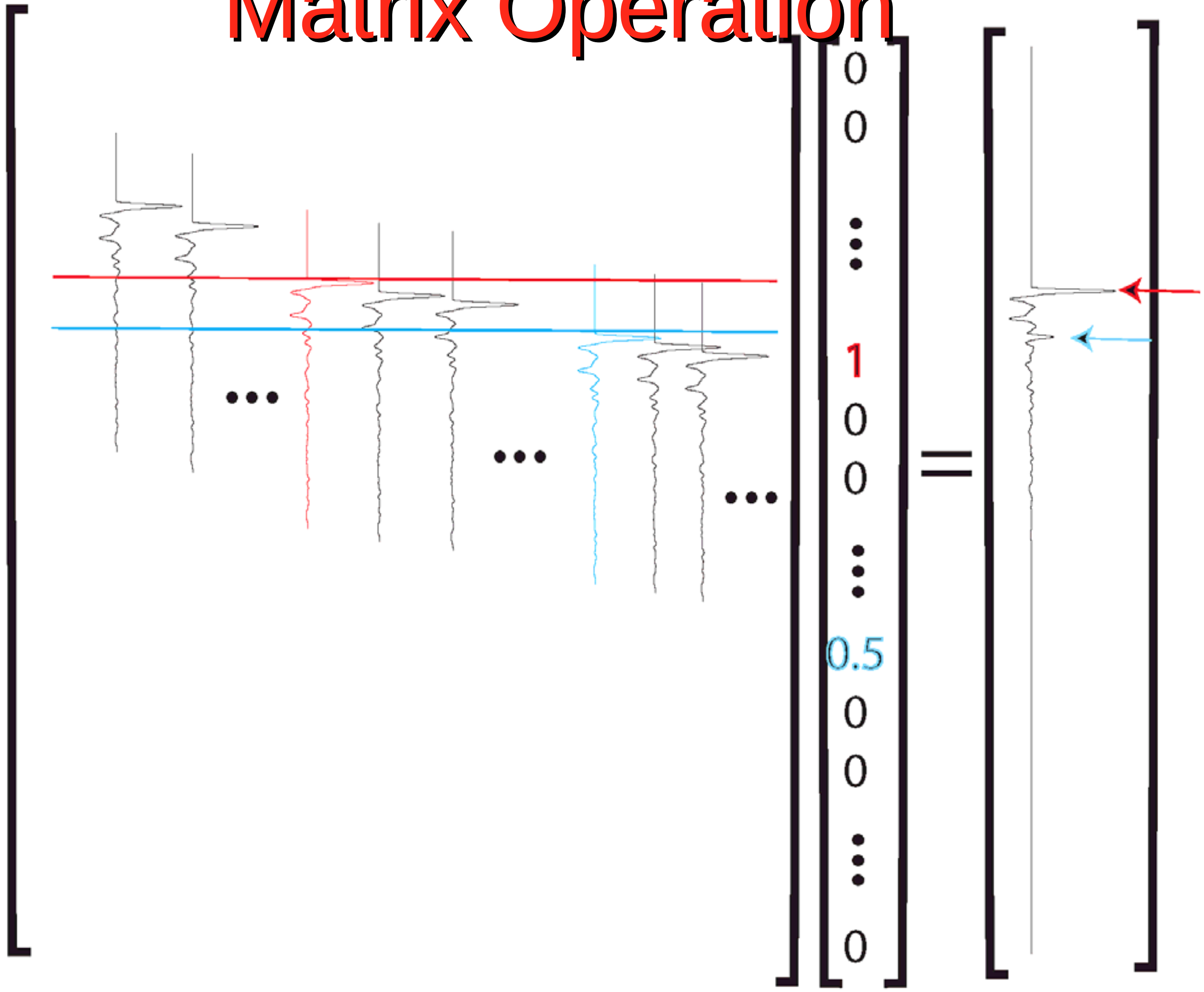
Time Domain Methods

- Cast problem as linear equations d=Si
- Some have preferred this approach to frequency domain methods because:
  - Full machinery of linear inverse theory available
  - More flexibility in regularization (e.g. Tikhonov regularization)
- Very high computational cost compared to frequency domain methods
- For this workshop provide useful insight and familiar framework for some

Convolution Fundamentals



Convolution as a Matrix Operation



$$\mathbf{s} \quad \mathbf{i} = \mathbf{d}$$

Time Domain Decon and Inversion

Since convolution $d = s \star i$ can be cast as

$$\mathbf{d} = \mathbf{S}\mathbf{i},$$

deconvolution can be viewed as an inverse problem. Choose your favorite recipe to construct a generalized inverse $\mathbf{S}^{-g}$ and we obtain

$$\begin{aligned} \hat{\mathbf{i}} &= \mathbf{S}^{-g}\mathbf{d} \\ &= \mathbf{S}^{-g}\mathbf{S}\mathbf{d} \\ &= \mathbf{R}\mathbf{i} \end{aligned}$$

where $\mathbf{R} = \mathbf{S}^{-g}\mathbf{S}$ is the familiar resolution matrix.

Time or Frequency Domain?

- Frequency domain method MUCH faster to compute (NlogN versus N^3 algorithm)
- Multitaper method (Park and Levin, 2000) is probably the best FD method
- Time domain approach likely has more unexplored options

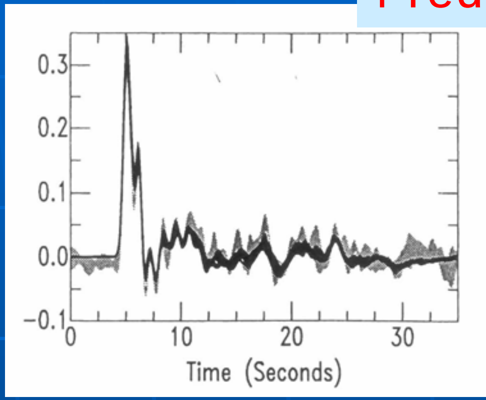
Some Fundamental Receiver Function Assumptions

- Incident wavefield is a pure P wave
  - Requires very weak scattering in lower mantle to be true; otherwise incident wavefield would not be pure P waves.
  - S wave receiver functions require a comparable assumption
- Single phase incident as a plane wave
  - Reason for focus on 30-90 degree distance
    - What happens between 10-30 degrees?
    - What happens beyond 90?
  - Limits time window for some intermediate depth events
  - Limits time window in 90+ range where PcP-P small
- Scattering in imaging volume is weak
  - More on this one later.

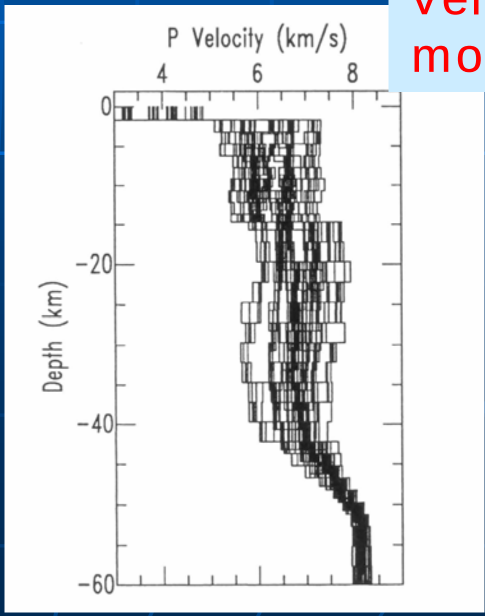
Inversion strategy for receiver function interpretation

- Approach of earliest paper using this technique
- Approach
  - Nonlinear inversion
  - Compute synthetics and synthetic receiver function
  - Minimize misfit between data and synthetic receiver functions
  - Variable minimization strategies (standard inversion issues)
- Key point: uses RF as data, NOT the impulse response

Actual and Predicted rf



Velocity models



Ammon et al. (1990)

Receiver Functions and Direct Imaging

- Harder problem than inversion approach because
  - Multiples are a BIG problem that is not an issue for inversion approach
  - A receiver function is NOT the radial impulse response – treating it as such is at best an approximation
  - Conventional rf decon discards P wavefield data.

P wave and Conventional Deconvolution

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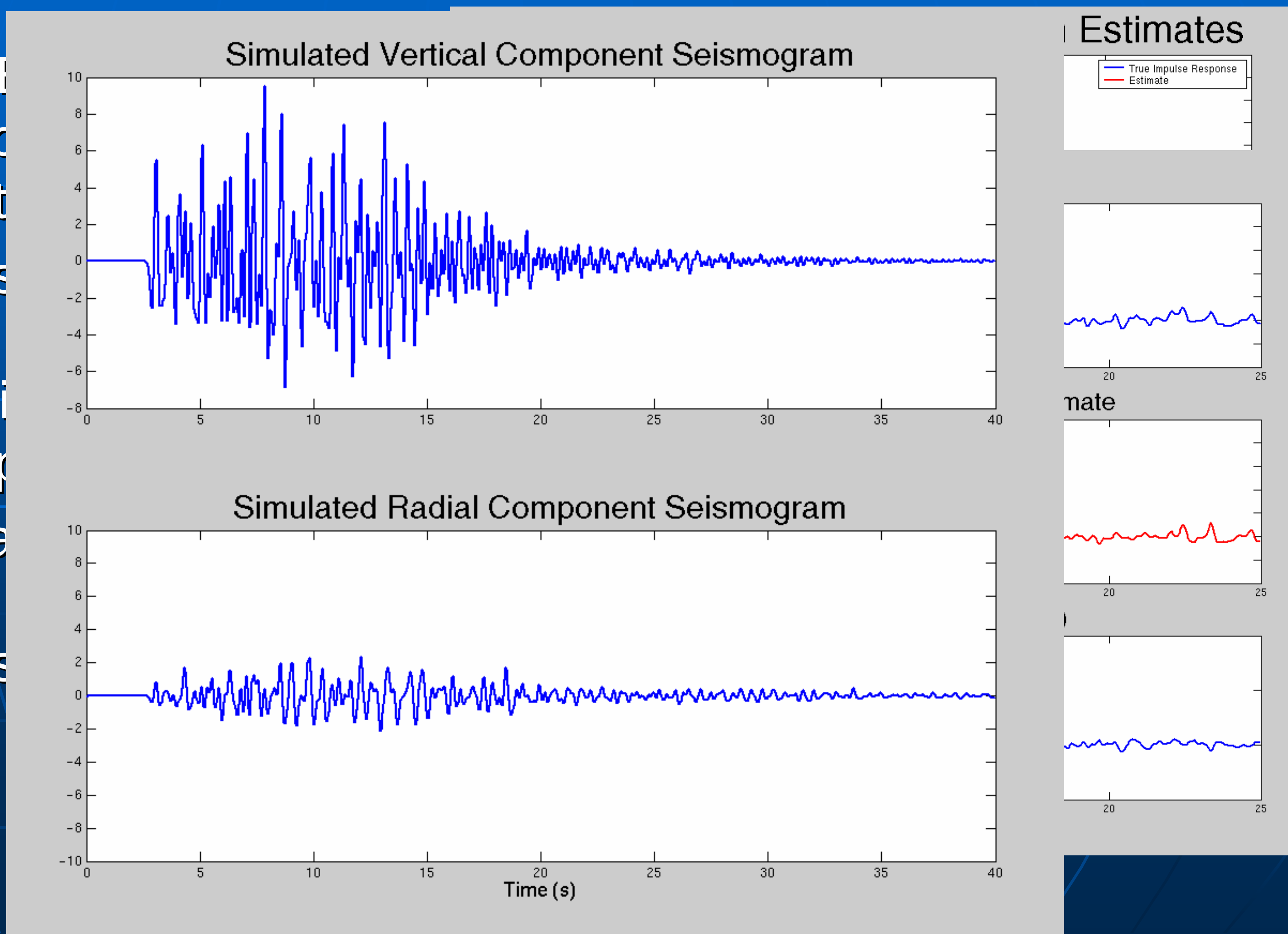


Multichannel Methods

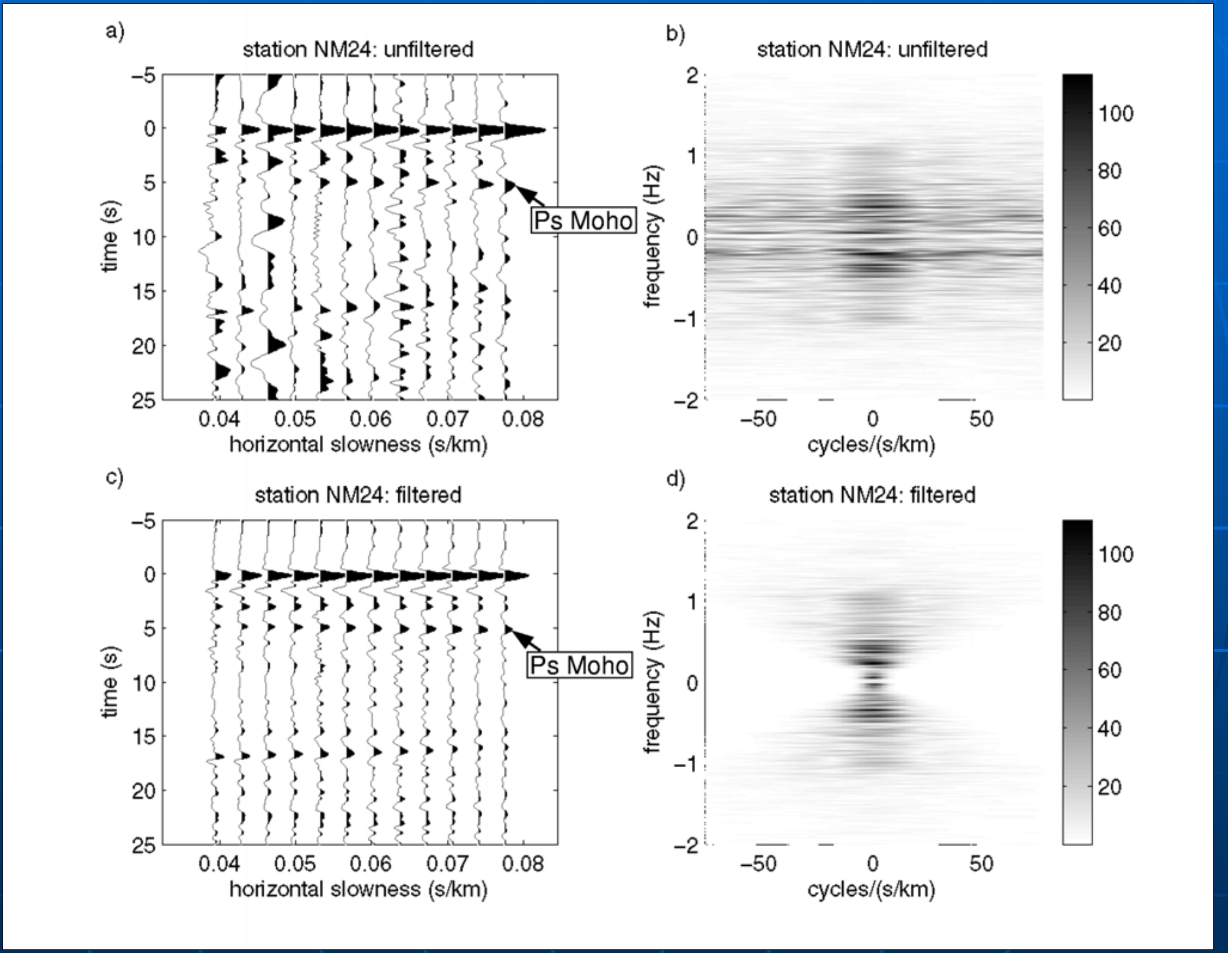
- Why multichannel?
  - Emergence of broadband array data (e.g. IRIS PASSCAL) in the past 15 years
  - Aim to exploit data redundancy
  - Learn from experience of reflection seismology
  - Some versions provide estimates of P impulse response
- Types:
  - Common receiver gathers
  - Common event gathers

Common receiver gather methods

- Most common approach: average moveout corrected RF estimates for each station
- Simultaneous time-domain deconvolution (Gurrola et al, 1995)
- Wilson and Aster (2005) f-k filtering (Subject of tutorial 2)

Receiver Function Imaging:

- Receiver functions are obtained for all earthquakes recorded at all stations.
- Sets of receiver functions arriving from various angles of incidence are jointly filtered to remove noise using array (FK) processing techniques.



Wilson and Aster, 2005

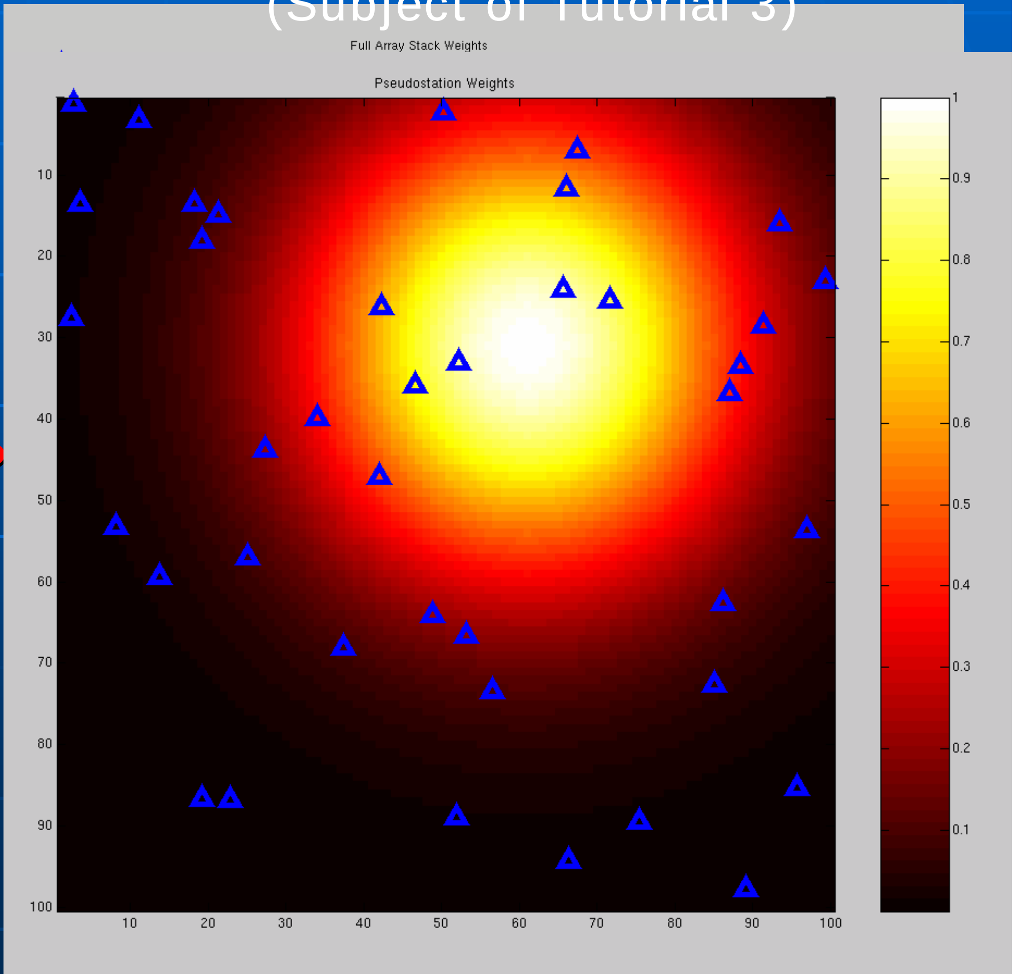
Common Event Gather Methods

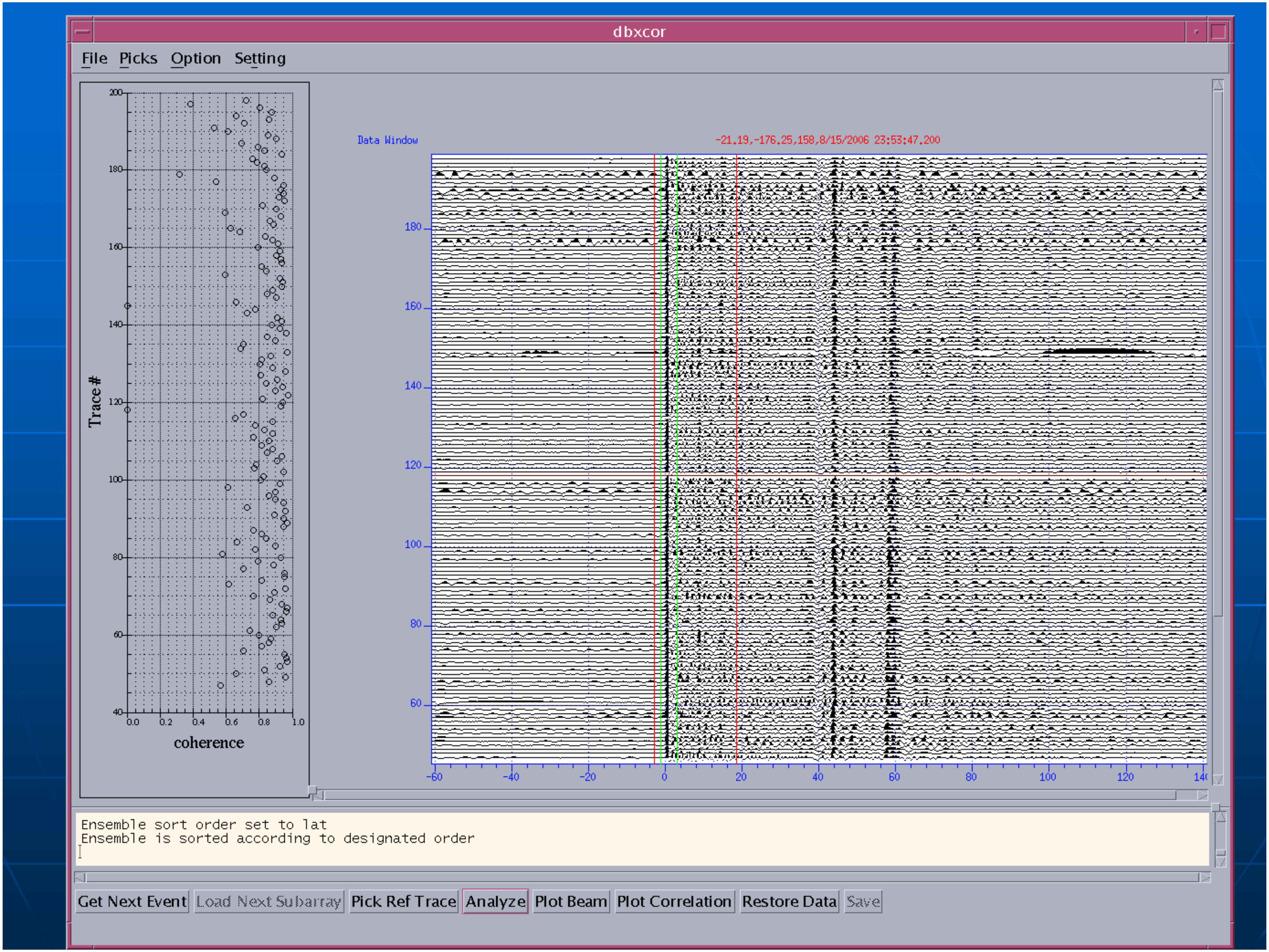
- Li and Nabalek (1999) method
- Principal component method (Bostock and Rondenay, 1999)
- Pseudostation stacking method (Neal and Pavlis, 1999,2000)

Common Event Gather Methods

- Common to all
  - Align to actual P arrival time (how?)
  - Use the stack for deconvolution (stack?)
- Differences
  - What defines the array?
  - Stacking algorithm
    - Simple mean
    - Principal component
    - Robust stack

Pseudostation Stack Method
Full Array Methods
(Subject of Tutorial 3)

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Ensemble sort order set to lat
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Analyze
Plot Beam
Plot Correlation
Restore Data
Save

Beyond Receiver Functions

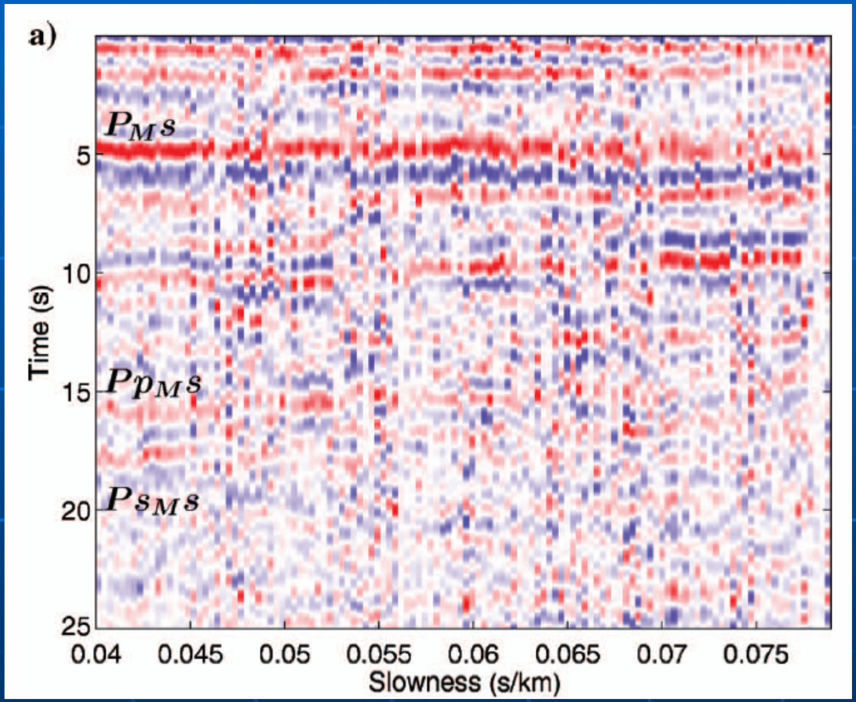
- Issues with receiver functions
  - Complicated by P multiples
  - Experience is the approach simply doesn't always work
  - All except array methods make estimating P reflection response impossible
  - Multiples/other modes can obscure structure and/or complicate interpretation (but can also be exploited).
- General approaches
  - Statistical approaches
  - Inverse scattering series
  - Model-based correction

Statistical Method

Baig and Bostock (2005), Mercier et al. (2006)

- Blind convolution conjecture
  - Assume wavelet is smooth
  - Assume impulse response functions to be estimated are minimum phase
- A common receiver gather method (some common ground with Gurrola et al, 1995)

Example: Mercier et al (2006)



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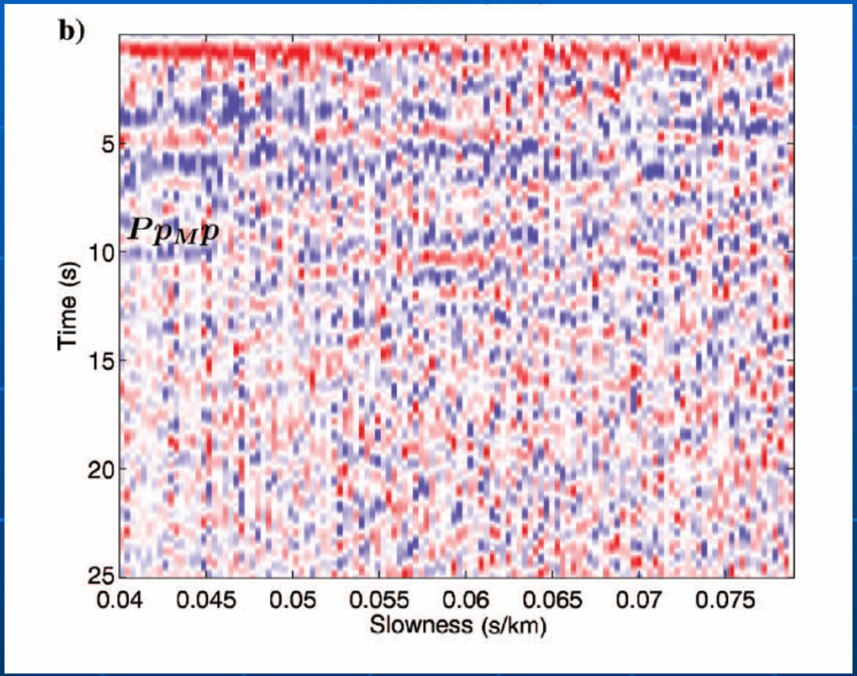

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Inverse Scattering Series Approach

- Does not rec

Relation of

(General Ca

$2\,\mathrm{Re}[R_r^{fs}(\mathbf{x}_A, \mathbf{x}$

Radially syr

$2\,\mathrm{Re}[R_r^{fs}(\omega)]$

Free Surface Multiple Removal Series

Reflection Series

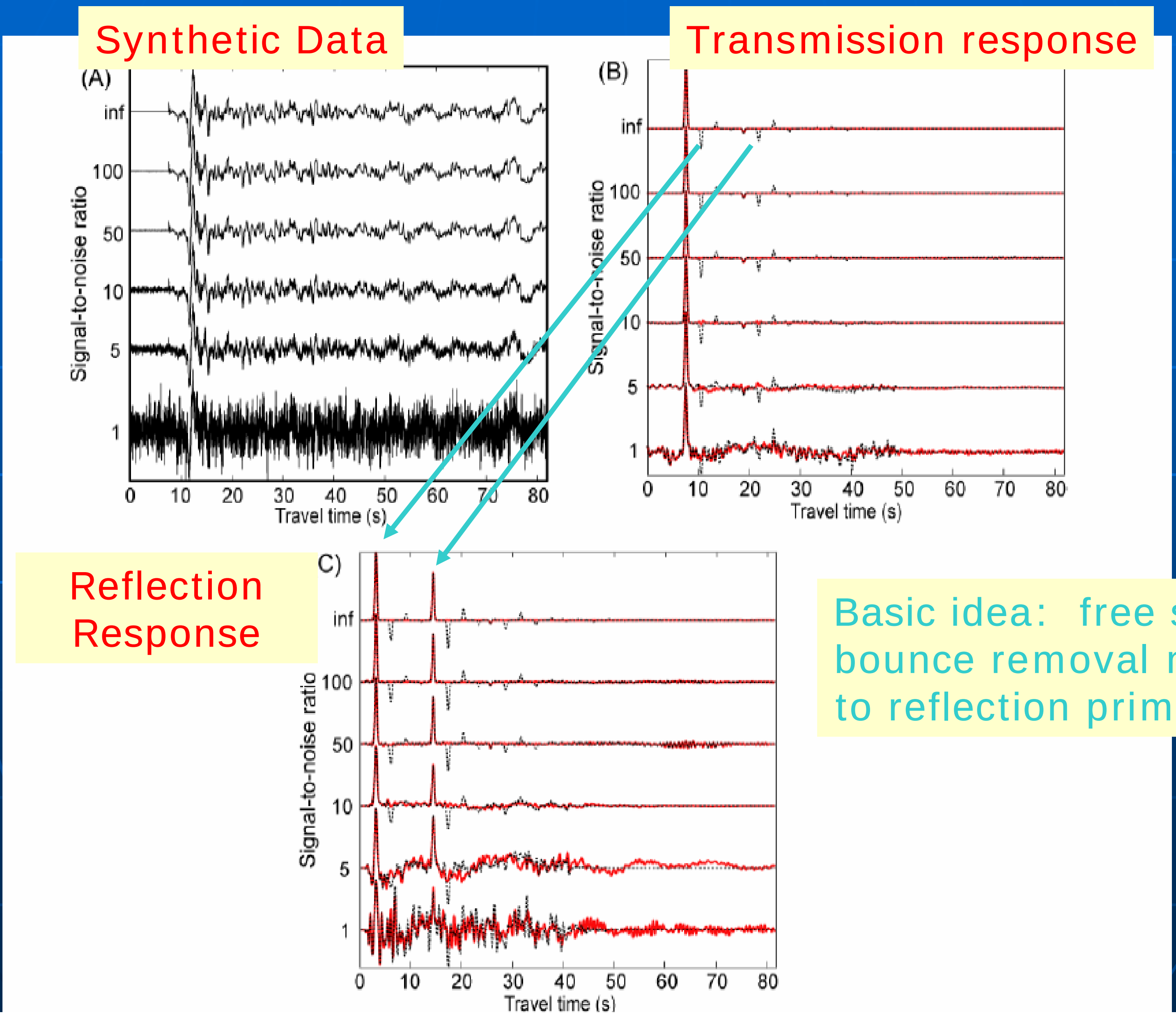
$$R_r(\omega) = R_r^{fs}(\omega)/(1 - R_r^{fs}(\omega))$$
$$= R_r^{fs}(\omega) + (R_r^{fs}(\omega))^2 + (R_r^{fs}(\omega))^3 + \cdots .$$

Transmission Geometry Series

$$T_t(\omega) = T_t^{fs}(\omega)/(1 - R_r^{fs}(\omega))$$
$$= T_t^{fs}(\omega)(1 + R_r^{fs}(\omega) + (R_r^{fs}(\omega))^2 + \cdots),$$

(Fan et al., 2006)

Example: Fan et al (2006)



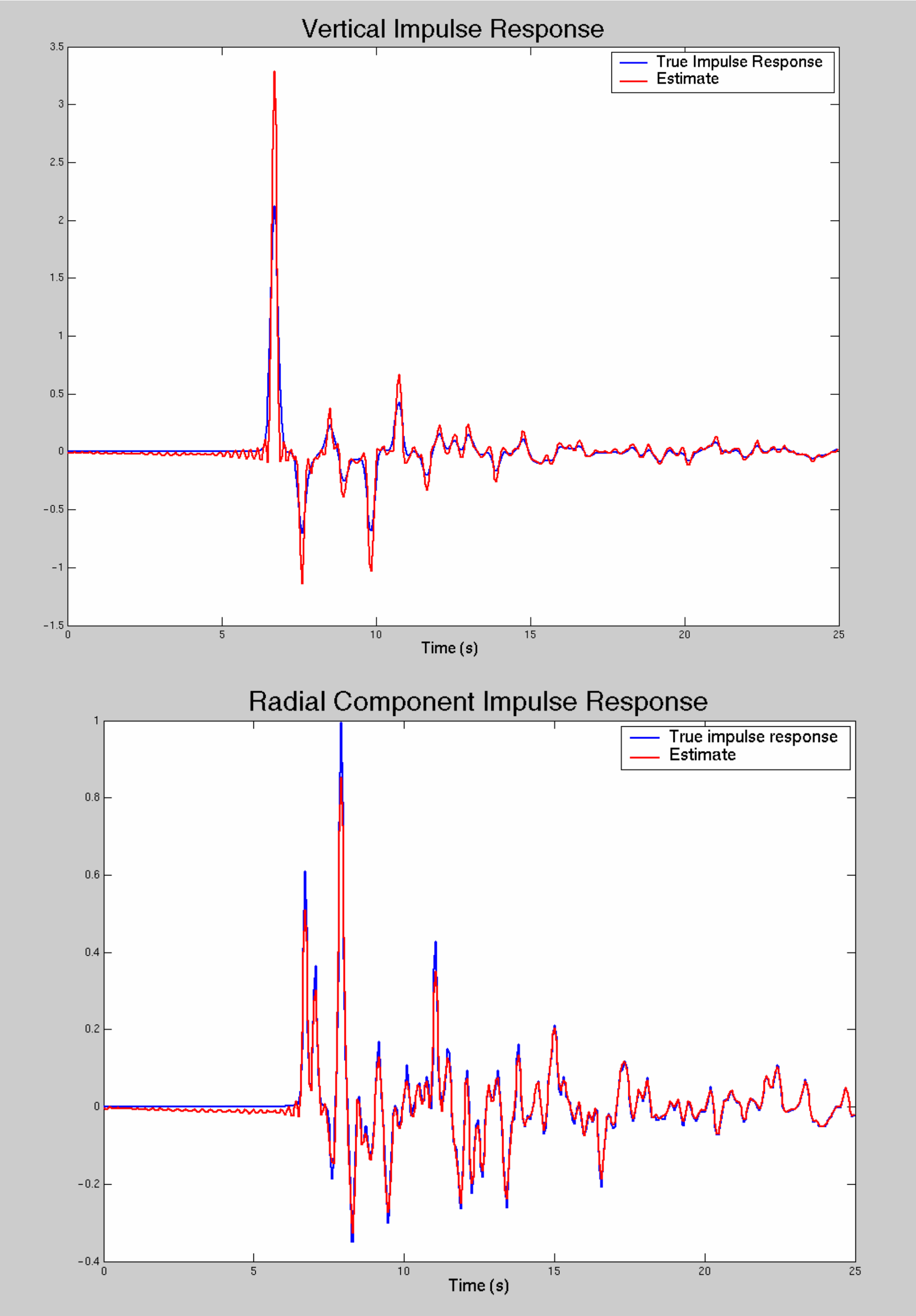
Basic idea: free surface bounce removal maps to reflection primary

Model-based Approach: Convolutional Model Theory

- KEY CONCEPT: the source wavelet is unknown AND a fundamental part of the problem
- data = source * impulse_response
- The source wavelet has a nonlinear relationship to the real Earth's velocity and density structure through the unknown impulse response function

How would we d
the ar

- Data=d=s*i_k (i=1,2,3)
- $s_{est}=i_k^{-1}$ *d (i=1,2,3)
- Use s_{est} to estimate impulse response
- BIG PROBLEM IN REAL WORLD: Don't know i_k which has to be computed from Earth structure



Inversion issues for discussion

- KEY POINT: The source wavelet is an unknown we need to estimate as a fundamental part of the problem
- Optimization conditions(?)
  - Single station: minimize difference between individual component wavelet estimates
  - Arrays: minimize difference between wavelet estimates from ALL components of ALL stations foreach event gather OR difference over a finite aperture
- Earth model parameterization?
  - Station by station 1d models
  - Fixed number of layers of variable thickness (1D synthetics foreach station)
  - Fully 3D models
- Construction strategy?
  - Linearize?
  - Directed search (e.g. genetic algorithms)?
  - Inverse scattering series?

Model-based approach: Theory for General Elastic Media

Using the elastic wave representation theorem it can be shown (Pavlis, in preparation)

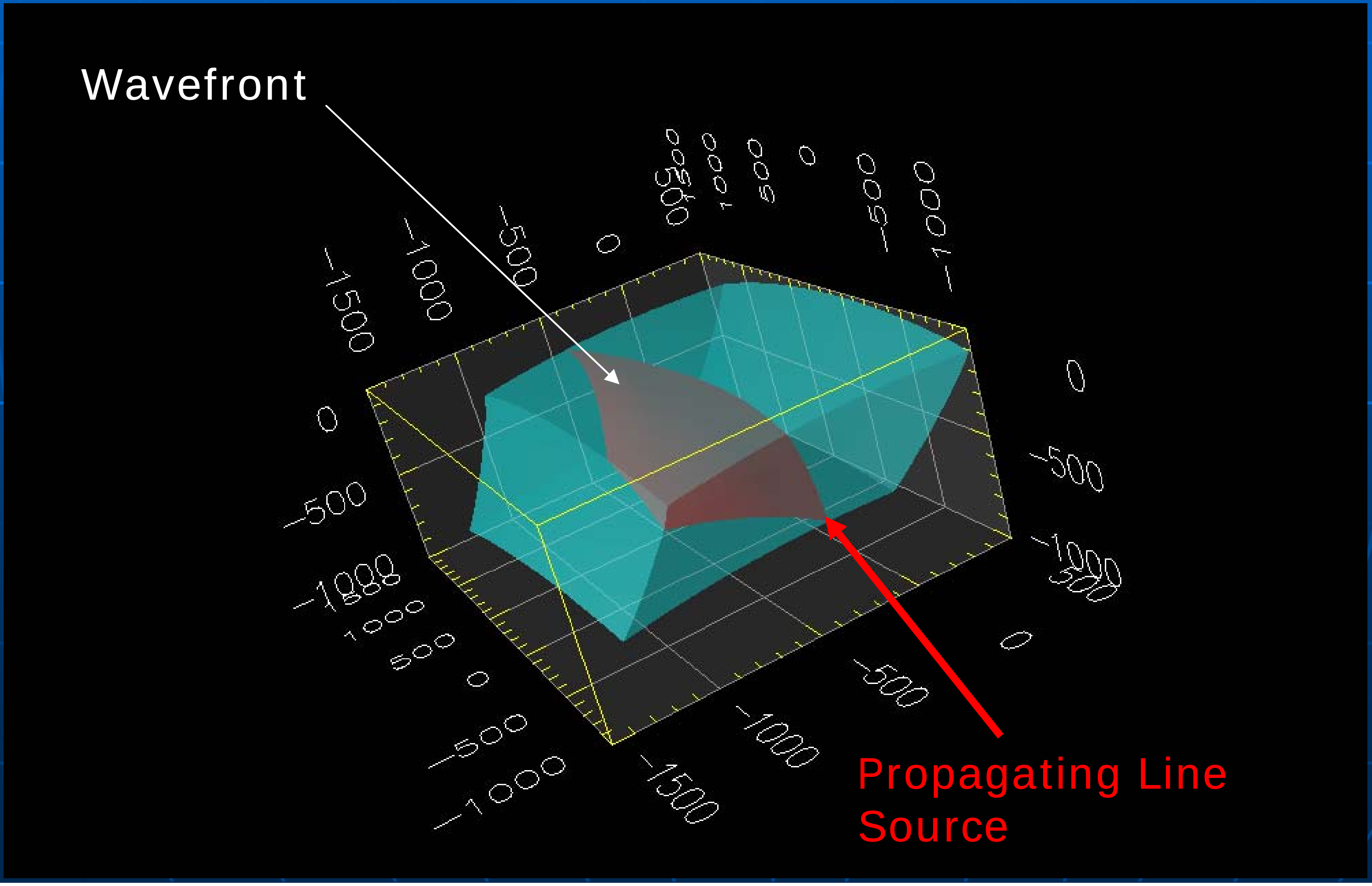
$$d_n = s \star i_n \tag{8}$$

where

$$i_n(\mathbf{x}, t, \mathbf{u}; \tau) = \int\int_{S_{bottom}} \overbrace{G_{np}(\mathbf{x}, t-\tau; \xi, 0)}^{\text{Propagator}} \overbrace{S_{p3}(\xi)\delta(\tau - \mathbf{u}\cdot\xi)}^{\text{Plane wave source}} d\xi_1 d\xi_2 \tag{9}$$

is the impulse response of the medium for the incident plane wave with slowness vector $\mathbf{u}$.

Plane wave source



Forward Modeling and Imaging

- 3D modeling problem requires plane wave synthetics
- Needed badly to explore limits of current impulse generation capabilities
- Needed badly to explore limits of current imaging methods

Summary of Key Points

- Receiver function estimation is a form of deconvolution
- Receiver functions are never the impulse response of the medium
  - Not a problem for 1D model inversion because RFs are data for inversion
  - Treating RFs as the impulse response will cause artifacts in all wavefield imaging AND limit resolution
- Approaches to do better
  - Multichannel methods
    - Common receiver methods
    - Common event methods
    - Statistical methods
  - Multiple removal
  - Solving for the source wavelet as part of the problem
  - Denser data
- Assertion: doing this right is one of the biggest barrier to progress with most data

Discussion points on forward modeling:

- What do we need to test imaging algorithms?
- What do we need to improve impulse response estimates?
- Can global synthetics provide a workable source wavelet?
- Requires implementation of a plane-wave source condition
  - Feasible?
  - How should it be done (sum G sources or produce family of plane waves?)
- Implementing a community model?

Other Discussion Points

- Questions for previous speakers?
- What are the key unsolved theoretical problems that limit what we can do in wavefield imaging?
- What are the main practical bottlenecks to progress for you?