SOME THERMAL CONSEQUENCES OF A GRAVITATIONALLY POWERED DYNAMO

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Abstract. It is argued that the most plausible source of power for the geodynamo is gravitational energy released by the growth of the solid inner core, and the essential features of this mechanism are outlined. The thermal regimes of the outer core which are possible if the dynamo is gravitationally powered are studied, and a number of interesting possibilities are found. First, if the liquidus gradient T_L' is less than the conduction gradient T_C', a slurry must occur in the fluid at the bottom of the outer core. Second, it is shown that compositionally driven convection can occur even if the actual temperature gradient T' is less than the adiabat T_A'. This allows the possibility that heat may be transferred radially inward by the motions. Hence there is no direct relation between the rate that heat is conducted outward in the core and the rate of heat transfer to the mantle. Third, it is found that Higgins and Kennedy's hypothesis does not preclude convective overturning driven by compositional buoyancy provided the thermal conductivity of the outer core is sufficiently large. Fourth, a slurry in the bulk of the outer core is stabilizing and incompatible with a convectively driven dynamo of any kind. It is argued that the gravitationally powered dynamo is possible only if the composition of the core is more metallic than the eutectic. The thermal evolution of the earth is considered, and it is found that if $T_A' < T_C'$, the heat transfer problems for the core and mantle decouple with conditions in the core leading to a prescribed temperature at the base of the mantle and the mantle then prescribing the heat flux which must emanate from the core. It is shown that if $T' \doteq T_L'$, a large flux of heat may flow from the core with virtually no change in temperature.

Introduction

Recent studies by Gubbins [1977], Loper and Roberts [1978], and Loper [1978] have revived Braginsky's [1963] idea that the geomagnetic dynamo is driven by gravitational settling associated with the growth of the solid inner core. The distinction between gravitationally driven convection and thermally driven convection has been ignored by dynamo theorists [e.g., Busse, 1975b, 1976] in their quest to solve the hydromagnetic dynamo problem, since the two are likely to yield similar fluid flow patterns. However, the possible thermal regimes in the earth's core compatible with gravitationally driven convection are quite different from those compatible with thermally driven convection. The primary purpose of this paper is to enumerate and elucidate the thermal regimes of the earth's core which are possible if the dynamo is gravitationally powered. Before doing so, we shall state the case for the gravitationally powered dynamo and describe its salient features.

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It is generally agreed that the earth's magnetic field is generated by a hydromagnetic dynamo within the fluid outer core. The field is sustained against ohmic losses by a transfer of kinetic energy into magnetic via dynamo action, but the ultimate source of power is not known. The most likely power sources are thermal convection, precession, and gravitational settling. We shall attempt to evaluate the plausibility of each of these prospective power sources by using two criteria. The first criterion concerns the rate at which energy is supplied to the fluid motions; the larger the figure, the more plausible the source. The accepted minimum, based upon estimates of ohmic losses in a 'small field' dynamo, is roughly 10^9 W, while a 'large field' dynamo needs approximately 10^{11} W. The second criterion concerns the inevitability of energy flow into the kinetic mode; if there appears to be no path for the energy other than into kinetic energy, that power source will be judged to be more plausible than one whose energy can be conducted away or converted into heat by some other means. We shall see that only one of the prospective power sources satisfies both criteria.

For some time the most popular candidate for the power source has been thermal convection [Bullard, 1949] driven either by radioactive decay of ^{40}K [Hall and Rama Murthy, 1971] or by the latent heat released by solidification of the inner core [Verhoogen, 1961]. Much of the popularity of thermal convection stems from the fact that it is well understood and has been widely studied, providing a strong theoretical base for dynamo models based on the thermal convection [Busse, 1973, 1975b, 1976; Soward, 1974]. However, this popularity does not have a direct bearing on the plausibility of thermal convection as a power source. Potassium 40, in conjunction with sulfur, appears to have been suggested as a core constituent principally to supply the heat needed for the thermally driven dynamo. Hall and Rama Murthy [1971] suggest a heat source as large as 10^{13} W. Stacey [1977a] also supports the view that potassium may occur in the core. However the presence of ^{40}K in the core does not appear to be dictated by the early chemical history of the earth. In fact, Ringwood [1977] has criticized the assumptions needed to get large amounts of ^{40}K into the core. An alternative driving force for thermal convection is the slow cooling and crystallization of the core. Verhoogen [1961] has shown that heat can be released at the rate of 10^{12} W if the earth cools at a rate of 12°C in 10^9 yr. An attractive feature of this mechanism is the fact that it is a natural consequence of the slow cooling of the earth over geologic time. However, a serious weakness of the thermally driven dynamo is its low thermal efficiency, stemming from the relatively high thermal conductivity of the core fluid. This allows much of the available heat to be conducted down the adiabat, effectively short-circuiting the thermal convection. Specifically, Metchnik et al. [1974] have estimated the efficiency to be zero if 2.5×10^{12} W or less is available and only 6.4% if 7.5×10^{12} W is available. In view of these low values of efficiency, thermal convection must be considered a doubtful source of power for the dynamo, although Stacey [1977b, p. 209] believes that the available power may be as large as 2.7×10^{11} W.

The idea that rotational kinetic energy of the earth may be supplied to the fluid motions by hydromagnetic torques arising from the precessional motions of the core and mantle has been advocated by Malkus [1963, 1968]. He has estimated that 2×10^{10} W may be fed directly into fluid motions, a figure corroborated by Stacey [1973]. However, these estimates have been criticized by Rochester et al. [1975], who place an upper bound on the power available from precession at 10^7 W, well below the accepted minimum of 10^9 W. A more fundamental objection raised by Loper [1975] is that since the energy must be fed into fluid motions via dissipative boundary layers, the transmission losses must be taken into account. In the laminar case these losses are effectively 100%, showing the laminar precessional dynamo to be impossible. Recently, Rochester [1977] has stated that the same argument may apply to the turbulent case also, casting grave doubts on the viability of the precessionally driven dynamo.

Gravitational settling associated with the growth of the solid inner core was first proposed as an energy source for the dynamo by Braginsky [1963]. The basic idea is that the solid inner core has formed by crystallization of a dense solid from the molten outer core as the earth gradually cooled over geological time. The amount of energy released by this process may be crudely estimated to be $gL(\Delta m)$, where L is the distance through which a mass Δm moves in a local gravitational field g. The mass Δm is roughly equal to the volume V of the solid inner core times the density jump $\Delta\rho$ at the inner-core boundary. Estimating $g = 7$ ms^{-2}, $L = 2 \times 10^6$m, $V = 7 \times 10^{18}$m^3, and $\Delta\rho = 0.5$ g/cm$^3 = 5 \times 10^2$ kg m^{-3}, then the total energy release is 5×10^{28} J. If this energy has been released uniformly over the lifetime of the earth, 4.5×10^9 yr, the rate of energy release is 3.6×10^{11} W, sufficient to sustain a large-field dynamo. The amount of energy released by this mechanism varies lineraly with the density contrast $\Delta\rho$, and the toroidal field strength varies as $(\Delta\rho)^{1/2}$ (see Loper [1978] for details). The value of $\Delta\rho = 0.5$ g/cm^3 is typical of estimates in the literature but Loper [1978] has shown that $\Delta\rho \doteq 2.5$ g/cm^3 is possible. As we shall explain in the following paragraphs, it appears that this energy must be released in the form of kinetic energy, providing an efficient power source for the dynamo. Therefore gravitational potential energy appears to be the most plausible source of energy for the geodynamo.

Let us assume that the earth accreted homogeneously, and that at some early date the core and mantle separated with a release of gravitational energy sufficient to melt the entire core. In the early stages of core formation the molten material was undoubtedly of a low-melting-point, multicomponent eutectic composition. Following Usselman [1975], we shall assume that some of the gravitational energy released by core formation melted additional metals, making the initial composition of the core more metallic than the eutectic. (In this simple discussion we shall neglect the dependence of the eutectic composition upon pressure and speak of a single eutectic composition for the entire core.) This is in contrast to Braginsky [1963], who assumed the core to be less metallic than the eutectic. (We shall see later that Braginsky's core model encounters difficulties.) The core is undoubtedly composed of a number of elements, but we shall characterize it simply as a binary alloy composed of a heavy metal (iron with some nickel) and a light nonmetal (sulfur, silicon, and/or oxygen).

Due to the vigorous motions associated with its formation, the core was initially in a well-mixed state with homogeneous composition and adiabatic temperature gradient. As the core cooled by transfer of heat to the mantle, the temperature gradient first intersected the liquidus at the center of the earth, and the solid inner core began to form. It is known from metallurgy that the solid which crystallizes from a binary alloy of noneutectic composition does not have the same composition as the liquid. Specifically, if the liquid is more metallic than the eutectic, then the solid is more metallic and hence more dense than the liquid, even if the change of density upon solidification is ignored. (It will be seen in a later section that the situation is more complicated if the liquid is less metallic than the eutectic, but the result is the same: a dense solid is formed.) As the inner core grows by accretion of dense solid crystallizing from the molten outer core, a layer of light material is left in the liquid near the inner core, creating an unstable condition. Since a large amount of latent heat is released by the solidification processes, the core fluid may be unstable due to thermal buoyancy as well as compositional buoyancy, but in the following section we shall consider regimes which are thermally stable as well as thermally unstable. In what follows we shall assume the net density gradient to be unstable, avoiding the possibility of salt fingering [Turner, 1974], which would short-circuit the gravitational drive. Thus we have the picture of convective overturning in the outer core driven primarily by the compositional buoyancy generated by the solidification process and perhaps in part by thermal buoyancy generated by the release of latent heat. It is virtually certain that within the rotating core these convective motions are of proper form and sufficient vigor to generate a magnetic field by dynamo action. In fact, since the magnetic diffusivity is much larger than the kinematic viscosity, the primary means of dissipating the energy fed into these motions is via ohmic dissipation. This accounts for the inherent efficiency of the gravitationally driven dynamo noted by Gubbins [1977].

While motions driven by compositional buoyancy and those driven by thermal buoyancy are undoubtedly very similar and lead to virtually the same kinematic dynamo problem, their effects upon the thermal regime of the core are quite different. Specifically, the motions driven by compositional buoyancy may be thought of as a mechanical mixing of the outer core which can occur even in the presence of a stabilizing thermal gradient. In the following section we shall investigate the implications of the gravitationally driven dynamo upon the thermal state of the core.

Possible Thermal Regimes

The various thermal regimes of the outer core may be characterized by numerical ordering of several gradients of temperature with pressure. These gradients are as follows: (1) the adiabatic gradient T_A' (the prime denoting differentiation with respect to pressure) which occurs upon vigorous mixing; (2) the liquidus gradient T_L' at constant composition, the outer core being nearly homogeneous in composition; (3) the conduction gradient T_C' necessary to remove the heat in the absence of motion; and (4) the actual gradient T'. The first three gradients shall be considered as given. The adiabatic gradient and liquidus gradient are determined by the thermodynamic properties of the fluid:

$$T_A' = \left(\frac{\partial V/\partial T}{\partial S/\partial T}\right)_{p,\xi} \qquad T_L' = \left(\frac{\Delta V}{\Delta S}\right)_{p,\xi}$$

where V is the specific volume, S the entropy, and ξ a measure of the composition; Δ denotes the change upon melting at constant pressure and composition. We shall consider only normal materials for which T_A' and T_L' are positive. The conductive gradient is governed by the thermal boundary conditions at the mantle-core boundary and is positive for a cooling core. For simplicity we shall assume all gradients to be constant, although this rules out the possibility that the outer core may be divided into distinct thermal regimes as suggested by Kennedy and Higgins [1973a].

The possiblt thermal regimes of the outer core will be depicted schematically on plots of temperature versus pressure. The liquid outer core and solid inner core are assumed to be in thermal equilibrium at their common boundary; that is, $T_1 = T_L$ at $p = p_1$, the pressure at the inner core boundary. Since convective motions are inhibited close to a solid boundary, we expect a thin conductive layer to exist in the fluid near the solid inner core, giving $T' = T_C'$ at $p = p_1$. As we shall see, this condition does not hold in all cases.

Within the outer core we must require that $T' \leq T_L'$ (i.e., $T \geq T_L$), else the outer core would be solid. Also T' must be bounded by T_A' and T_C', as we shall now explain. If $T_A' < T_C'$, the fluid is thermally unstable, and the subsequent convection is characterized by $T' \doteq T_A'$. On the other hand, if $T_C' < T_A'$, the fluid is thermally stable, and the mixing forced by compositional buoyancy drives T' toward T_A', while thermal conduction drives T' toward T_C', giving $T_C' < T' < T_A'$. The result of these constraints is that the thermal regimes of $T_L' < T_A' < T_C'$ and $T_L' < T_C' < T_A'$ are not possible, since they result in a solid outer core. There remain four possible thermal regimes for the outer core:

Regime A	$T_A' < T_C' < T_L'$
Regime B	$T_A' < T_L' < T_C'$
Regime C	$T_C' < T_A' < T_L'$
Regime D	$T_C' < T_L' < T_A'$

We shall consider each of these in detail.

Regime A: $T_A' < T_C' < T_L'$

The solid freezes directly onto the inner core, and the latent heat released there is removed by a thin conductive layer in which the temperature remains above the liquidus; see Fig. 1. The fluid is buoyant both compositionally and thermally. Heat is transferred radially outward by the convective motions in the familiar manner. This regime is closest to the conventional picture associated with the thermally driven dynamo.

Regime B: $T_A' < T_L' < T_C'$

In this regime the solid cannot freeze directly onto the inner core because a conductive layer to remove the latent heat cannot be constructed. By hypothesis, $T_L' < T_C'$ in such a layer, implying that it is frozen solid. This dilemma is avoided by the formation of a slurry of solid particles suspended in the liquid phase directly above the inner-core boundary [Loper and Roberts, 1978]. The latent heat released by the freezing of particles raises the temperature from the adiabat to the liquidus as shown in Fig. 2. Since the particles are heavier than the fluid by virtue of their compositional difference, they tend to fall radially inward onto the surface of the solid inner core, contributing to its growth, although some solidification still occurs at the inner-core boundary. The deficit of particles in the slurry layer, together with the excess of light material released at the boundary makes the fluid compositionally buoyant, driving the convective motions which sustain the dynamo. As the layer overturns, a fraction of the fluid moving downward freezes, reestablishing the slurry layer, and the process continues. With a portion of the latent heat released throughout the layer, rather than directly at the inner-core boundary as in regime A, the conductive gradient at the boundary is reduced to be identical to the liquidus gradient.

Regime C: $T_C' < T_A' < T_L'$

Now the thermal conductivity of the fluid is sufficiently large that the thermal component of the density gradient tends to stabilize the fluid. We will assume that the compositional component of the density gradient is destabilizing and sufficiently strong to overcome the stabilizing effect of the thermal component. Thus the core fluid overturns and is in effect mixed mechanically. Again a thin conductive layer exists close to the inner-core boundary as shown in Fig. 3. The opposing effects of conduction and mixing keep T' between T_C' and T_A'. The temperature is everywhere above the liquidus, so that no slurry forms. Actually, the picture will

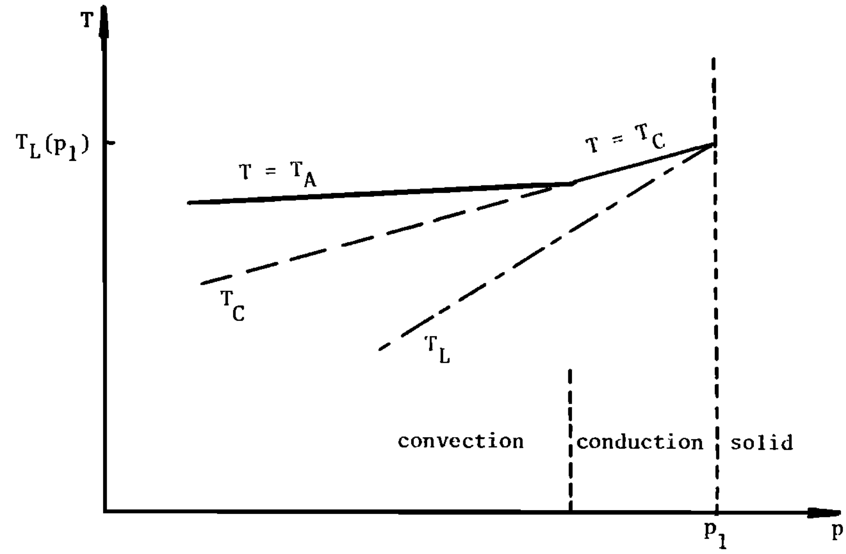

Fig. 1. A schematic representation of thermal regime A: $T_A' < T_C' < T_L'$. The fluid is both compositionally and thermally buoyant. The inner core grows by direct freezing on its surface.

not be as simple as that depicted in Fig. 3. Upward plumes will tend to follow the adiabat more closely than shown, while descending material will be closer to the conduction temperature. In order to present the ideas in their simplest possible format, these refinements are ignored. A number of papers [Frazer, 1973; Verhoogen, 1973; Stacey, 1975] have assumed that the temperature gradient cannot be less than adiabatic if convective overturning is to occur. The present argument shows this assumption to be invalid.

This regime has one unusual thermal property resulting from the mechanical mixing. Since $T_C' < T'$, more heat is conducted down the temperature gradient with motion than in the static case. The excess of heat conducted radially outward cannot, by supposition, be transferred to the mantle. It must instead be carried radially inward by the convective motions. This is in contrast to motions driven by thermal buoyancy in which the convective heat transfer is invariably radially outward. In other words, mixing in the presence of an unstable temperature gradient (as in regimes A and B discussed previously) results in the convection of heat radially outward, while mixing in the presence of a stable temperature gradient (as in the present regime) results in the convection of heat radially inward. An important consequence of this idea is that there is no direct relation between the rate that heat is conducted outward in the outer core and the rate of heat transfer to the mantle. This removes a difficulty which has caused some concern in the literature [Stacey, 1972; Kennedy and Higgins, 1973b].

If the thermal component of the density gradient is destabilizing, convection is driven by both compositional and thermal buoyancy, and

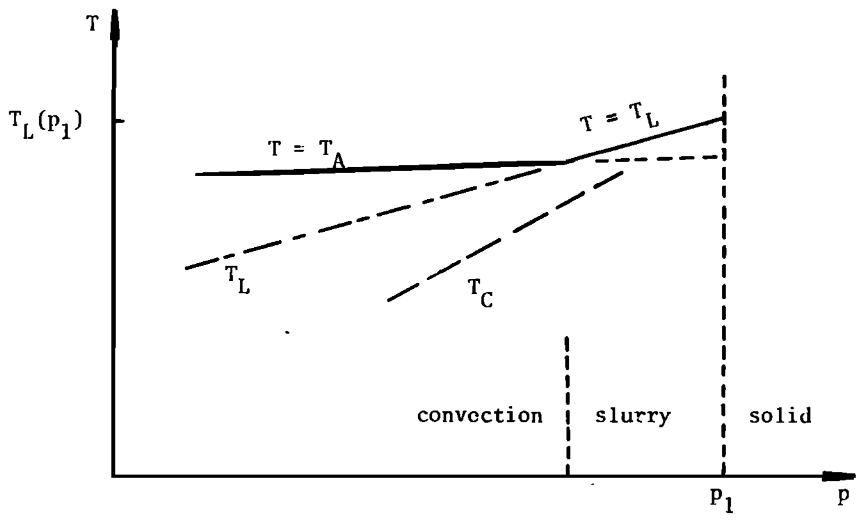

Fig. 2. A schematic representation of thermal regime B: $T_A' < T_L' < T_C'$. A slurry layer must form at the bottom of the outer core. The inner core grows by sedimentation of solid particles.

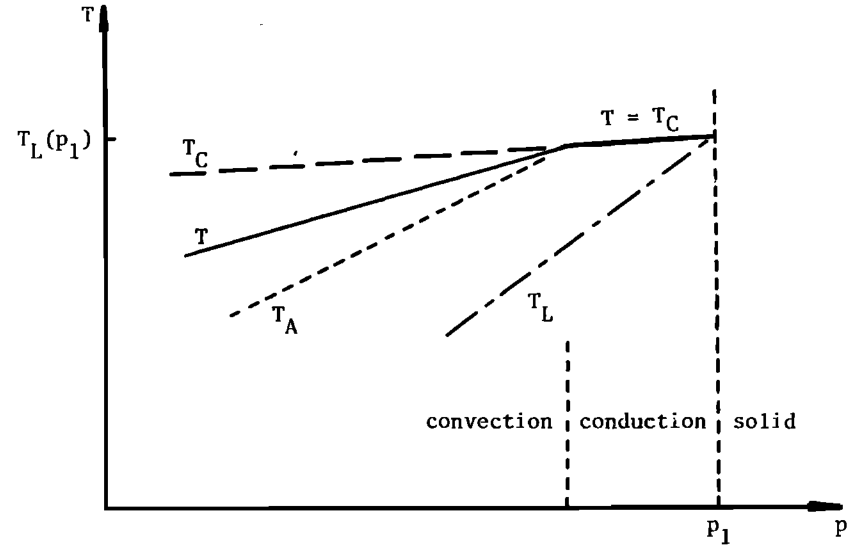

Fig. 3. A schematic representation of thermal regime C: $T_C' < T_A' < T_L'$. The fluid is thermally stable but overturning is driven by compositional buoyancy. The convective motions carry heat radially inward.

estimates of the energy available to drive the dynamo based upon compositional effects only are lower bounds. On the other hand, if the thermal gradients are stabilizing, the fluid motions driven by compositional buoyancy must do work to transfer the heat radially inward, and less energy is available to drive the dynamo. In fact, if the stabilizing thermal gradient just cancels the destabilizing compositional gradient, making the fluid neutrally buoyant, no gravitational energy is available to drive the dynamo. (If the thermal gradient is stronger still, we have the regime in which salt fingering can occur.) Since convective overturning is suppressed for such strongly stabilizing thermal gradients, but the dynamo is an observed fact, we must assume that compositional buoyancy dominates thermal effects within the core if it is in regime C.

Regime D: $T_C' < T_L' < T_A'$

This regime is of particular interest because Higgins and Kennedy [1971] have claimed it to be valid for the outer core. Their claim has caused great concern because it has been interpreted as implying that the core is stably stratified. If this were true, then all dynamo mechanisms face grave difficulties, since radial motion is an essential ingredient [Busse, 1975a]. Higgins and Kennedy's hypothesis does indeed imply a stably stratified core if convection is driven by thermal buoyancy. However, as we shall see, this is not necessarily the case for convection driven by compositional buoyancy; it is possible to have convective overturning even if their hypothesis is valid. Discussion of this regime is facilitated if it is divided into two, depending upon

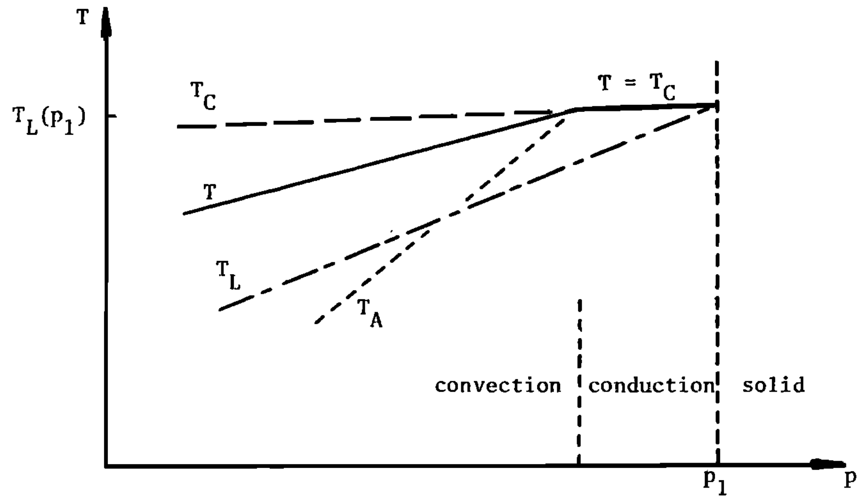

Fig. 4. A schematic representation of thermal regime D1: $T_C' < T' < T_L' < T_A'$. This regime is very similar to C.

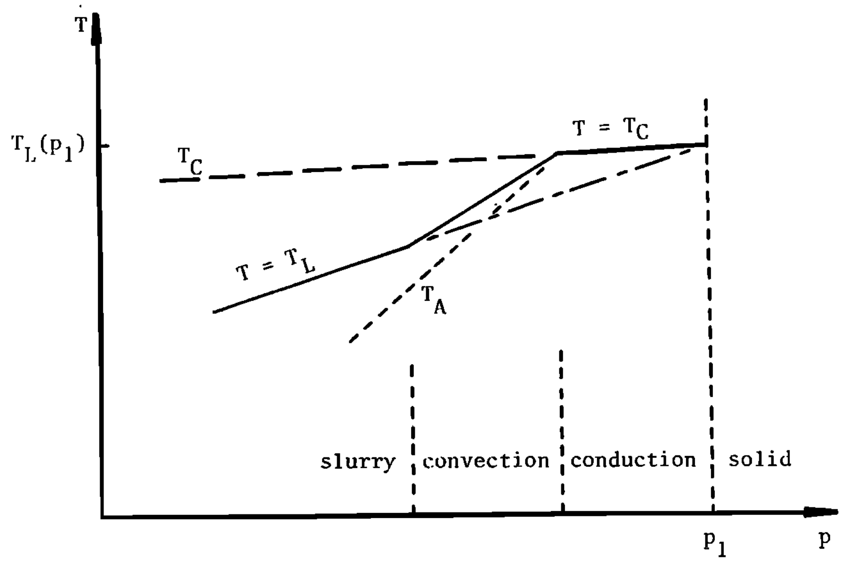

Fig. 5. A schematic representation of thermal regime D2: $T_C' < T' = T_L' < T_A'$. A slurry occurs in the outer core. Irreversible processes generate stable gradients which inhibit radial motion.

the relative magnitude of the effects of conduction and convection.

Regime D1: High thermal conductivity

If the thermal conductivity is sufficiently high that

$$T_C' < T' < T_L' < T_A'$$

we have the situation shown in Fig. 4. This is essentially the same as regime C discussed previously. Convection is driven by compositional buoyancy despite the stabilizing effect of the thermal gradients. One difference between the present regime and regime C is the possibility that a slurry may form in a strong plume which behaves essentially adiabatically. This regime will be quite difficult to model and may require the nonequilibrium slurry theory developed by Loper and Roberts [1978].

Regime D2: Low thermal conductivity

If the thermal conductivity is too small to maintain $T' < T_L'$, then the regime

$$T_C' < T' = T_L' < T_A'$$

holds for the bulk of the outer core as shown in Fig. 5. A thin conductive layer occurs near the inner-core boundary, and a similarly thin convective layer occurs immediately above it, while the bulk of the core is filled with a slurry as envisaged by Busse [1972] and Malkus [1973]. If irreversible processes such as conduction of heat and gravitational settling of particles were absent, the slurry would be neutrally buoyant [Busse, 1972; Loper and Roberts, 1978]. However, thermal conduction leads to a stable thermal gradient, while the gravitational settling of the dense, metal-rich particles creates a stable compositional gradient. Furthermore, the settling of particles acts to transfer additional heat radially outward by releasing latent heat at a given level in the core then melting to reabsorb heat at a lower level, further enhancing the stable thermal gradient. Altogether, the slurry would be quite stable and convective motions restricted to a thin layer near the inner-core boundary. It is doubtful that a dynamo could operate successfully in this regime.

Recent work by Stacey and Irvine [1977; see also Stacey, 1977a] concerning Lindemann's law and the Grüneisen parameter has led them to conclude that Kennedy and Higgin's hypothesis is invalid. Assuming this to be the case, the core is not in regime D.

We conclude this section with a discussion of the gravitationally powered dynamo acting in a fluid whose composotion is less metallic than the eutectic as proposed by Braginsky [1963]. One difficulty with this model is that the solid which forms initially is less metallic and hence less dense than the liquid. This light solid cannot freeze directly onto the inner core, since it would result in an inner core less dense than the surrounding liquid, a very unstable situation. Rather we must assume that the metal-poor particles float upward, leaving a thin metal-rich layer at the bottom of the outer core. Since the metal is the more dense constituent, this layer is stably stratified. This layer will evolve compositionally until the fluid in contact with the solid inner core reaches the eutectic composition. Further cooling causes the solid inner core to grow by the freezing of a dense eutectic solid onto its surface. Simultaneously, the entire layer moves radially outward by freezing metal-poor particles in its interior which float upward to the top of the layer. The particles then melt, forming light liquid which drives the convective overturning in the bulk of the outer core. Since the composition of the fluid within this layer varies with depth, it shall be referred to as the variable-composition layer (VCL). Within this layer the fluid temperature lies on the liquidus. With the composition of the fluid being less metallic than the eutectic, the liquidus temperature decreases as the metal

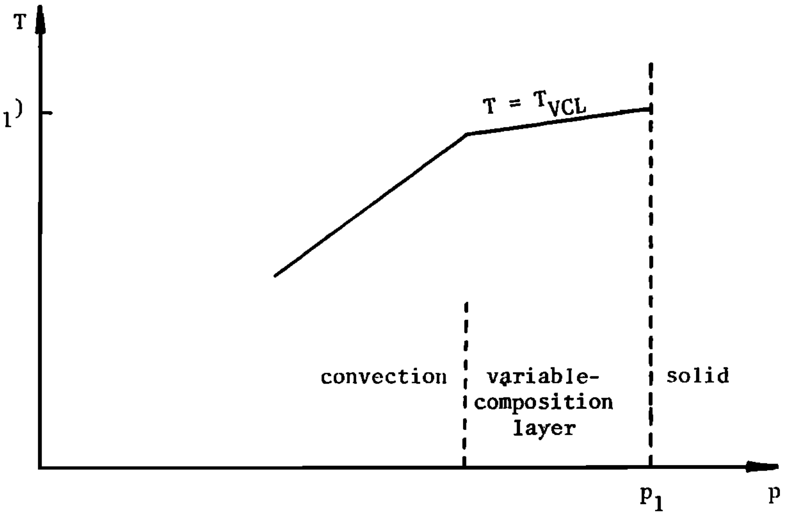

Fig. 6. A schematic representation of the thermal regime associated with Braginsky's model of the core. A variable-composition layer must form at the bottom of the outer core.

concentration increases. This causes the liquidus gradient in the VCL to flatten as shown schematically in Fig. 6. It is even conceivable that the gradient reverses in the VCL.

The picture presented so far appears consistent and plausible, but Braginsky's model encounters difficulties when the heat transfer across the VCL is analyzed. Since the VCL is strongly stabilized by the compositional gradient, heat can be removed only by conduction. In particular, the latent heat released by the freezing of the eutectic solid at the inner-core boundary must be removed by conduction. To make matters worse, the light particles which drift radially outward in the VCL act to pump heat downward into the VCL by releasing latent heat within the layer as they freeze, then absorbing it at the top as they melt. The heat pumped inward in this fashion must also be conducted out of the VCL. The heat conduction problem is further exacerbated by the fact that the temperature gradient within the VCL is relatively flat as explained in the previous paragraph. This makes Braginsky's model either very implausible or, as in regime B, impossible.

This completes the discussion of the possible thermal regimes of the earth's core. We now consider the factors which control the rate of thermal evolution of the earth.

Thermal Evolution of the Earth

The rate of thermal evolution of the earth is controlled principally by the physical properties of the fluid in the outer core and of the mantle, as we shall now explain. By hypothesis the temperature T_1 at the inner-core boundary is equal to the liquidus temperature, while the liquidus temperature at that radius is determined by the composition of the alloy and the pressure p_1 at the inner-core boundary. The temperature T_2 at the mantle-core boundary (MCB) is given by

$$T_2 = T_1 + \int_{p_1}^{p_2} T' dp$$

where p_2 is the pressure at the MCB and T' is the actual temperature gradient.

If the outer core is in thermal regime A or B, the temperature gradient is very close to the adiabat: $T' \doteq T_A'$. But the adiabatic gradient is determined by the properties of the materials composing the alloy and the composition of the alloy. Thus the temperature at the MCB depends only upon the composition of the outer core and the pressure at the inner core. These two factors change very slowly as the solid inner core grows. However, for any time short in comparison with the lifetime of the earth, the temperature at the MCB may be considered as fixed and given, independent of the rate of heat transfer. This gives the mantle a fixed temperature at its lower boundary. The temperature of its upper boundary is essentially the mean surface temperature of the earth determined by a balance between solar insolation and radiative cooling to outer space. Since these fluxes are larger than the heat flux out of the earth by a factor of 10^3, the upper temperature is also fixed, independent of the rate of heat transfer. Thus the mantle may be modeled as a spherical shell of viscoelastic material with prescribed temperatures on its two boundaries. This, together with appropriate mechanical boundary conditions, gives a well-posed heat transfer problem for the mantle. Presumably, this will yield convection in the mantle, but in any case the solution to the mantle problem will result in a specific heat flux at the lower boundary of the mantle. This heat flux then serves as a thermal boundary condition at the top of the core and governs the rate of thermal evolution and consequently the rate of growth of the solid inner core. Thus for regimes A and B the rate of thermal evolution is determined solely by the properties of the core and mantle. Further, the heat transfer problems for the core and mantle are decoupled.

If the outer core is in thermal regime C or D1, the mean temperature gradient is determined by the balance between the competing effects of conduction and mixing, and hence is dependent in part upon the rate of cooling. This causes the heat transfer problems for the core and mantle to be coupled, but the net result should be the same: the rate of thermal evolution of the earth is determined by the properties of the core and mantle.

Let us assume for the sake of discussion that the core is in regime A or B and that a heat flux of 3×10^{12} W, say, is required to cross the MCB. This heat comes from at least four sources: heat capacity of the core, latent heat of solidification as the solid core grows, gravitational energy due to sedimentation appearing as ohmic heating in the core, and compressional heating. The latter heat source is relatively small and may be neglected [Loper, 1978]. Also neglected in this balance is any heating due to radioactivity in the core. The relative importance of the effects due to latent heat and gravitational energy release depends, in part, upon the density difference between liquid and solid. Braginsky [1963] has argued that the contribution from gravitation exceeds that due to latent heat by a factor of 8 while Loper [1978] finds a maximum ratio of 5. More specifically, the gravitational energy released per unit mass of solid is given by $\kappa(\Delta\rho)$, where κ is a numerical factor equal to 760 J m^3 kg^{-2} and $\Delta\rho$ is the density jump at the inner-core boundary. Assuming a latent heat of 100 cal/g $\doteq 4 \times 10^5$ J/kg, the ratio of energy available from gravitational settling to that from latent heating is 2×10^{-3} $(\Delta\rho)m^3$ kg^{-1}. The two are of equal magnitude if $\Delta\rho \doteq 0.5 \times 10^3$ kg/m^3 = 0.5 g/cm^3 with gravitational energy being dominant if $\Delta\rho$ is larger. However, due to the efficiency factor discussed in the opening section, gravitational settling is far more effective in sustaining the magnetic field than is the latent heat. The relative importance of the heat available from the heat capacity of the core and that due to solidification and sedimentation depends upon the relative magnitudes of the liquidus gradient and the actual gradient. This stems from the fact that the temperature at the inner-core boundary must remain on the liquidus as the solid inner core grows. If the two gradients are nearly equal in magnitude, the solid core grows with very little overall cooling of the core, while if they are very disparate, the core cools significantly as the solid core grows. More specifically, the temperature within the outer core is dependent upon the pressure p_1 at the inner-core boundary by the relation

$$T(p,p_1) = T_L(p_1) + \int_{p_1}^{p} T'(p)dp$$

Differentiation with respect to p_1 gives

$$\partial T/\partial p_1 = (T_L' - T')_{p=p_1}$$

This may be readily converted into radial gradients by use of the hydrostatic equation. The relative contributions to the heat flux out of the core due to heat capacity, latent heat, and gravitational energy may be expressed quantitatively as

$$\Delta Q = M_c C_p \left(\frac{\partial T_L}{\partial r} - \frac{\partial T}{\partial r}\right)_{r=r_1} + (\Delta H + \kappa\Delta\rho) 4\pi r_1^2 \rho$$

where M_c is the mass of the whole core, C_p is the heat capacity of the core, and ΔH is the latent heat of fusion. This is a generalization of (5) of Verhoogen [1961] taking into account the gravitational energy and the fact that the core is not isothermal. Verhoogen [1961] estimated that the contributions from the heat capacity and latent heat are roughly equal, but the above equation shows that the contribution from the heat capacity may be smaller than he estimated. In other words, a significant amount of heat may flow from the core, as a result of the growth of the solid inner core, with virtually no change in the mean temperature of the core. This is particularly possible in regimes B and D1. This implies that the heat flux from the core is effectively constant in time; consequently, the strength of the dynamo is also constant. This observation agrees well with paleomagnetic studies which show that the strength of the earth's magnetic field has been roughly constant for several billion years.

The gravitational dynamo is powered by the growth of the solid inner core over the lifetime of the earth. Therefore the thermal regime of the earth should be stable over very long periods of time. In other words, a particular thermal regime is the result of a steady-state heat-flux balance which persists indefinitely; there is no relaxation to a neutral state. The conduction gradient T_C' is the most instrumental in determining the thermal regime of the core. This gradient is proportional to the rate of heat transfer to the mantle and inversely proportional to the thermal conductivity of the core. Current estimates of these are of such magnitude and sufficient uncertainty to make accurate determination of the actual thermal regime of the core very difficult.

The growth of the metal-rich solid inner core causes the composition of the liquid outer core to evolve toward the eutectic. Once the eutectic composition is reached, the gravitational dynamo will cease to function because the solid which freezes from an eutectic liquid has the same composition as the liquid. Once this happens, the dynamo will drop sharply in vigor and perhaps even fail entirely. Since no such event has been found in the paleomagnetic records, it appears safe to assume the core composition has not reached the eutectic. Studies by Loper [1978] suggest that gravitational dynamo may be able to function far into the future.

Summary and Conclusions

In the introductory section we attempted to evaluate the plausibility of three proposed power sources for the geodynamo: thermal convection, precession, and gravitational settling. The plausibility was judged using two criteria: can sufficient power be fed to the dynamo by the proposed mechanism and must the power be fed into the magnetic field. The most plausible power source appears to be gravitational settling. The essential features of this power source were outlined.

The primary goal of the paper was to list and describe the possible thermal regimes of the outer core which are compatible with the gravitationally powered dynamo. This has been done in the second section, where four possible regimes are considered. The first, in which $T_A' < T_C' < T_L'$, is the simplest and possesses no unusual features. The second regime with $T_A' < T_L' < T_C'$ is similar to the first except that a slurry layer must occur at the bottom of the outer core. In In each of these regimes the fluid is both thermally and compositionally unstable. This is in contrast to the third possible regime, $T_C' < T_A' < T_L'$, in which the thermal gradient tends to stabilize the fluid. However, overturning is driven by the stronger compositional buoyancy despite the fact that $T' < T_A'$. This introduces the possibility that heat may be transported radially inward by the convection driven by compositional buoyancy. Consequently, there is no direct relation between the rate that heat is conducted outward in the outer core and the rate of heat transfer to the mantle. The fourth regime, $T_C' < T_L' < T_A'$, allows compositionally driven convection provided the thermal conductivity is sufficiently large that $T_C' < T' < T_L' < T_A'$. This possibility appears to have been overlooked by Higgins and Kennedy [1971]. It was found that a slurry in the bulk of the outer core is incompatible with overturning because transport processes produce both thermal and compositional gradients which tend to stabilize the fluid.

Due to the nature of heat conduction in a spherical geometry, the conduction gradient T_C' is likely to be steeper near the inner core than at the mantle-core boundary. Therefore it is possible that the outer core is not in a single thermal regime but is in regime A or B near the inner core and in regime C or D1 near the mantle.

In discussing the various thermal regimes we assumed the composition of the core fluid to be more metallic than the eutectic so that a dense metal-rich solid formed upon freezing. The possibility that the core fluid may be less metallic than the eutectic was investigated, and it was found that a layer of variable composition must form at the bottom of the outer core. Difficulties associated with the removal of heat from this layer led us to the conclusion that a metal-poor composition for the core is unlikely.

The thermal evolution of the earth was considered, and it was noted that if $T_A' < T_C'$, the heat transfer problems for the core and mantle are decoupled with conditions in the core leading to a prescribed temperature at the base of the mantle and the mantle in turn prescribing the heat flux which must emanate from the core. The relative amounts of heat supplied by the heat capacity of the core, the latent heat of solidification, and the gravitational energy released were considered. It was found that if $T_A' = T_L'$, a significant flux of heat may flow from the core to the mantle with virtually no change in the overall temperature of the earth.

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