Bulletin of the Seismological Society of America. Vol. 53, No. 1, pp. 27–37. January, 1963

COMPARATIVE SPECTRA OF MICROSEISMS AND SWELL

By R. A. Haubrich, W. H. Munk and F. E. Snodgrass

ABSTRACT

Spectra of seismic and ocean wave recordings near San Diego, California, show closely related features. The wave spectra consist of a sharp peak whose frequency, $f(t)$, increases linearly with time and consistent with the expected dispersive behaviour from a source at 6150 nautical miles (presumably a storm in the Ross Sea). The seismic spectra show peaks at $f(t)$ and at $2\,f(t)$; the double frequency peak contains 100 times the energy of the peak at the primary frequency. A comparison between the peak frequencies and band widths of the seismic and ocean wave spectra, and an estimate of the direction and beam width of the seismic radiation, leads to the following conclusions: that the microseismic generation area is predominantly *local,* being confined to a distance of 100 miles up or down the coast. For the primary frequencies the generative strip is presumably confined to shallow water; for the double frequencies it extends 200 miles seaward.

Introduction

We have studied many spectra of the microseismic band typically centered at 150 millicycles per second (Haubrich and Iyer, 1962). At frequencies below this band we had found a relatively uniform level of about 10^{-6} $(\mu/\mathrm{sec})^2$/mcps without interesting structure. Following a suggestion by Sutton (1962) that this background was the result of perturbations of the seismometer weight by turbulent convection, we followed the Lamont procedure of producing a stable stratification by heating the air near the top of the seismometer case. As a result of this simple modification, the noise level has been lowered by 20 *db*, and this has made possible a detailed comparison between seismic and wave spectra over the entire frequency interval 30 to 300 mcps. The seismic spectra have peaks both at the frequency of the ocean waves and at twice that frequency. Some of the results described here bear a close resemblance to unpublished findings at Lamont kindly made available to us by Oliver and Page (1962).

Figure 1 shows the locations of the recording stations. Seismic and ocean waves are recorded digitally with a very large dynamic range, and this in turn permits us to obtain spectra of satisfactory resolution. We shall show how this improved resolution can be exploited to derive some information concerning the generation of microseisms: there is nothing new about the existence of microseisms having the frequency and twice the frequency of prominent ocean waves. Those of double frequency contain more energy and have been known for a longer time. Spectra showing a 2:1 relation with ocean waves are given by Darbyshire (1950) and Haq (1954). Seismic waves having the "primary frequency" of ocean waves associated with east coast hurricanes were observed by Oliver and Ewing (1957); both wave systems showed the characteristic dispersive increase in frequency with time. Related observations are due to Pomeroy (1959). Oliver (1962) describes extremely low frequency (37 mcps) microseisms with the characteristic dispersive behavior and attributes these to waves from an intense storm in the South Atlantic.

Description of Seismic Spectra

Figure 2 shows the situation for the seismic spectra at the start of a particular sequence to be discussed in detail. The primary peak which dominates the subsequent development is barely discernible at 40 mcps. A broad peak centered at 70 mcps, and an intense broad peak at 150 mcps dominate the picture. The intensity drops appreciably at higher frequencies. The relatively broad intense peak between 130 and 170 mcps has been found on many previous occasions. It is the feature

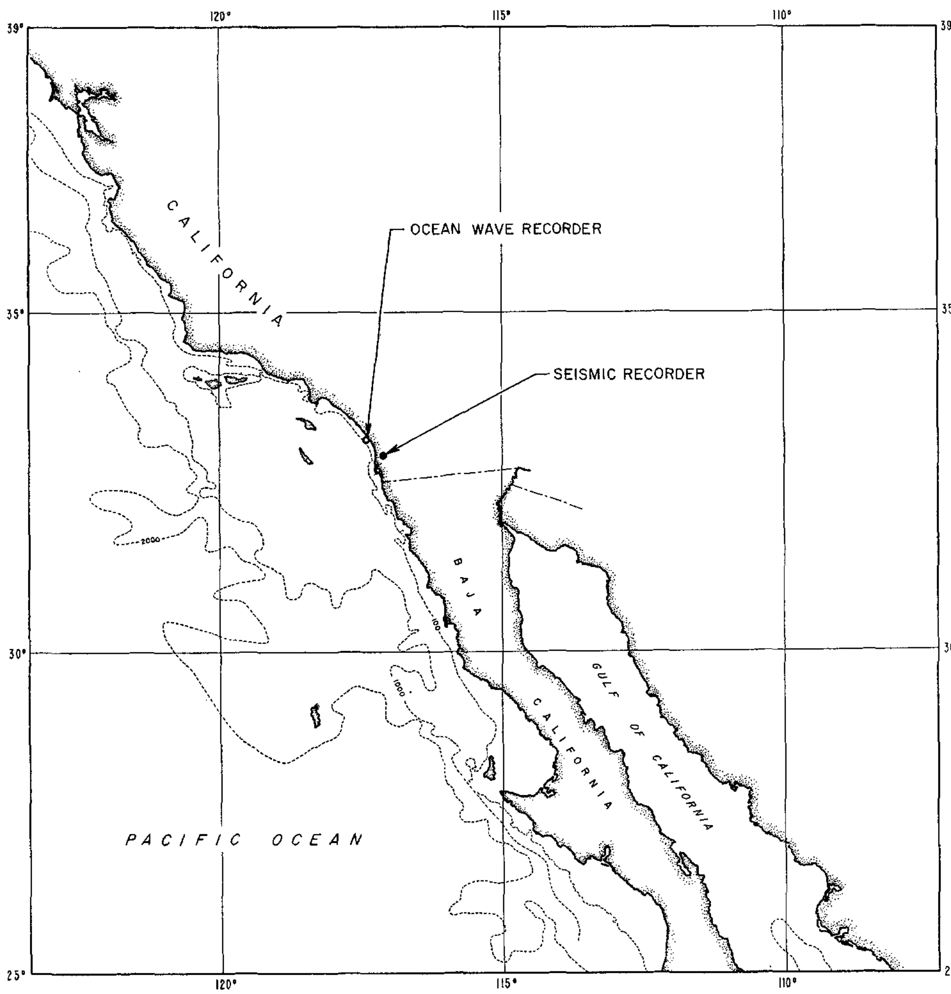

FIG. 1. The locations of the seismic station (solid point) and the wave station (open point). The continental borderland is shoreward of the 1000 fathom line.

ordinarily identified with the 7-second microseisms. This "7-second hum" is not characterized by pronounced dispersive shifts.

The lower curves give the direction of arrival of the seismic waves, and an indication of beam width. Let

$$C_{EN} + iQ_{EN}$$

designate the complex cross-spectrum between the EW and NS components of ground motion, and similarly for the vertical component UD. The direction of

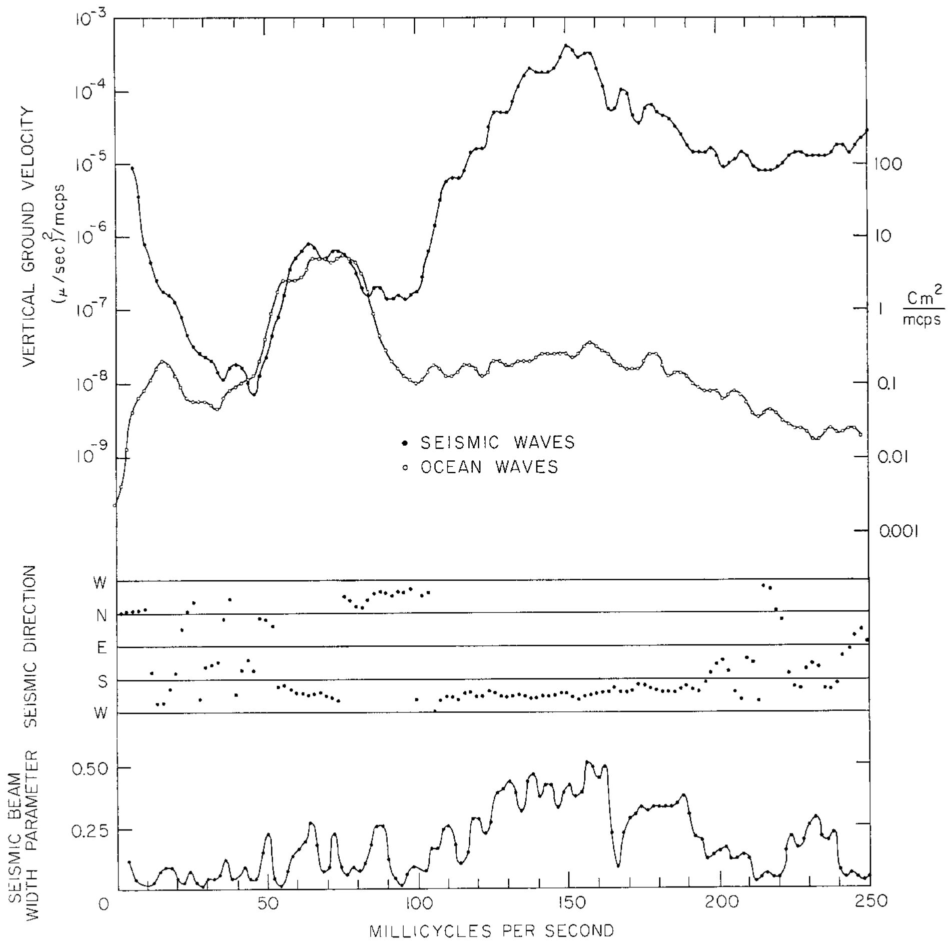

FIG. 2. *Top* Comparative spectra of microseisms (solid points, left scale) and ocean swell (open points, right scale) on the morning of 19 May 1962, GCT. The vertical ground velocity is in units of (microns/second)^2/millicycle per second. To convert to cgs units, 1 $(\mu/\text{sec})^2/$mcps $= 10^{-5}$ cm^2 sec^{-1}. The wave elevation spectrum is in units of cm^2/mcps. To convert to cgs units, 1 cm^2/mcps $= 10^3$ cm^2/cps $= 10^3$ cm^2 sec. To convert this to bottom pressure spectra, 1 cm^2/mcps is nearly equivalent to 10^9 (dynes/cm^2)^2 sec $= 10^9$ grams^2 cm^{-2} sec^{-3}. *Center* The direction *from* which the microseisms are coming. *Bottom* Beam width parameter.

arrival, θ, is computed assuming a single source of Rayleigh waves, according to the relation

$$\tan\theta = Q_{NU}/Q_{EU}\,.$$

A parameter characteristic of beam width is

$$B = \frac{Q_{NU}^2 + Q_{EU}^2}{C_{UU}(C_{NN} + C_{EE})}$$

where C_{UU}, C_{NN}, C_{EE} are the power spectra of vertical, NS and EW components, respectively. For a pencil beam of Rayleigh waves with no other energy present,

$B = 1$; for isotropic radiation $B = 0$. The frequency dependence of θ, B and of all C's and Q's is understood. Note that the seismic radiation at both primary and double frequency is predominantly from the southwest, and that the beam width at double frequency is narrower than in the primary frequency band. The spectra are based on one-hour records and contain 28 degrees of freedom.

The situation two days later is shown in figure 3. The seismic spectrum consists of three principal features: (i) a peak at 55 mcps, (ii) a peak at about double this

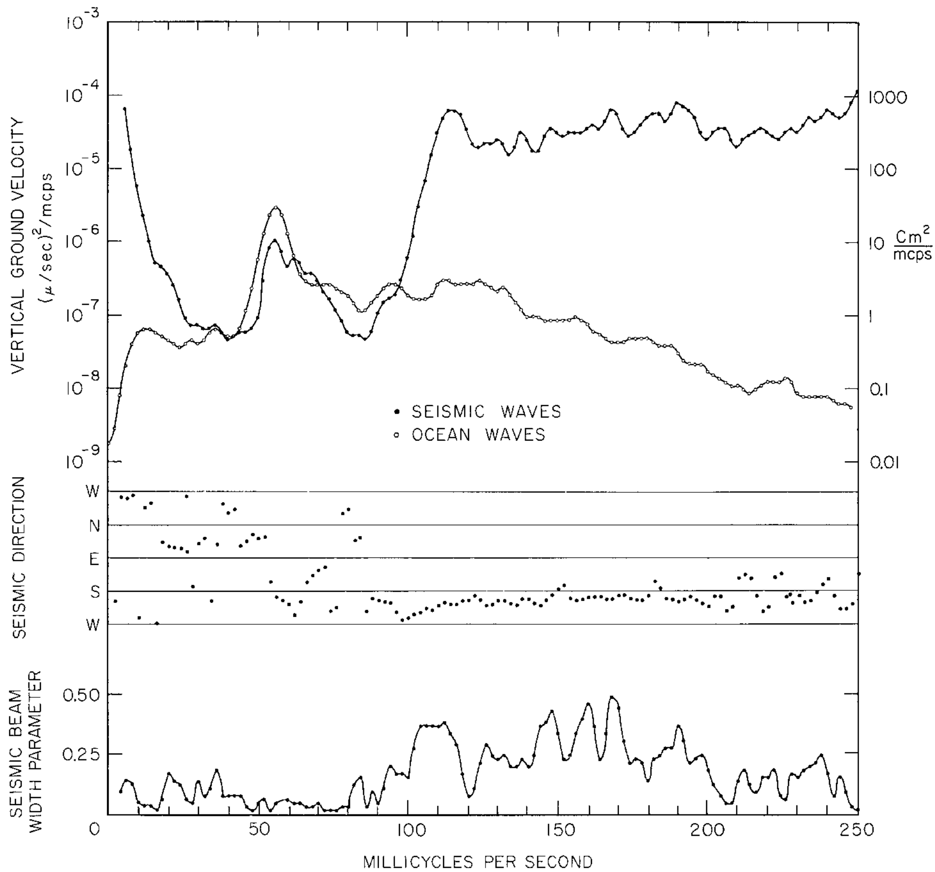

FIG. 3. Comparative spectra on the afternoon of 21 May 1962. For legend, see figure 2.

frequency, and approximately 100 times as intense, superimposed on (iii) a plateau of high and relative uniform intensity to at least 250 mcps.

Figure 4 shows the situation 12 hours later. The primary and double frequency have somewhat increased and the peaks intensified.

Figure 5 shows the peak frequencies and intensities during the entire development of this particular sequence.

Thus we have been able to resolve a narrow dispersive peak moving across the broad peak of the 7-second hum in the direction of increasing frequency. This suggests that the 7-second hum is the result, at least in part, of the smearing of many such peaks.

The Primary Frequencies

The progressive shift in the primary frequencies immediately suggests a close relation to ocean swell. Accordingly, concurrent measurements of ocean waves were taken at Camp Pendleton 26 miles to the northwest of the seismic installation (figure 1), and these records were subjected to the same analyses as that of the seismic records (figures 2, 3, and 4). The resemblance is obvious, except for a side

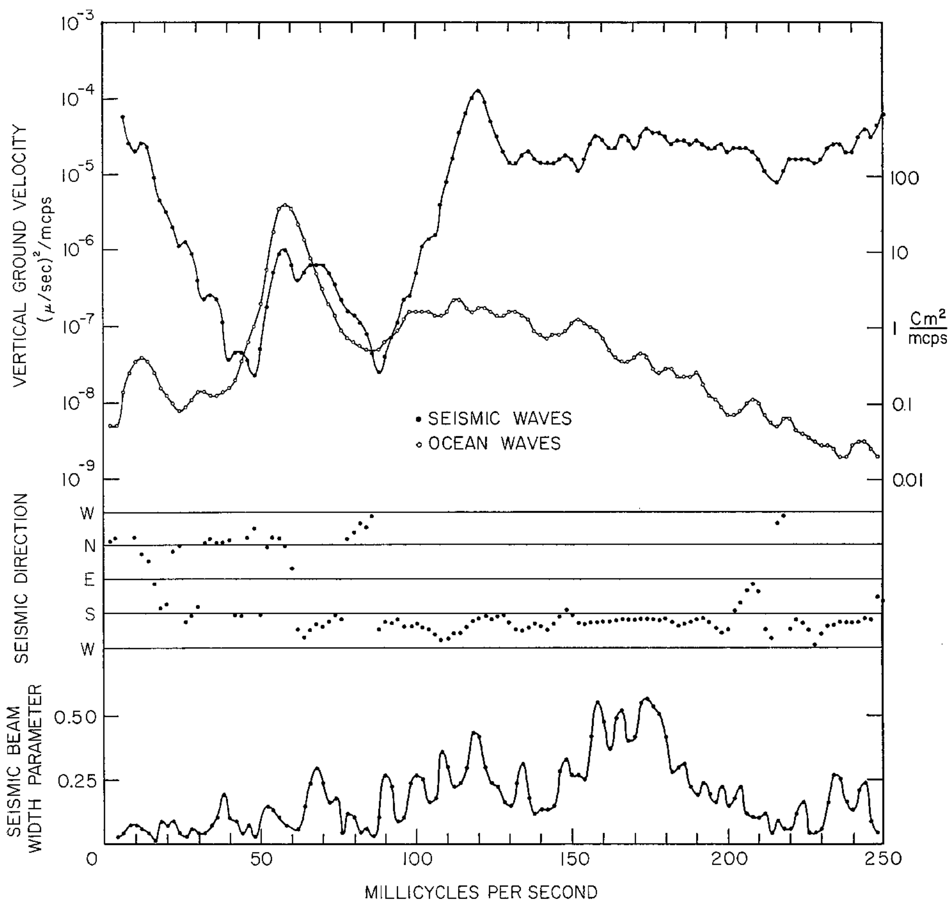

Fig. 4. The situation during the early morning of 22 May 1962. For legend, see figure 2.

peak in the seismic spectra that is not contained in the wave spectra. This particular feature is to be discussed subsequently. The frequency of the principal primary peaks in the wave and seismic spectra when plotted against time lies along a single line (*PF* in figure 5) and establishes that the two phenomena are closely linked.

Let $V(f)$ designate the group velocity of ocean waves appropriate to some frequency f. A wave packet containing frequencies centered on f will have travelled a distance r from a storm area in a time t at velocity $V(f)$. According to classical deep water theory

$$V(f) = \frac{r}{t} = \frac{g}{4\pi f} \tag{1}$$

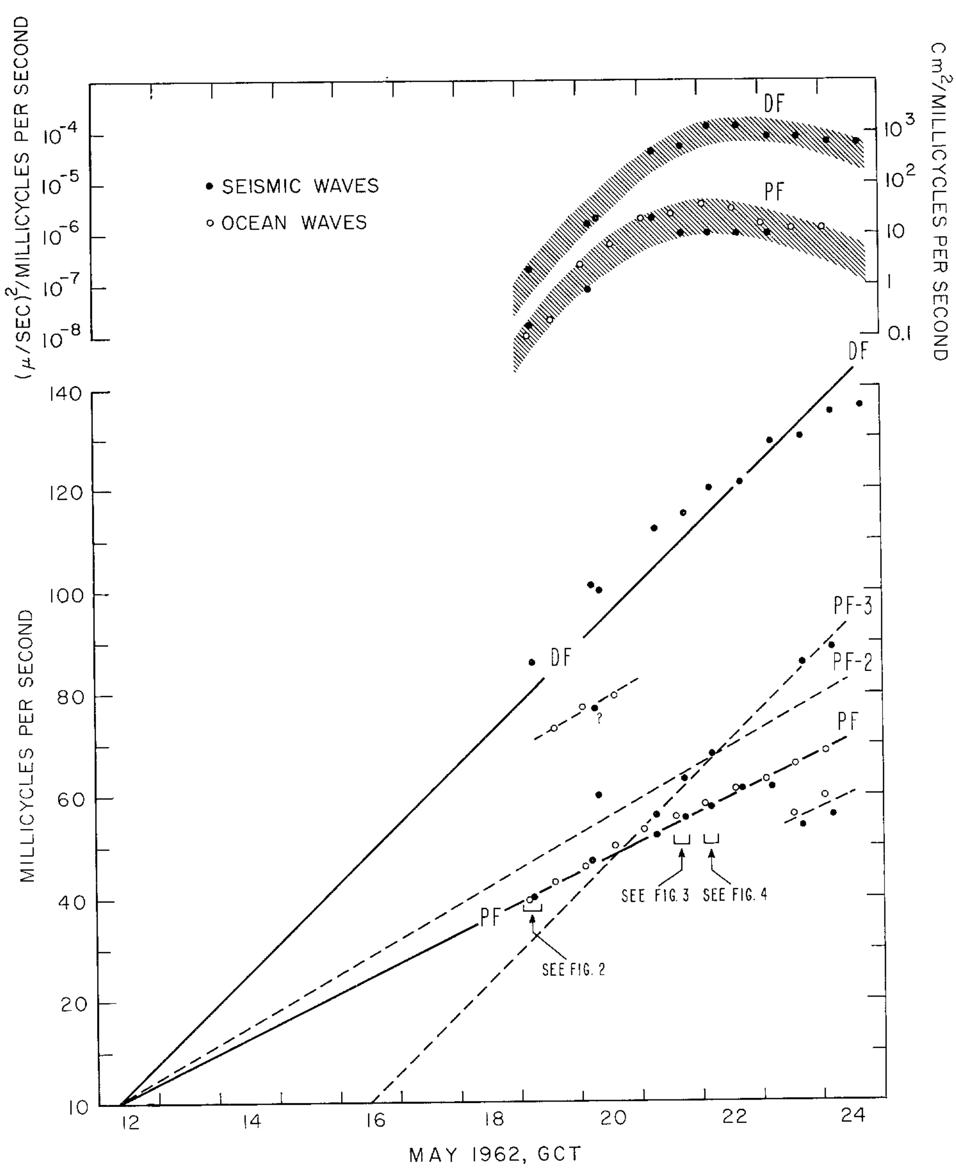

FIG. 5. The lower diagram shows the dispersive variation in frequency of the spectral peaks of the seismic records (solid points) and ocean wave records (open points). Frequencies of the waves and the Primary Frequencies of the seismic motion follow the line marked PF; the line marked DF is drawn for Double this Frequency. The two bands on the upper diagram designate the peak intensities of the primary frequency band and ocean waves, on the one hand, and of the double frequency band on the other hand.

so that

$$f = \frac{g}{4\pi} \frac{1}{r} t.$$

Thus at a fixed location, r, frequency increases linearly with time. The intercept of this line $f(t)$ establishes the time of origin, and the slope of the line varies inversely with the distance, r, of the source. The line marked PF is consistent with the following values:

time origin at 00^{h} 12 May 1962, GCT

distance $r = 6150$ nautical miles.

On the basis of directional studies of ocean waves (Munk, Miller, Snodgrass, and Barber, 1962) we suspect a source region in the Ross Sea near 150° W, 60° S.

The frequencies of the microseismic peaks and swell peaks agree within 2 per cent. Accordingly the values of r agree within 2 per cent, so that the microseisms are produced locally, that is, within 100 miles up or down the coast from La Jolla.

This is in agreement also with the observed fact that the seismic spectral peak is not appreciably broadened as compared to the ocean wave spectral peak. Both peaks are remarkably narrow and not properly resolved in the figures, but a high resolution analysis (not shown) gave a width of $\frac{1}{14}$ the central frequency at the half-power point ($Q = 14$). If the seismic waves were generated along a continuous coastal strip of length l, then the peak would be broadened by an amount of the order (l/r) times the central frequency. For $l = 100$ miles, the broadening amounts to $100/6150 \approx 2\%$ as compared to the observed width of 7 per cent, and is thus barely discernible.

Finally, the observations indicate that the radiation in the primary peak subtends a very broad angle. This would be in agreement with a source strip one hundred miles long and centered at La Jolla.

We conclude that the evidence concerning the primary seismic frequencies, their rate of drift, the sharpness of the associated peak and the lack of directivity are consistent with generation by ocean waves along a coastal strip something like 100 miles in length and centered at the coastal point opposite the wave station.

The Side Peak

The seismic spectra of 21 and 22 May show a side peak of slightly higher frequency than the principal primary peak; the side peak is definitely not contained in the associated ocean wave spectra. The side peak is associated with some directivity (figure 4) and the indicated direction is from the southwest.

We suggest two hypotheses: the side peak is associated with the same storm as the principal peak, and represents the generation of seismic waves at some beaches nearer the source (PF-2 in figure 5); the indicated distance from the source is then 5200 nautical miles, as compared to 6150 for the principal peak. A storm in the Ross Sea and the conversion of some ocean wave energy to seismic wave energy in the Hawaiian Islands (in addition to California beaches) is consistent with these numbers. An alternate explanation is that the side peak is associated with waves on a distant coast from a different storm. The construction PF-3 is consistent with a storm of 16 May at a distance of 2050 miles from the unknown beaches where the seismic waves were generated by the ocean waves.

Further experience should help us decide whether we are dealing with multiple regions, local and distant, where swells from a single storm produce microseisms, or whether we are dealing with multiple storms.

The Double Frequencies

The line *DF* in figure 5 is drawn to give, at any time, twice the frequency of line *PF*. The observed frequencies follow the line sufficiently closely to demonstrate a relation to the primary frequencies, though there are significant departures. The evidence supports the Longuet-Higgins theory of microseism generation by oppositely directed waves (Longuet-Higgins, 1950). The frequency and relative narrowness of the spectral peaks at the double frequencies demonstrates, as in the case of the primary frequencies, that the source region is fairly concentrated in the ocean areas near the seismic station.

We can estimate the width of the strip in which the microseisms are generated. If it extended too far off shore, then there would be no frequency overlap between incident and reflected waves. Thus the width of the strip is proportional to the width of the spectral peak, and this, in turn, is associated with the extent of the storm in time and space.

Let $f_0(t)$ designate the center frequency of the ocean waves as they reach some coastal point, P, at a distance r_0 from the source. Now consider a point at sea a distance Δ from P and lying between P and the storm. To reach this offshore point, the incident waves have travelled a distance $r_0 - \Delta$, and the reflected waves (now travelling back towards the source) a distance $r_0 + \Delta$. The total travel times are the same, and the required group velocities are then

$$V_{\text{incident}} = \frac{r_0 - \Delta}{t} = \frac{g}{4\pi(f_0 + \delta f)}$$

$$V_{\text{reflected}} = \frac{r_0 + \Delta}{t} = \frac{g}{4\pi(f_0 - \delta f)}$$

so that the center frequencies of incident and reflected waves differ by an amount

$$2\delta f = 2f_0(\Delta/r_0).$$

To assure a reasonable frequency overlap between incident and reflected energy requires a spectral width of the order

$$\frac{1}{Q} = \frac{2\delta f}{f} = 2\,\frac{\Delta}{r_0}$$

For a given spectral width, microseisms are then generated in a strip (ignoring all trigonometric factors)

$$\Delta = \tfrac{1}{2} r_0 Q^{-1}$$

where Q^{-1} is of the order (storm fetch/storm distance), or (storm duration/travel time t), whichever is larger. In the former case we derive the simple result $\Delta = \frac{1}{2}$ (storm fetch). Setting $r_0 = 6150$ nautical miles and $Q = 14$ gives $\Delta = 220$ nautical miles. In the case of distant, compact, and short-lived storms the strip is narrower; for nearby storms of large size and duration, the strip is wider.

The observations indicate a fair degree of directivity in the secondary peaks, the seismic waves arriving from the southwest, that is, from roughly normal to shore. The result might be accounted for as follows: Rayleigh waves of double frequency experience a marked increase in phase velocity as they travel from an oceanic to a continental region. (At the primary frequencies this effect is small on account of the greater wave length.) The foregoing numerical values indicate that a large fraction of the generation of microseisms takes place in the open sea. For glancing incidence, the waves are refractively turned (totally reflected) back to sea. Only waves arriving from approximately normal to shore can reach the continent.

Relative Intensities

We are now in a position to compare quantitatively the peak intensities of the primary frequencies and double frequencies in the microseisms (figure 5). The double frequencies are more intense. Typical intensity ratios are 10:1 near the lower frequencies, increasing to 100:1 near the high frequencies. In the latter case the double frequencies are superposed upon a broad plateau, and a correction for this might lower the ratio by a factor of two.

A comparison of the spectra of vertical ground velocity and ocean wave amplitude at the primary frequencies leads to the ratio

$$\frac{10^{-6}(\mu/\mathrm{sec})^2/\mathrm{millicycle}}{10\ \mathrm{cm}^2/\mathrm{millicycles}} = 10^{-15}\ \mathrm{sec}^{-2}.$$

It might be more meaningful to compare the spectra of vertical displacement, ground and sea surface. The displacement spectrum is obtained from the velocity spectrum by dividing by $4\pi^2f^2$. This raises the microseismic spectrum at the primary frequencies relative to that at double frequencies by a factor of 4, and improves the agreement between the two spectra. The ratio in the intensities is roughly 10^{-14}, so that the ground is displaced vertically by 10^{-7} times the vertical displacement of the sea surface.

The simplest hypothesis attributes the microseisms of primary frequency to oscillatory pressure sources on the bottom along the coast line, with some prescribed strength per unit length of coast. The problem cannot be set up as that of an infinite line source along the coast. The "short-crestedness" of the waves limits the correlation parallel to shore to a distance of the order λ/ϕ, where λ is the wave length and ϕ the beam width of the incoming ocean swell. For very small values of ϕ the longshore correlation may be limited by irregularities in the coast line. In the direction normal to shore the effect is essentially limited to a strip for which the depth is less than $\lambda/4$.

A number of factors enter in considering the relative intensities of the primary and double frequency radiation. (i) The relative widths of the generation zones favor the double frequencies by perhaps 1000:1. (ii) The double frequencies involve a second order effect which is of the order of the mean-square slope of the waves times the primary effect. (iii) The double frequencies are reduced in the ratio of the coefficient of energy reflection. Measurements at San Clemente Island indicate a value of 20%. (iv) Finally there is a mismatch in the lengths λ_0 and λ_s of the ocean and seismic waves of equal frequency. For the primary frequencies the ratio λ_0/λ_s is 500 m./50,000 m., or 0.01; for the secondary frequencies the pertinent ratio is $Q(\lambda_0/\lambda_s) = 14 \times 500/\frac{1}{2}\ 50{,}000 = 0.28$.

The 7-second Hum

The situation of 19 May 1962 (figure 2) is rather typical with respect to the "7-second hum". The peak extends from 120 to 170 mcps, without significant narrow peaks superimposed, dropping off sharply (by 35 db) at the low frequencies, and gently (by 15 db) at the high frequencies. But here again the evidence points towards an important local effect. The primary spectrum shows three peaks, and all of these are present in the wave spectrum. The peak at 40 mcps (which is the main topic of this note) is actually much clearer on the seismograms than the wave records. Two peaks, at 66 and 76 mcps, show up in both spectra, and their intensities bear the same ratios as those plotted on top of figure 5. Most of the primary energy lies between 60 and 85 mcps, as compared to the limits 120 to 170 mcps for the 7-second hum. The ratio between the intensities at primary and double frequency is also not far from what we found previously for the narrow peak. Primary and double frequencies both come from southwest.

One is tempted to conclude that the discussion pertaining to the narrow peak associated with the storm of 12 May applies also to the more typically complex situation. If this is the case, then the 7-second hum is largely the result of local wave activity, and represents a blurred picture of the simple story told by a single, predominant narrow peak. This is not to imply that the effect of distant wave action may not predominate at times, nor can we discount the role of the oceanic wave guide in shaping the microseismic spectrum. The seismic noise problem is a complex one, and it is not constructive to insist that a single explanation accounts for all things at all places at all times.

The fact that the ocean wave spectrum over a vast range of frequencies is at its lowest between 3 and 30 mcps suggests a similar trough in the seismic noise spectrum. The observed rise below 30 mcps can probably be attributed to instrumental noise, and an effort to improve the instrument response in this range may be rewarding.

Acknowledgment

This research was supported by Contract No. AF 49(638)-905 of the Air Force Office of Scientific Research as part of the Advanced Research Projects Agency's Vela Uniform Program.

References

Darbyshire, J.
1950, "Identification of Microseismic Activity with Sea Waves," *Proc. Roy. Soc. A.*, 202: 439–448.

Haq, Kazi Emanul
1954, *The Nature and Origin of Microseisms*, Thesis for Doctor of Science, Dept. of Geology and Geophysics, Mass. Inst. of Tech.

Haubrich, R. A. and H. M. Iyer
1962, "A Digital Seismograph System for Measuring Earth Noise," *Bull. Seism. Soc. Am.*, 52: 87–93.

Longuet-Higgins, M. S.
1950, "A Theory of the Origin of Microseisms," *Phil. Trans. Roy. Soc., A.*, 243: 1–35.
Munk, Miller, Snodgrass and Barber
1962, "Directional Studies of Ocean Waves," submitted to *Phil. Trans.*
Oliver, J.
1962, "A Worldwide Storm of Microseisms with Periods of about 27 Seconds," *Bull. Seism. Soc. Am.*, (in press).
Oliver, J. and M. Ewing
1957, "Microseisms in the 11 to 18 Second Period Range," *Bull. Seism. Soc. Am.*, 47: 111–127.
Oliver, J., and R. Page
1962, "Concurrent Storms of Long and Ultra-long Period Microseisms", (in press).
Pomeroy, Paul
1959, "Background and Storm Microseisms in the Period Range 11–12 Seconds," *J. of Geophys. Res.*, 64: 1120.
Sutton, George H.
1962, "Note on Long-Period Noise in Seismographs," *J. of Geophys. Res.*, 67: 2082–2084.

La Jolla Laboratories
Institute of Geophysics & Planetary Physics
University of California
La Jolla, California

Manuscript received July 11, 1962.