Rheology in Earth Forces

Introduction

1. Rheology --- study of the deformation and flow of matter.
2. Continuum concept vs. localized failure, e.g.,
    a. Fracture of rocks
    b. Sliding on preexisting faults
3. Three basic types of continuum deformation:
    a. Elastic
    b. Viscous
    c. Plastic

4. Combinations of all three types

What behavior do the faults in the road cut exhibit? (attempt to answer this below)

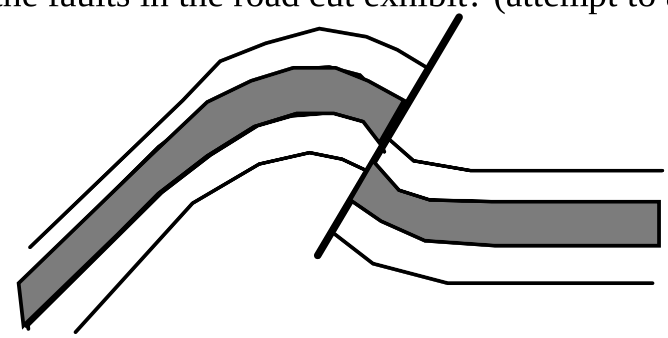

Figure 1. Fault in roadcut on Hwy. 30.

Stress Tensor

Let the 3 mutually perpendicular planes be orthogonal to the 3 axes of a orthogonal coordinate system (Figure 2). Here we introduce a notation for the decomposition of the stress vector on the three planes. We use tensor notation . The "*i*" subscript refers to the plane normal to the "*i*" axis. The "*j*" subscript refers to the coordinate direction of the stress.

Stresses that act along the normal to a plane are called normal stresses (σ_{ij} , $i = j$).
Stresses that lie in the plane are called shear stresses (σ_{ij} , $i \neq j$).

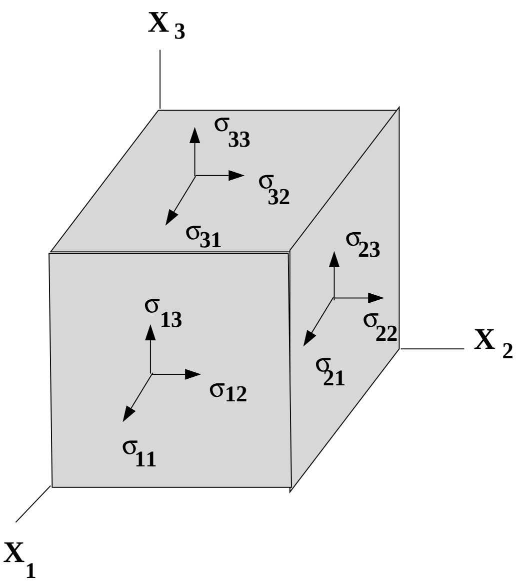

Figure 2. Stresses acting at a point in a rock on 3 mutually perpendicular planes.

Stresses are just a convenient way of looking at all the forces acting at a single point in a rock. The definition does not depend on the rheology of the rock at all. What are the shear stresses and what are the normal stresses?

Elastic Behavior

What is an elastic material? Do the rocks in the road cut behave elastically?

Two parameters are necessary to define an elastic material, Young's modulus, *E*, and Poisson's ratio, ν.

Young's modulus: Consider the case of uniaxial stress, σ_1, where only a single normal stress is acting on a material (imagine a stress acting along the axis of a bar). When we pull or press on the solid bar its length changes. The relative length change (longitudinal strain), $\varepsilon_1 = \Delta L/L$ is proportional to the applied stress. For an elastic material

$$\sigma_1 = E\varepsilon_1$$

So *E* is just the constant of proportionality. A defining property of an elastic material is that the strain will return to zero when the stress is removed.

Poisson's Ratio: A simple interpretation of Poisson's ratio is that is gives the transverse strain for axial loading. Consider the uniaxial situation again: Poisson's ratio is the lateral contraction per unit breadth divided by the longitudinal extension per unit length.

The general relationship between stress and strain (Generalized Hooke's law) in an elastic medium is given by

$$\sigma_{ij} = \frac{E\nu}{(1+\nu)(1-2\nu)}\varepsilon_{ii}\delta_{ij} + \frac{E}{(1+\nu)}\varepsilon_{ij}$$

where $\delta_{ij} = 0 = i \neq j$, otherwise = 1 if $i = j$. If we invert this equation, we obtain

$$\varepsilon_{ii} = \frac{1}{E}\left[\sigma_{ii} - \nu\left(\sigma_{jj} + \sigma_{kk}\right)\right]$$

$$\varepsilon_{ij} = \frac{(1+\nu)}{E}\sigma_{ij}$$

Note that when $i \neq j$, then we are talking about shear stress or strain, and when $i = j$, then we are referring to *normal* stress or strain. Note the role of Poisson's ratio ν in the normal strain; it is a response to transverse stress.

Elastic behavior is important in seismic waves, elastic support of loads, flexure, etc.

Plastic Behavior

What is a plastic material?

In a typical stress-strain diagram, the elastic range is that region of low to moderate stress where a constant slope is exhibited. We say that the material exhibits an elastic range and a plastic range.

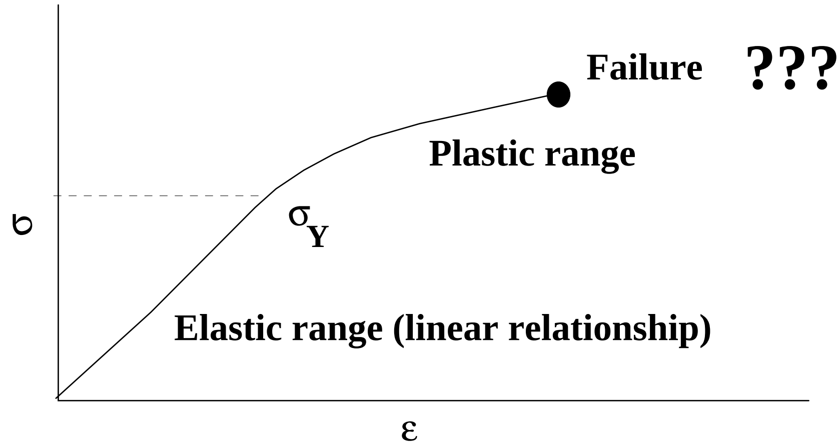

Figure 3. Stress-strain relationship.

The simplest type of plastic behavior is perfectly-plastic, where the stress never rises above the yield stress. A material can be perfectly plastic without an elastic regime.

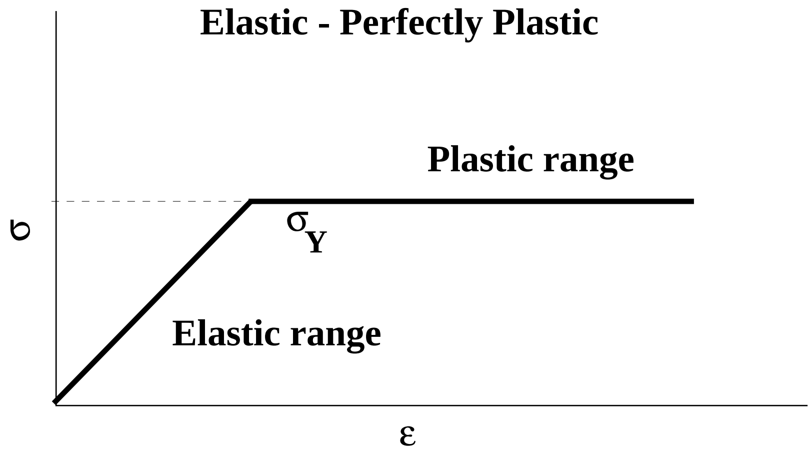

Figure 4. Elastic, perfectly-plastic material.

Some materials show an increase in yield stress with increasing strain. This phenomenon is known as strain hardening.

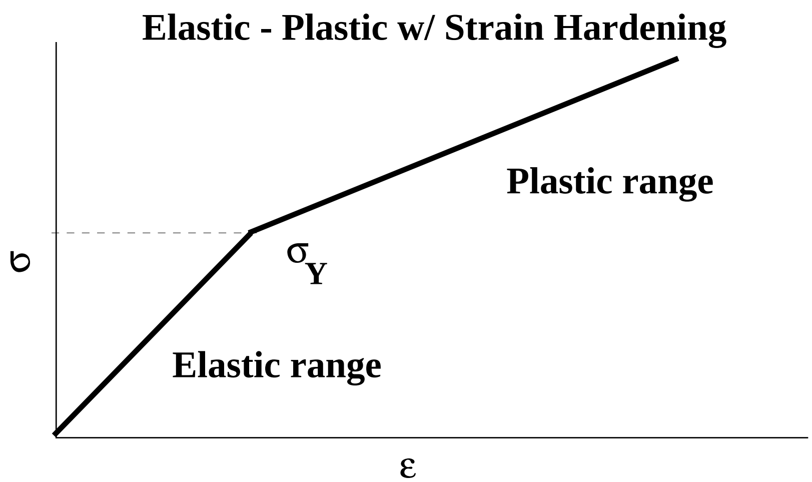

Figure 5. Strain hardening.

Plastic behavior of material is an engineering idealization to describe a material that is actually creeping. If fact, plastic behavior is a subset of viscous flow when the strain rate is independent of time. Normally, time does not enter into elastic-plastic problem. An elastic-plastic description of the lithosphere has been very successful.

Plastic deformation and failure of rocks: faulted crust as a plastic medium.

Creep or Viscous flow

What is viscosity?

Post glacial Rebound; How can the mantle be both elastic and viscous? The sagging window pane analogy.

A proper definition of viscosity is

$$\sigma'_{ij} = 2\eta\dot{\varepsilon}'_{ij}$$

where the primes ($'$) denotes deviatoric quantities. What does that mean? Also $\dot{\varepsilon} = d\varepsilon / dt =$ strain rate. When η is not a function of stress, it is called a *Newtonian* (or linear) viscosity; otherwise the term *non-Newtonian* is used. In general, viscosity is a derived quantity and the more fundamental relationship is between strain rate and stress, which is usually loosely written as

$$\dot{\varepsilon}_s = A\sigma_s^n$$

where the subscript "*s*" denotes "shear" and the relationship is meant to describe steady state creep. Here

$$A = A(P, T, \text{material properties})$$

(Pressure, Temperature) and n = 1 describes Newtonian flow and n > 1 describes non-Newtonian flow.

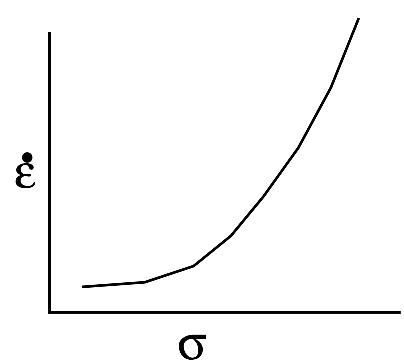

Figure 6. Non-Newtonian stress strain relationship.

Deformation Mechanisms

Creep deformation of rocks is understood on an atomic scale as having to do with the temperature activated migration of defects within individual crystals and dislocation of crystal planes. That is a vast body of literature on this subject, which is beyond the scope of the discussion here.

Lithosphere

<u>What is the lithosphere?</u>

Elastic lithosphere
Mechanical lithosphere
Seismic lithosphere
Thermal lithosphere

<u>Asthenosphere</u>

Viscoelastic Behavior

Viscoelastic material has both viscous and elastic elements of behavior. It is a good way to describe the lithosphere. A way of describing material that has both vicious and elastic behavior is by the way of mechanical analogs. While these analogs offer only a qualitative description of the actual behavior of rocks, they can be useful to describe rheological phenomena. We consider two basic elements.

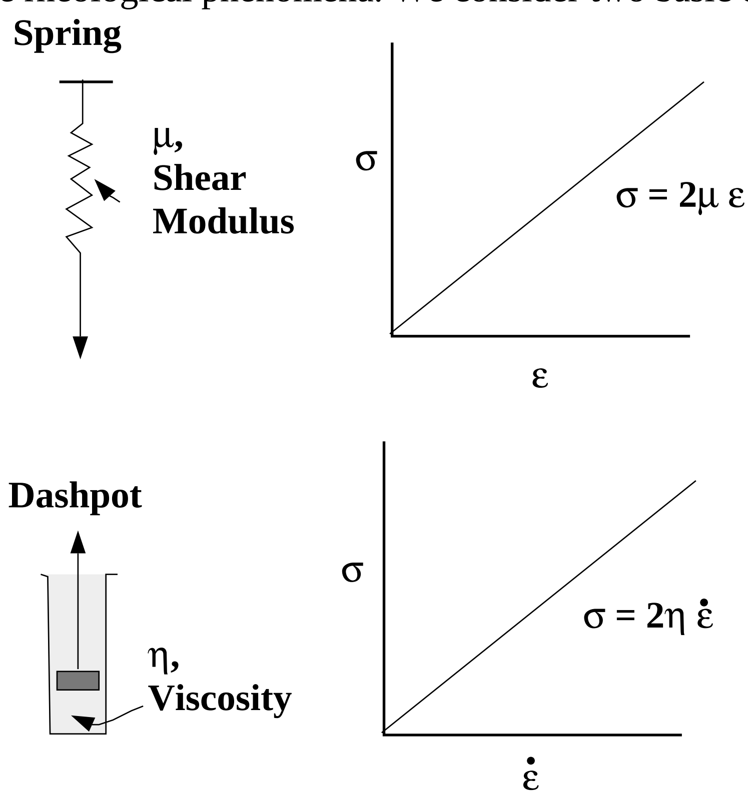

Figure 7. Mechanical analogs to rock behavior.

The elastic model is a perfect spring obeying linear stress-strain laws. The dashpot is a loosely fitting piston which moves through a cylindrical tube filled with a viscous fluid. The piston moves at a rate which varies inversely as the viscosity.

<u>A Maxwell viscoelastic model</u>

A common model applied to rocks is that of a "Maxwell" solid. Such a material behaves elastically on a short time scale and viscously on a long time scale.

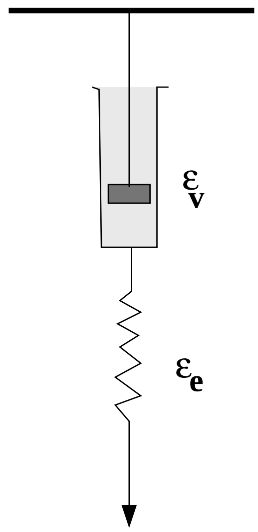

Figure 8. Maxwell model of rock.

If a constant stress is applied to a Maxwell solid, then there will be an instantaneous elastic response followed by a viscous response.

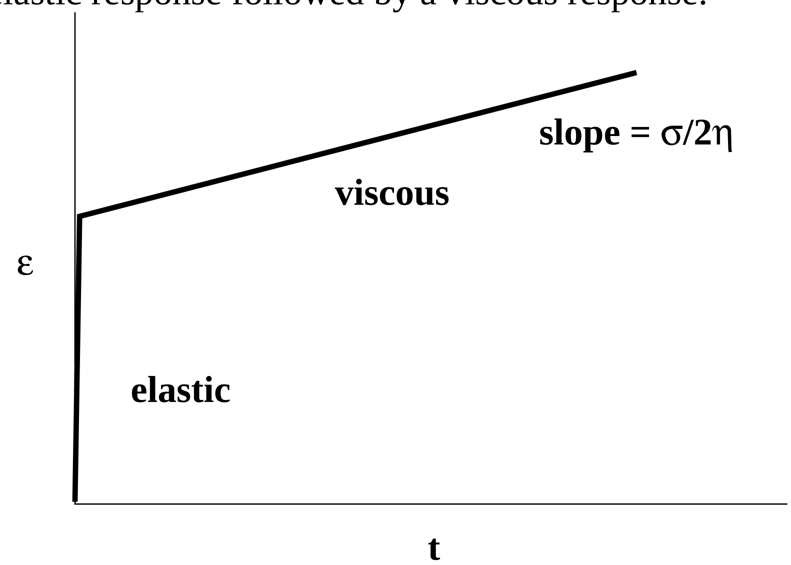

Figure 9. Strain vs. time for a Maxwell model.

The separation in time between elastic and viscous behavior is given by the Maxwell time:

$$\tau_{ve} = \frac{\eta}{E}$$

Thus the instantaneous elastic strain is followed by a steady state linear viscous deformation. When the load is removed, only the elastic strain is recovered.

The application of a Maxwell model to the lithosphere is quite useful in describing the base of the elastic lithosphere. The Maxwell time divides the elastic from the viscous regime. When we talk about an elastic lithosphere, we should ask "On what time scale is it elastic": 10^6 years? 4.6 x 10^9 years?

Very old (~1 Ga) stress systems on Earth are sometimes present in rocks of the upper lithosphere: the presence of these residual stresses shows that the upper lithosphere either has a finite yield strength or a Maxwell time on the order of at least the age of the rocks.

Strength Envelope Diagram for the Lithosphere

We wish to determine the "strength" of the lithosphere. That is, at any given depth, what is the stress that when applied will lead to some type of deformation. We assume the upper portion of the lithosphere is highly faulted and the lower part is subject to viscous creep.

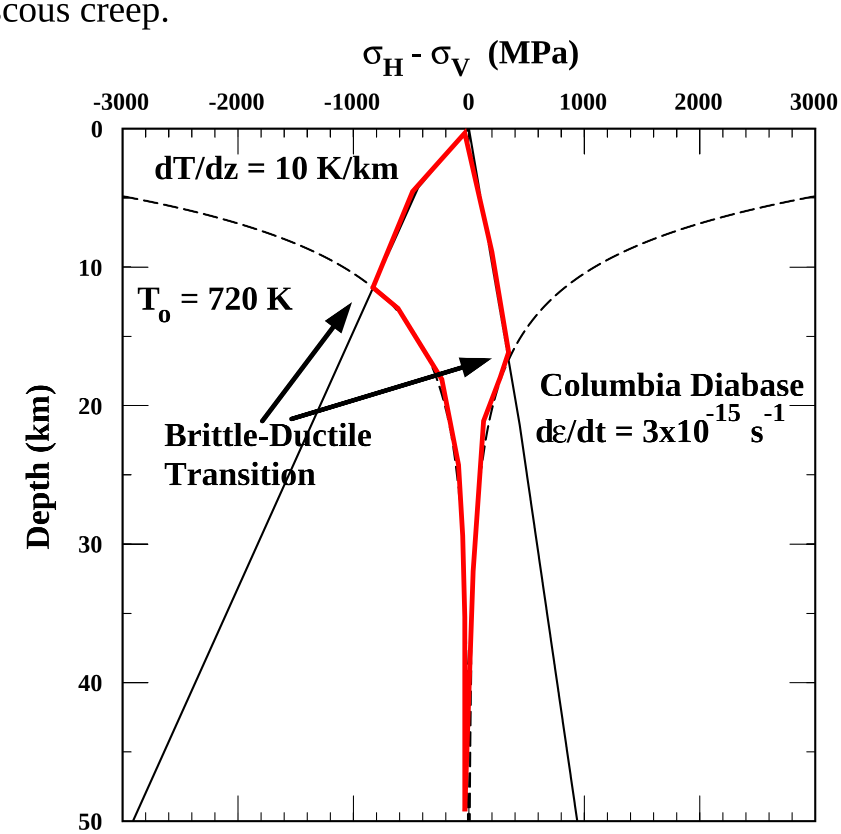

Figure 10. Strength Envelope Diagram.