Forces, Work, Energy, and Power

What are some sources of *energy* in the Earth?

Gravity field, radioactivity, kinetic energy

Where did the Earth derive this energy?

What are some important *Earth forces*?

Pressure, gravity, force of flowing fluids

How can gravity be both a force and a form of energy?

What is the most important source of energy for geological processes?

Easy: Radioactivity

What is the most important Earth Force?

Easy: Gravity

Where do earthquakes fit into this?

Force (Newtons = N)

Newton's 2nd law:

Force = rate of change of momentum and is a **vector** (boldface)

$$\mathbf{F} = \frac{d(m\mathbf{v})}{dt} = m\mathbf{a} \tag{1}$$

Example: Let $\mathbf{a} = \mathbf{g_0}$, then $\boldsymbol{F} = m\mathbf{g_0}$, where $\mathbf{g_0}$ is the gravitational acceleration of the Earth. What do we mean by the *gravitational acceleration of the Earth*?

If you leap off a tall building you will (assuming you are not Superman) accelerate with magnitude g_0, experience a gravitational force of magnitude **F**. Or if you step on a scale, you will provide a force that is equal to mass times gravity; in this case the force is called "weight". What is the direction of the force? It is in the direction of a gravity field.

Newton's law of universal gravitation says that 2 masses, m_1 and m_2, are attracted to each other with a force proportional to the products of their masses and inversely proportional to the distance squared between them:

$$\mathbf{F} = G\frac{m_1 m_2}{r^2}\mathbf{u}_r \tag{2}$$

Here $\mathbf{u}_r$ is a unit vector along a line between the two masses and G is a constant of proportionality called the "Universal Gravitational Constant". The masses are effectively "point masses". Now let $m_2 = m_E$, the mass of the Earth, and let m_1 = your mass, so

$$\mathbf{F} = G\frac{m_1 m_E}{r^2}\mathbf{u}_r = m_1\mathbf{g}_0 \tag{3}$$

or

$$\mathbf{g}_0 = G\frac{m_E}{r^2}\mathbf{u}_r \tag{4}$$

We seem to have to assume that all of the Earth's mass is concentrated at its center. But it turns out that the attraction of gravity for a homogeneous sphere is the same as a point mass. So we can state that the mass of the Earth is equal to the volume of a sphere with radius (R_E) equal to that of the Earth times the Earth's mean density ($\bar{\rho}$).

$$m_E = \tfrac{4}{3}\pi R_E^3 \bar{\rho} \tag{5}$$

So if you are standing on the surface of the Earth, then

$$\mathbf{g}_0 = \tfrac{4}{3}\pi R_E G \bar{\rho}\mathbf{u}_r \tag{6}$$

How can we measure the gravity field of the Earth? A simple pendulum when set in motion will have a period T

$$T = 2\pi\sqrt{\frac{L}{g_0}} \tag{7}$$

Note without the **bold**, we simply mean the magnitude of a vector. Here L is the length of the pendulum. So with a pendulum you can determine g_0 and estimate the mean density of the Earth, $\bar{\rho}$. What happened when this experiment was performed and the mean density turned out to be 5500 kg/m^3 = 5.5 gm/cm^3? You can turn this around and calculate:

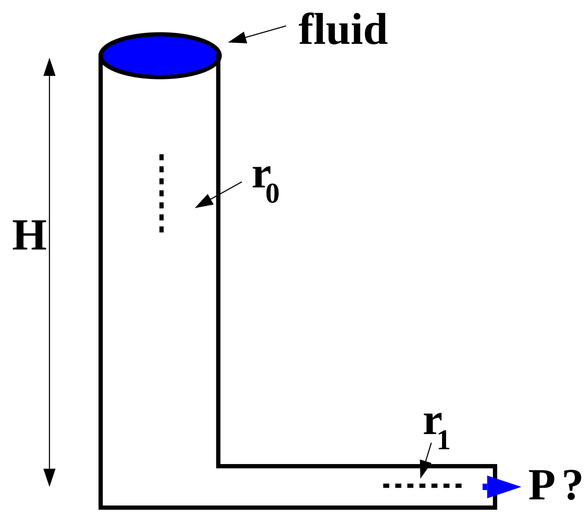
$$g_0 = \tfrac{4}{3}\pi R_E G\bar{\rho}$$
$$= \tfrac{4}{3}\pi \cdot 6.6371\text{x}10^6\,\text{m} \cdot 6.6726\text{x}10^{-11}\text{m}^3\ \text{kg}^{-1}s^{-2} \cdot 5500\text{kg m}^{-3} \qquad (8)$$
$$= 9.8\ \text{m s}^{-2}$$

Remember that g_0 is an acceleration but provides a force to a mass via

$$\mathbf{F} = m\mathbf{g}_0$$

Other Kinds of Forces:

Pressure



$$\text{Mass in column} = \pi r_0^2 H \rho_{\text{H}_2\text{O}} = M_c$$

$$\text{Force in column} = M_c g_0 = \pi r_0^2 H \rho_{\text{H}_2\text{O}} g_0 \quad \text{[Newtons (N)]}$$

Pressure = force/unit area

$$P = \frac{\pi r_0^2 \rho_{\text{H}_2\text{O}} g_0 H}{\pi r_0^2} = \rho_{\text{H}_2\text{O}} g_0 H \ \frac{\text{Newtons (N)}}{\text{m}^2} \qquad (9)$$
$$= \text{Pascals (Pa)}$$

Pressure at outlet is

$$P_{\text{out}} = \frac{\pi r_0^2 \rho_{\text{H}_2\text{O}} g_0 H}{\pi r_1^2} = \rho_{\text{H}_2\text{O}} g_0 H \left(\frac{r_0}{r_1}\right)^2 \qquad (10)$$

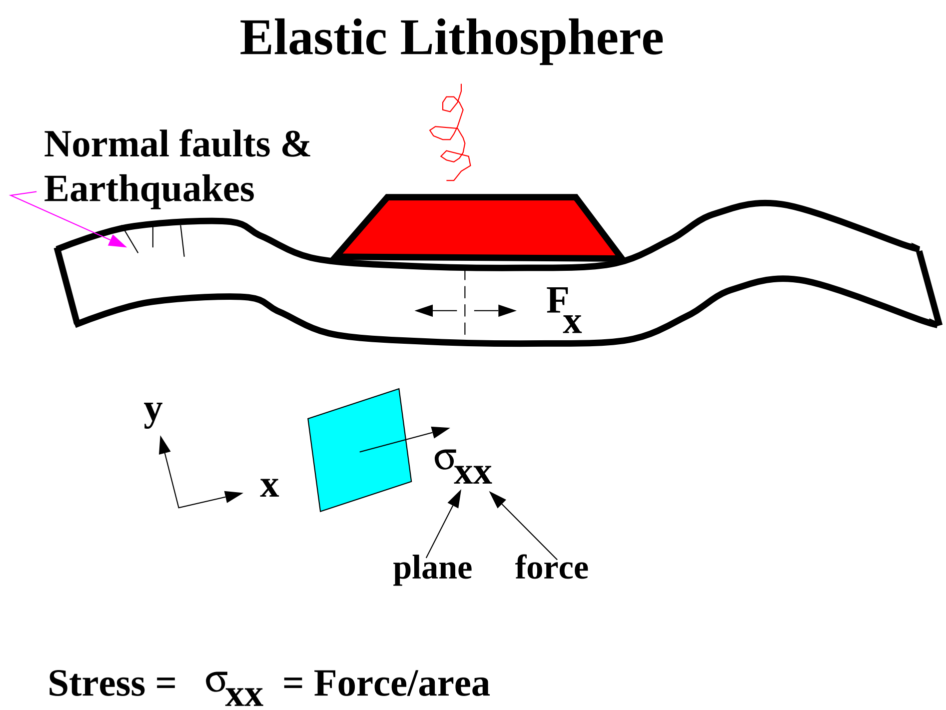
Often we assume that the pressure in the Earth is "hydrostatic", and in all directions at depth z is given by:

$$P = \rho_{\text{rock}} g_0 z \tag{11}$$

Why would rocks in the Earth behave like a hydrostatic fluid?

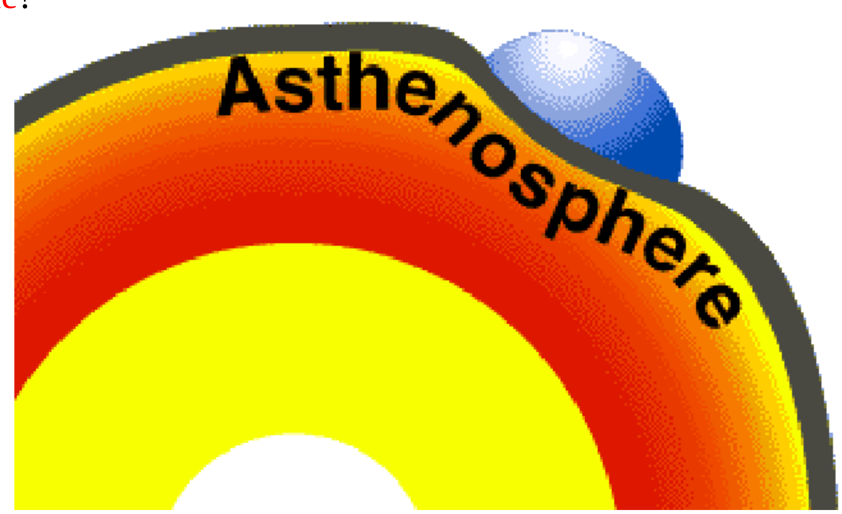
Elastic Forces

What is a lithosphere? What do we mean by elastic?



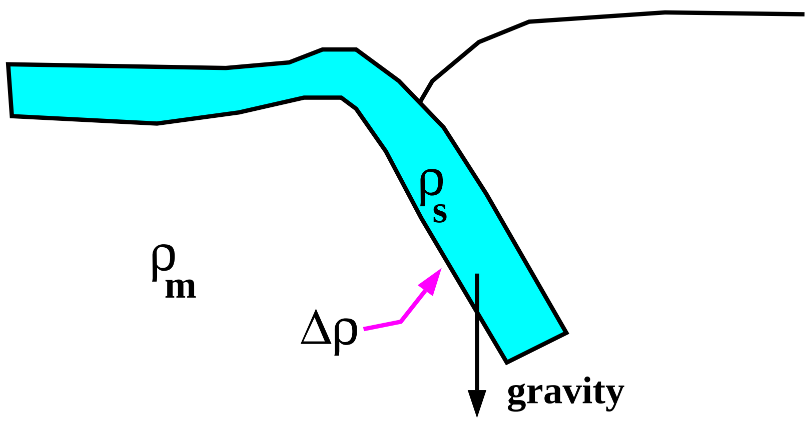
What is the stress σ_{xx}?

What about the mechanical lithosphere, the seismic lithosphere, the thermal lithosphere? What about the asthenosphere, the low velocity zone, the low viscosity zone?



Buoyancy Forces

Subducting Slab



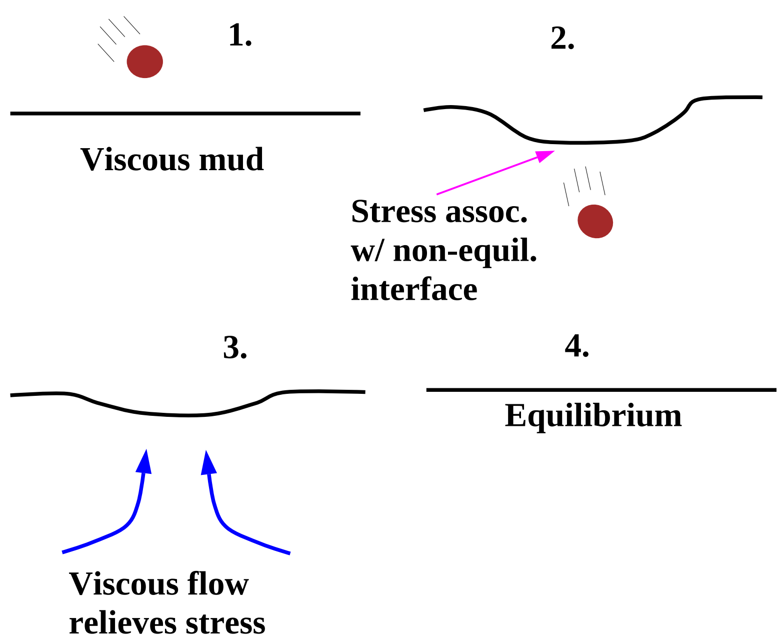
The slab provides a downward force:

$$\mathbf{F} = (\text{volume of slab}) \cdot \Delta\rho \mathbf{g}_0 \tag{12}$$

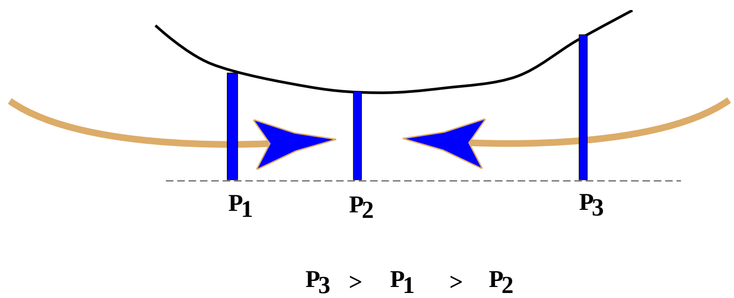
Why $\Delta\rho$? What is buoyancy? Gravity seems to be driving this system, but what is the real source of energy?

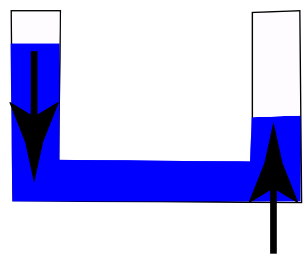
Viscous Forces

Density interfaces in fluid will tend towards horizontal (= perpendicular to gravity vector).



This can be understood in the context of differential pressures that lead to horizontal stresses that drive flow. Flow will always strive to make fluid in "columns" of equal height. <u>How is the viscosity of the mantle determined?</u>





Viscous Flow

Viscous folding

Where does force come from?

Conclusion: Gravity is the force driving almost everything, but heat sources in the Earth are the ultimate culprits.

Energy (Joules = J)

Thermal energy

(Temperature in Kelvin; K = C +273^{o})

Heat a mass, *m*, of rock by an amount ΔT, then the thermal energy is

$$E = mC_p \Delta T \tag{13}$$

where C_p is specific heat = amount of energy it takes to raise 1 kg of stuff by 1 degree K. The subscript "p" denotes that temperature is changed at constant pressure. The units of C_p are obvious:

$$E = \text{kg} \cdot \frac{\text{J}}{\text{kg K}} \text{K} = \text{J}$$

Note also that

$$E = \rho C_p \Delta T = \frac{\text{kg}}{\text{m}^3} \cdot \frac{\text{J}}{\text{kg K}} \text{K} = \frac{\text{J}}{\text{m}^3}$$

is thermal energy per unit volume.

Coefficient of thermal expansion

The coefficient of thermal expansion is extremely important as it ultimately relates thermal energy to gravity forces. Almost every geological material will expand on heating and contract on cooling (exception = ??). We measure this with a coefficient of thermal expansion, α. We ask: What is the change in density of a rock due to a temperature increase ΔT? What is the functional relationship $\rho = f(\rho_0, \Delta T)$ where ρ_0 is the density at the "reference state" and ρ is the new density after an increase in temperature, ΔT? Upon heating, the mass of a rock stays the same, but the volume increases. Consider a cube of dimension L_0 on a side. In one direction, upon heating, the length is increased by

$$L = L_0 \left(1 + \alpha_l \Delta T\right) \tag{14}$$

where α_l is the *linear coefficient of thermal expansion* and is simply a measure of the relative change in length per degree of temperature change. The new cube volume must then be:

$$V = L_0^3 \left(1 + \alpha_l \Delta T\right)^3 \approx L_0^3 \left(1 + 3\alpha_l \Delta T\right) \equiv L_0^3 \left(1 + \alpha \Delta T\right) \tag{15}$$

The approximation can be made if $\alpha_l \Delta T$ is small compared to one. We also define the *volume coefficient of thermal expansion*, α, as 3 times the linear coefficient. In terms of density:

$$\rho_0 = \frac{m}{L_0^3} \tag{16}$$

and

$$\rho = \frac{m}{L^3} = \frac{m}{L_0^3\left(1+\alpha\Delta T\right)} \approx \rho_0\left(1-\alpha\Delta T\right) \tag{17}$$

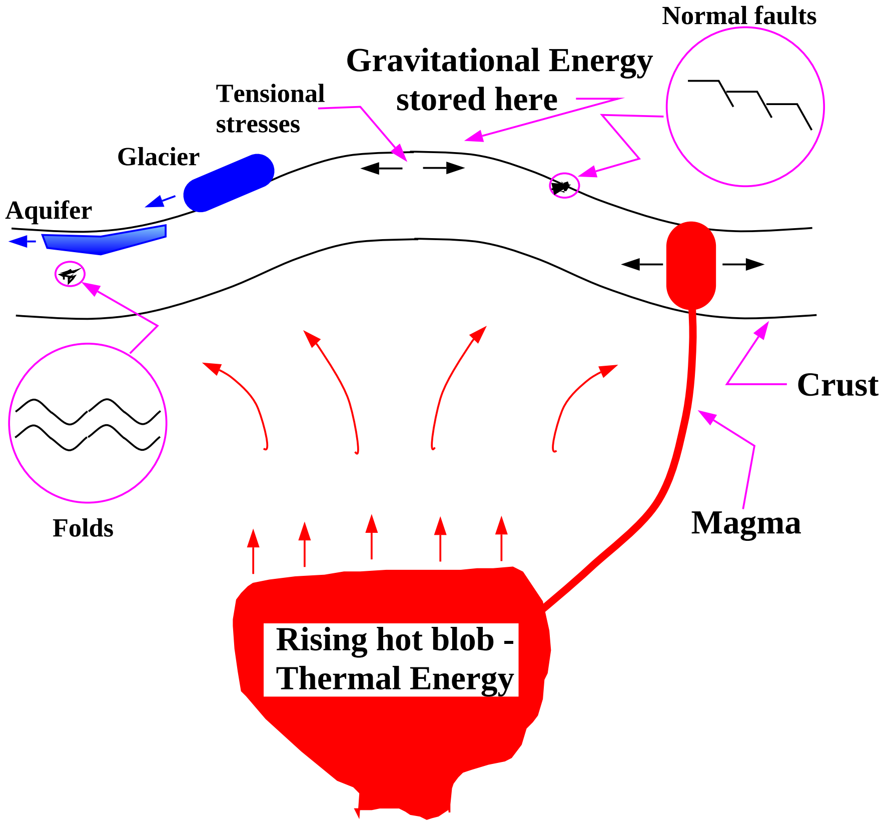
The approximation comes about by using a two-term *Taylor series expansion*, which is made assuming $\alpha\Delta T$ is small. For most rocks, $\alpha \approx 3 \times 10^{-5}$ K^{-1}, so even with an increase in temperature of 1000 K, $\alpha\Delta T$ is only about 0.03. Now consider a blob of hot stuff in the Earth of volume V. The density contrast between the blob and its surroundings is given by

$$\Delta\rho = \left[\rho_0\right] - \left[\rho_0\left(1-\alpha\Delta T\right)\right] = \rho_0\alpha\Delta T \tag{18}$$

and there will be a net upward buoyancy force

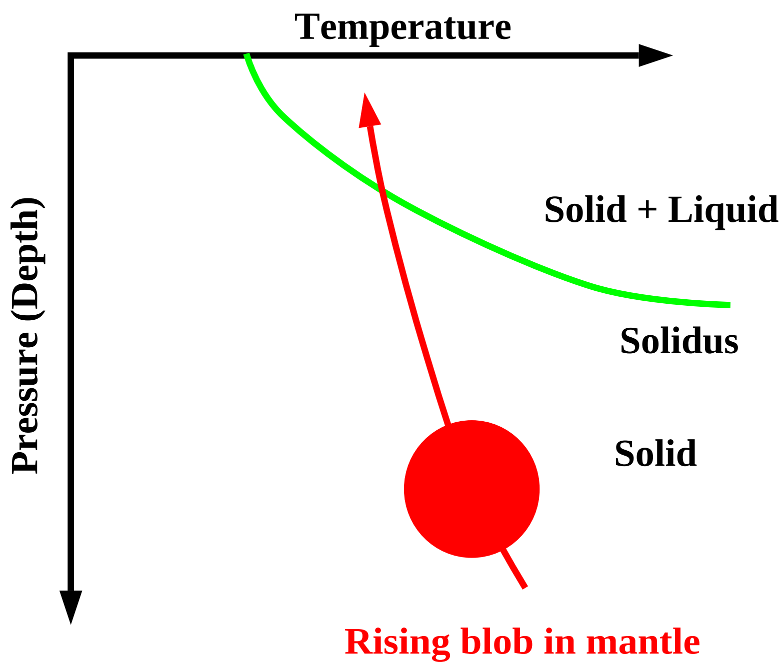
$$\mathbf{F} = m\mathbf{g}_0 = V\rho_0\alpha\Delta T\mathbf{g}_0 \tag{19}$$

We have turned thermal energy into a gravity (buoyancy) force!! The big picture is shown below.



Thermal energy gives rise to gravity energy, which in turn leads to various kinds of geological forces.

There is another source of buoyancy in geological materials: Rocks melt! Melts are almost always less dense than the rocks they come from, so they tend to rise buoyantly. This assumes that they have a pathway, such as fractures, to do so. Sometimes melts move upward by assimilation of the *country rock*. The temperature at which a rock first begins to melt is called the *solidus temperature* or just plain *solidus*. This depends on pressure; in general, the higher the pressure, the higher the solidus temperature. Pressure increases with depth in the Earth and is pretty well represented by $P = \rho g_0 z$, where z is depth in the Earth. (There's that assertion again that the Earth behaves like a hydrostatic fluid!)



Pressure-Release Partial Melting

In the figure above, a hot blob of solid material is rising buoyantly in the Earth. We can plot a curve that shows solidus temperature versus pressure (or depth). At first it is too deep for melting to occur. Then the blob crosses the solidus and melting can commence because of the lower pressure. This process is called *pressure release partial melting*. It is a very important process in the earth. For Example, it is responsible for the creation of oceanic crust.

Gravitational Energy

Work = Force x distance = Energy. Between points 1 and 2 on a path *S*:

$$W = \int_1^2 \mathbf{F} \cdot d\mathbf{S} \qquad (20)$$

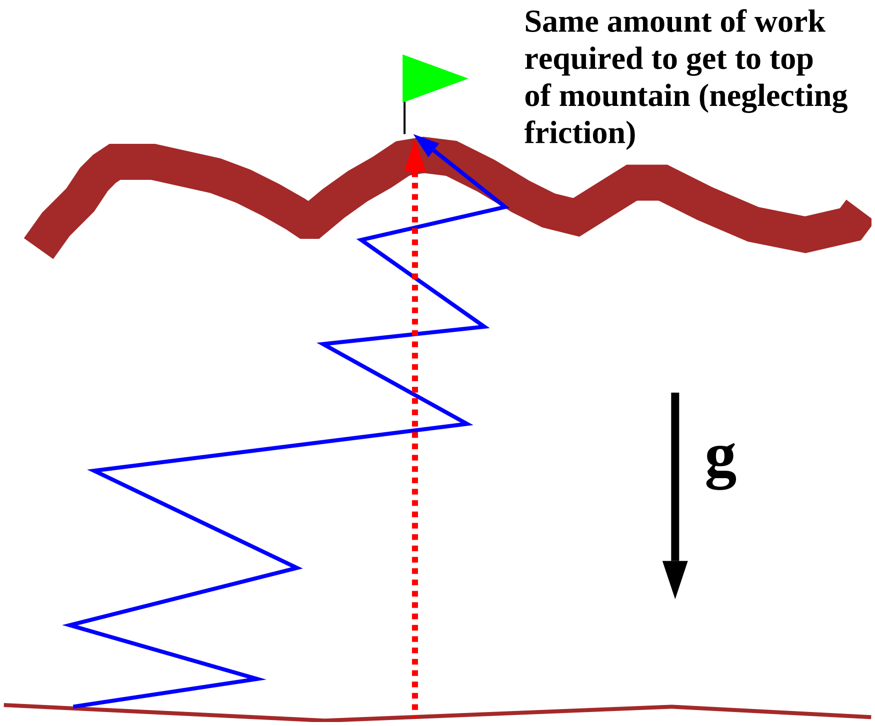
Why is their a dot product in the integrand? Now let $\mathbf{F} = m\mathbf{g_0}$

$$W = \int_1^2 m\mathbf{g}_0 \cdot d\mathbf{S} \qquad (21)$$

Let's define Φ as work per unit mass in a gravity field. This is called *gravitational potential energy* or just *gravitational potential*.

$$\Phi = \frac{W}{m} = \int_1^2 \mathbf{g}_0 \cdot d\mathbf{S} \qquad (22)$$

In general, Φ belongs to a general class of functions called *potential fields*. (**What is a field?**) You will discover that potential fields permeate my part of the course. **THEY ARE EVERYWHERE!** For every potential field, there is a corresponding force field. A most important property of potential fields is that it does not matter what path you take when integrating from 1 to 2. The value of the integral depends only on the end points of the integration. This is illustrated physically in the figure below.



We might as well write Earth's gravity as a scalar function with only a *z* component. (Gravity is a vector with *x*, *y*, and *z* components. But we are free to choose a coordinate system such that the *z* axis coincides with the gravity vector. Then that will be the only component.) The potential energy gained in going from level $z = 0$ to the top of the mountain at $z = H$ is then found by the simplest path:

$$\Phi = \int_0^H g_0 dz \qquad (23)$$

When you climb the hill, you have to expend work to overcome the force of the gravity field. You have gained gravitational potential energy. If forces in the Earth can somehow create a hill or mountain, then the hill or mountain has "stored" gravitational potential energy. This energy is available to do geological mischief!

Now it turns out that it does not matter which path you take to get to the top of the mount: you expend the same amount of energy (i.e., you have to do the same amount of work). A *potential* field is defined as one in which the work required to move around in the presence of the field does not depend on the path we choose to take. Therefore, the gravity field must be a potential field.

To proceed, we have to introduce the concept of *partial derivatives*.

There are many instances in the study of Earth Forces where a quantity is a function of more than one parameter. Consider a scalar function $w = f(x, y)$ (e.g., let w be the topography, defined at every point on a reference plane; the reference plane is normally sea level). We can make a contour map of this function in the x-y plane. Also, we can take the derivative of the function in any desired direction with vector calculus (= directional derivative = slope). Most important are the partial derivatives, which tell how the function varies with respect to changes in only one of its controlling variables: In the x direction, define:

$$\frac{\partial w}{\partial x} = \lim_{\Delta x \to 0} \frac{f(x + \Delta x, y) - f(x, y)}{\Delta x} \tag{24}$$

Similarly, in the y direction the derivative is:

$$\frac{\partial w}{\partial y} = \lim_{\Delta y \to 0} \frac{f(x, y + \Delta y) - f(x, y)}{\Delta y} \tag{25}$$

We can use partial derivatives to talk about the change of a function with direction in terms of a vector. For some arbitrary scalar $\varphi(x, y, z)$ we can define a vector, **F** in terms of components along the three Cartesian directions **i**, **j**, **k**:

$$\mathbf{F} = \frac{\partial \varphi}{\partial x}\mathbf{i} + \frac{\partial \varphi}{\partial y}\mathbf{j} + \frac{\partial \varphi}{\partial z}\mathbf{k} \tag{26}$$

made up of the components of change of φ in the x, y, and z directions; i.e., the directional derivatives in the x, y, and z, directions. In fact, **F** gives the magnitude and direction of the maximum change of φ. In two dimensions, if φ is topography, then **F** gives the maximum slope and its direction. The operation on φ to obtain **F** has a special name: ***F*** *is the gradient of* φ, denoted by

$$\mathbf{F} = \nabla\varphi \tag{27}$$

where ∇ is known as the "del" operator (or "that funny upside down triangle"). As a "hungry" operator, we write

$$\nabla = \left(\frac{\partial}{\partial x}\mathbf{i} + \frac{\partial}{\partial y}\mathbf{j} + \frac{\partial}{\partial z}\mathbf{k} \right) \tag{28}$$

The gradient is an extremely important concept here; e.g., water flow is driven by pressure gradients, and heat flow is proportional to the gradient of temperature.

Back to gravity. There is a nice theorem that says that if we integrate the gravity field along a path S between 1 and 2

$$\Phi = \int_1^2 \mathbf{g} \cdot d\mathbf{S} \tag{29}$$

and Φ is a potential field (i.e., the integral is independent of path) then

$$\mathbf{g} = \nabla\Phi \tag{30}$$

Two points here: this works for any gravity field, not just the mean or average field $\mathbf{g_0}$, and this theorem works for any potential field. We will see this relationship over and over in Earth Forces for many different kinds of potential fields.

Think of these *vector calculus* operations as just a shorthand notation for some differential operators that have direction. I do not expect to you to become proficient in vector calculus. I do expect you to understand what the operations mean physically.

We can return to the scalar average field, g_0 This is the average attraction of the Earth, which by definition is constant in any horizontal direction, but varies in the vertical direction. This is because it is given by GM/R^2. It's potential must be

$$\Phi = g_0 z \tag{31}$$

How do we know that? Because

$$g_0\mathbf{k} = \nabla\Phi \tag{32}$$

or

$$\mathbf{g}_0 = \nabla\Phi = \left(\frac{\partial\Phi}{\partial x}\mathbf{i} + \frac{\partial\Phi}{\partial y}\mathbf{j} + \frac{\partial\Phi}{\partial z}\mathbf{k}\right) \tag{33}$$

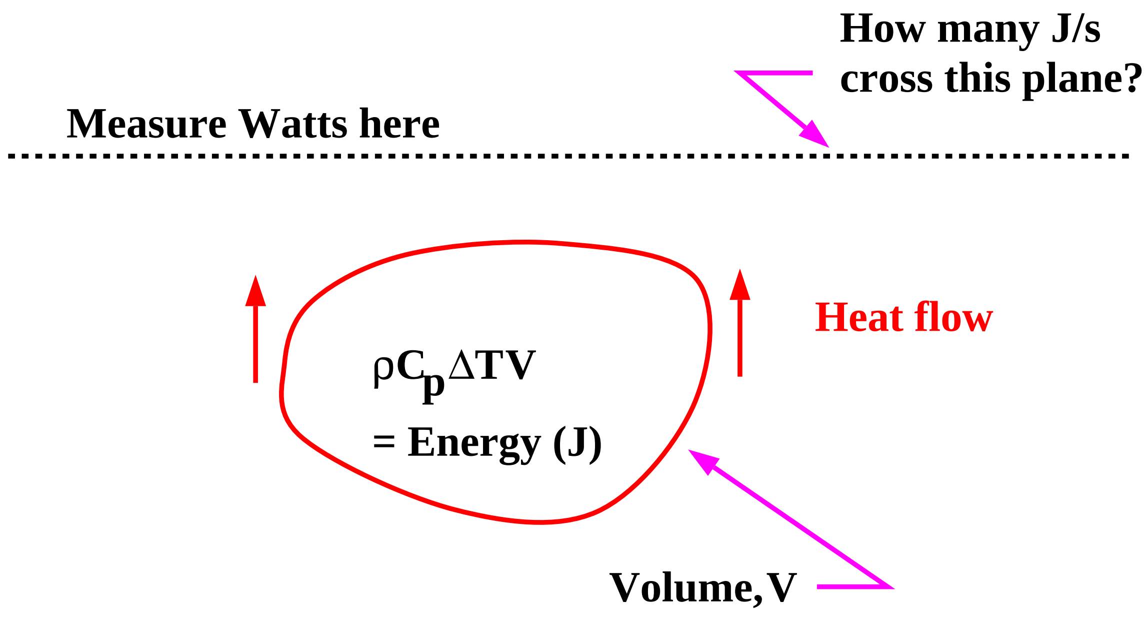
But Φ can only vary in the vertical direction, so

$$\begin{aligned} \mathbf{g}_0 = g_0\mathbf{k} = \nabla\Phi &= \left(0\mathbf{i} + 0\mathbf{j} + \frac{\partial\Phi}{\partial z}\mathbf{k}\right) \\ &= \frac{\partial(g_0 z)}{\partial z}\mathbf{k} = g_0\mathbf{k} \end{aligned} \tag{34}$$

Since Φ is gravitational potential energy per unit mass, then the potential energy gain for a mass m increasing its elevation by z is mg_0z, and the gain in potential energy per unit volume is $\rho g_0 z$. So if you know your mass (or average density), then you can figure out how much potential energy you will gain by climbing to the top of the mountain.

What about power? Power (energy per unit time = J/s = Watts = W)

Concept of power as the "motion" of energy:



The basic quantity that indicates how much thermal energy (the unity of energy is the *Joule, J*) is contained in a substance is the *specific heat*, C_p. The subscript "p" indicates that the quantity is measured at constant pressure. The units of C_p are Joules per kilogram per degree Kelvin (J/kg-K). So if the temperature of a substance is raised ΔT, then 1 kg of this stuff increases its energy by $C_p\Delta T$. If the density of this material is ρ, then one meter cubed increases its energy by $\rho C_p\Delta T$. And a volume V increases its energy by $\rho C_p\Delta T\ V$. If this blob of stuff crosses some boundary at a given rate, then a certain number of J/s (= Power = watts = W) cross the boundary. This is the major way the interior of the Earth transports heat!

One example of heat transport:

Assume radiogenic isotopes in the Earth produce Q W/m^3. What is the heat flux or flow across the surface of the Earth? Total generated is

$$Q \cdot \tfrac{4}{3}\pi R_E^3 \tag{35}$$

Then the flux (something flowing across a real or imaginary boundary per unit cross sectional area) is

$$q = \frac{Q \cdot \tfrac{4}{3}\pi R_E^3}{4\pi R_E^2} = Q\frac{R_E}{3} \quad \text{W m}^{-2} \tag{36}$$

Heat flow at the surface of the earth is usually expressed as mW/m^2. The average for the Earth is about 82 mW/m^2. **Is this equal to the total heat generated by isotopes?** No! **Why not?**

Across the outer part of the Earth, heat is transported by *conduction*. The rate is expressed by *Fourier's Law*:

$$\mathbf{q} = -K\nabla T \tag{37}$$

where K is the thermal conductivity. Oops, there's that gradient operator again.

What are the different ways that heat is transported?