Convection in the Earth and Other Planets

Introduction

What causes convection?

Unstable buoyancy

Region must act like a fluid.

How can you find out whether unstable buoyancy will be maintained (metastability)?

How can you find out whether or not fluid motion will be maintained?

The Lipton Cup-A-Soup experiment:

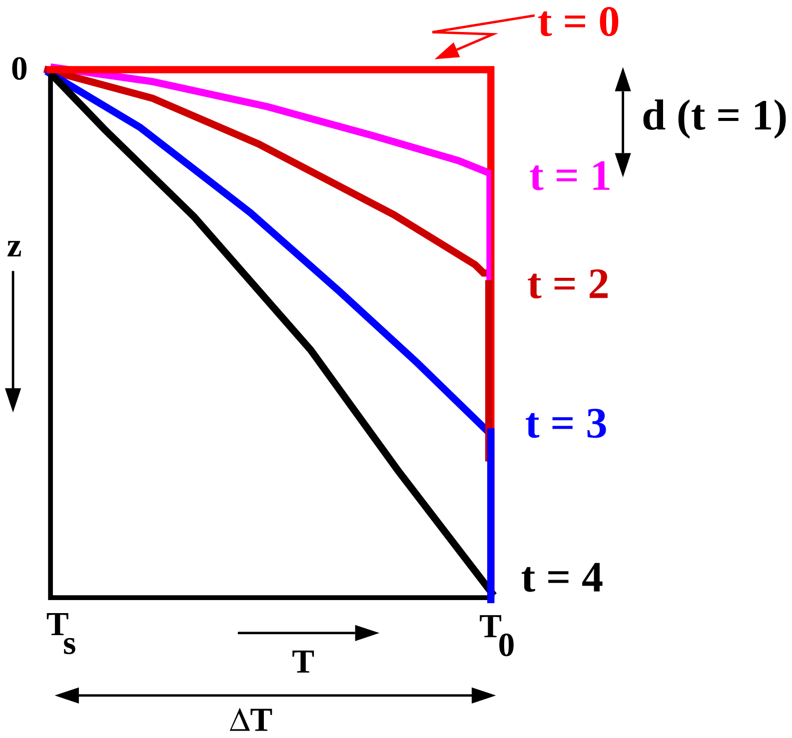


Figure 1

The temperature in the cup is given approximately by

$$T(z,t) = T_0 + (T_s - T_0)\,\mathrm{erfc}\left(z/\sqrt{4\kappa t}\right)$$

where T_0 is the original temperature (non-boiling) of the soup after you remove it from a burner and T_s is the surface temperature of the soup. Obviously, the cooling Cup-A-Soup develops a temperature gradient, ΔT, and a region near the top, $d(t)$, develops with a strong temperature gradient. This region becomes buoyantly unstable and starts to convect. The noodles act as keen tracers of the convective motion. There are several cells across the cup, which implies that convection does not extend to the bottom of the cup because convection cells tend to be equidimensional. **This may be exactly analogous to the primary mechanism for driving convection in the core!**

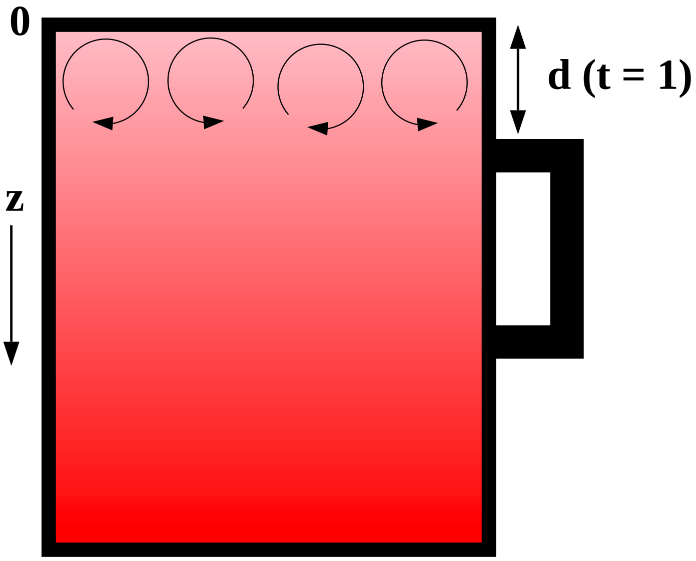


Figure 2

Convection takes place when hot material replaces cold material at the top and the cold material then sinks downward at a sufficient rate to keep up with the flow timescale of the problem. This is governed by the amount of time it takes for the top of the cell to cool off sufficiently (by diffusion) to become buoyantly unstable and sink.

Convection is a natural phenomenon that occurs in many situations: In the mantle and core, in the ocean, in the atmosphere, in fluids in the crust, etc. **It is the**

primary way in which the Earth transports its thermal energy.

Convection Behavior

Whether or not convection takes place depends on the *Rayleigh number*, Ra:

$$\mathrm{Ra} = \frac{\rho_0 g_0 \alpha \Delta T d^3}{\kappa \eta} = \frac{\rho_0 g_0 \alpha \Delta T d}{\kappa \eta / d^2} = \frac{\text{Buoyant Pressure}}{\text{Viscous Prssure}}$$

where ρ_0 = density, g_0 = gravity, α = volume coefficient of thermal expansion, ΔT = vertical temperature contrast across convecting system, d = vertical dimension of convecting system, κ = thermal diffusivity, and η = viscosity.

The higher value of Ra, the more likely for the buoyant movement of fluid to overcome viscous “resistance”. **Note also that both terms in denominator of Ra inhibit convection. Why?**

An alternative form of the Rayleigh number is in terms of heat production Q and thermal conductivity, K:

$$\mathrm{Ra} = \frac{\rho_0 g_0 \alpha Q d^5}{\kappa \eta K}$$

How can we tell if the mantle will actually convect? We see that the Rayleigh number is the ratio of buoyant pressure, tending to encourage flow, and viscous pressure, tending to resist flow. Obviously, there are Rayleigh numbers so low that viscous pressure wins and convection does not take place. And there are Rayleigh numbers so high that convection takes place readily. There is obviously a value of Ra, called the *critical Rayleigh number*, Ra_c, which is the boundary between these two regimes. That is, when $\mathrm{Ra} = \mathrm{Ra}_c$, then convection is just barely possible. The first thing one wants to ask when examining a natural system is the following: Does Ra exceed Ra_c? The critical Rayleigh number for convection in planets is about 1000. What one does is plug in values for d, Q, η, κ, etc. for a region in question to estimate Ra, and then see if the calculated value exceeds Ra_c. The determination of mantle viscosity from glacial rebound data allowed an accurate estimate of the Rayleigh number of the Earth's mantle (other factors in Ra were much better known). The mantle was found to be unstable to convection. The same is true for the mantles of the other terrestrial planets. **For the Earth's mantle, Ra is at least 100,000 times critical!!**

We test for stability against convection by introducing a small velocity disturbance of wavelength λ into a fluid of vertical extent *d*. We see if the disturbance grows in amplitude (convection develops) or dies out (convection does not develop). Figure 3 below shows the critical Rayleigh number plotted against the ratio of $2\pi d/\lambda$.

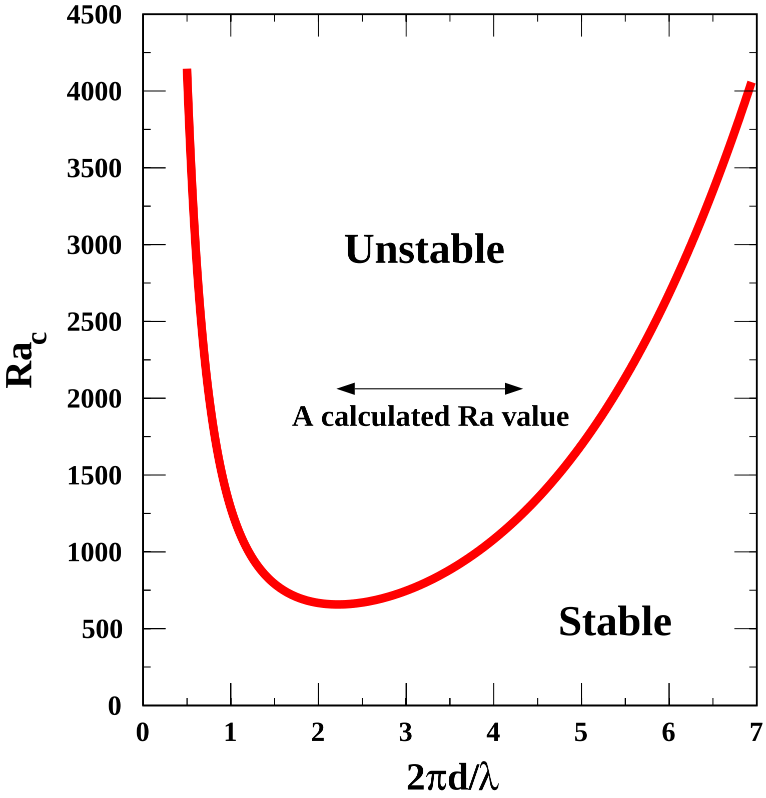


Figure 3.

The Structure of Mantle Convection

Below is schematic diagram for convection by heating from below. The temperature distribution is characterized by hot ascending plumes and cold descending plumes. The center of the region is nearly isothermal.

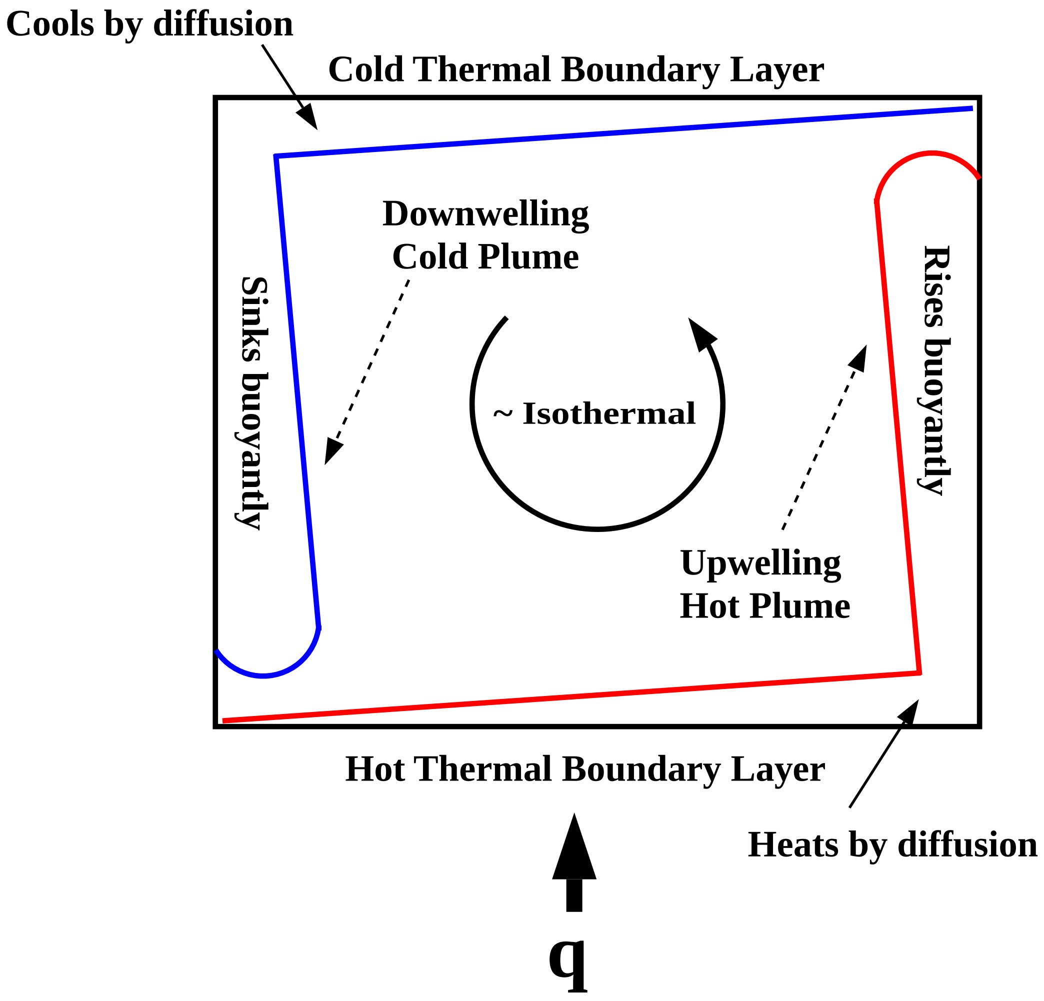
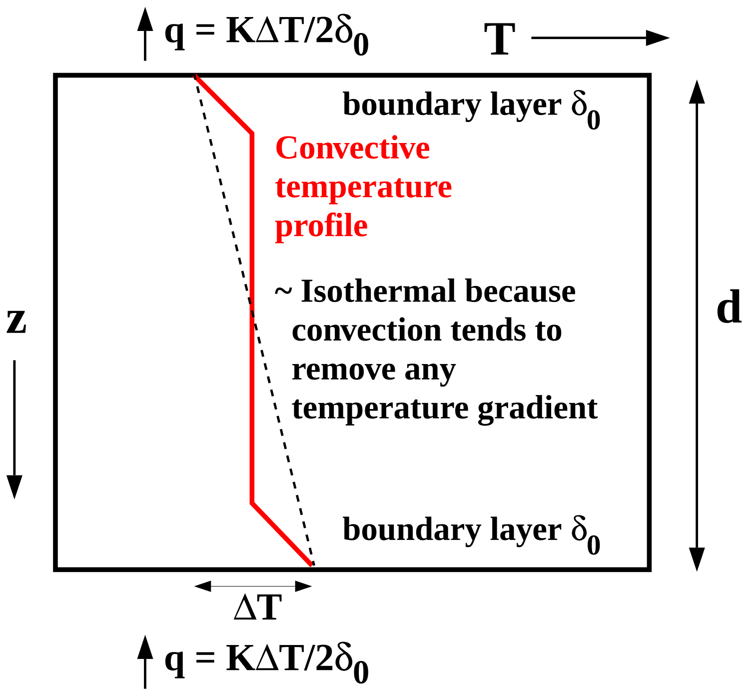


Figure 4

When a horizontal average of temperature is taken in the convection solution above, the *thermal boundary layers* are obvious (Figure 5). A thermal boundary layer exists wherever there is a boundary that heat cannot be transported across convectively. The core-mantle boundary has a thermal boundary layer (from which plumes arise), and the lithosphere, at the outer boundary of the solid Earth, is another thermal boundary layer. **Heat is transported conductively across thermal boundary layers.**



Conductive heat flux:
$q = K\, \Delta T/d$

Figure 5

In all of the terrestrial planets, except Earth, convection cannot break the surface (and cause plate tectonics). Convection takes place beneath a *stagnant lid* (Figure 6).

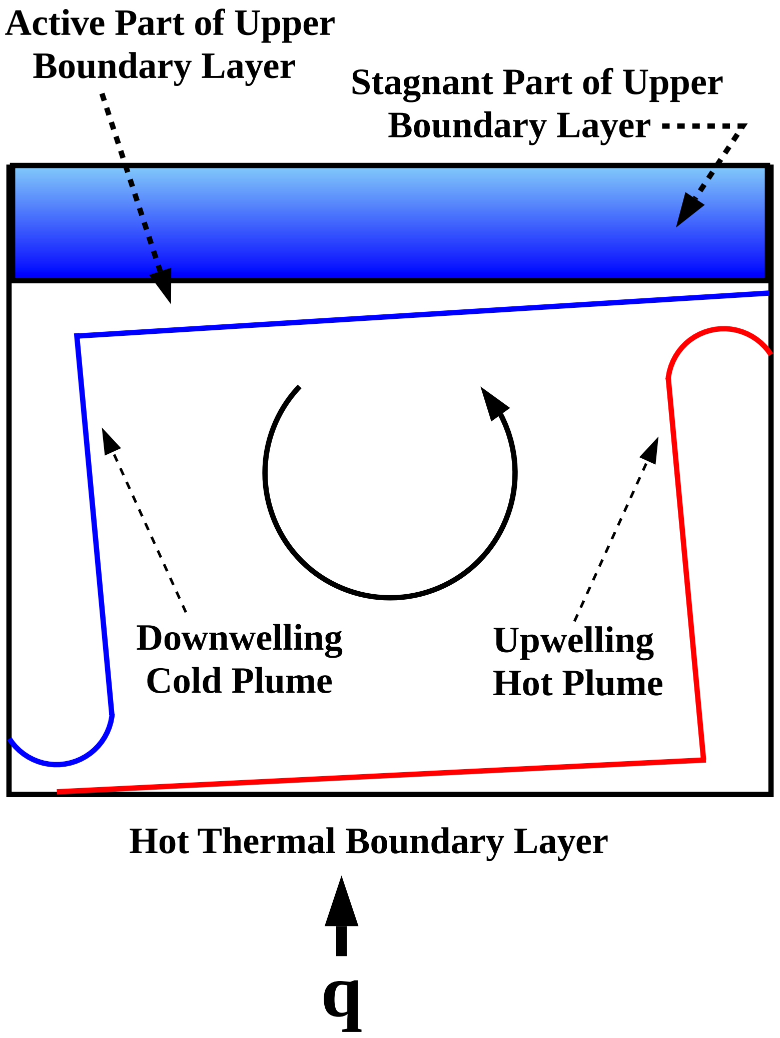


Figure 6

Below is a sketch (Figure 7) of convection from internal heating. Because there is no heat coming across the lower boundary, there is no thermal boundary layer at the bottom, and no hot rising plumes. Most geoscientists agree that mantle convection is driven almost entirely by internal heating with a small amount (< 10%) driven by heat coming across the core-mantle boundary.

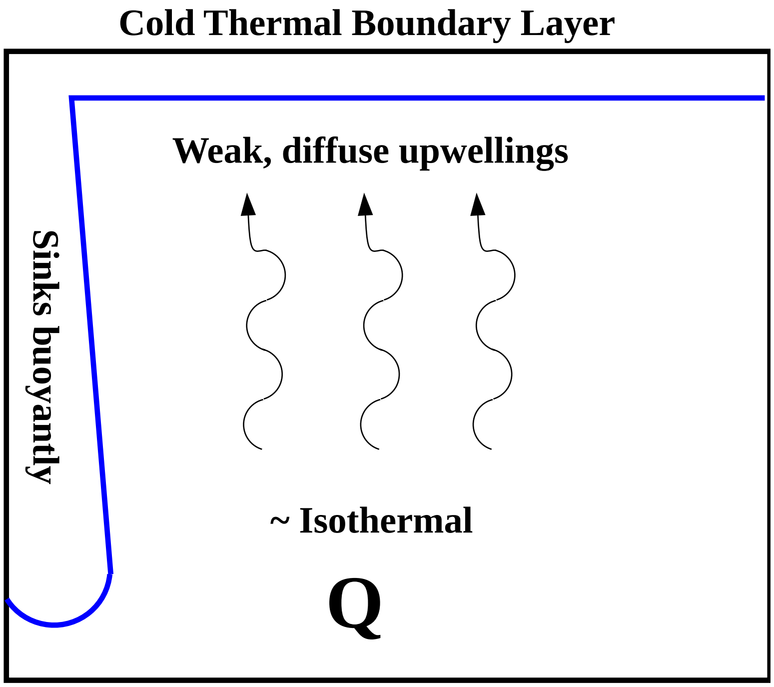


Figure 7