

EPSc 353 – Earth Forces
Plate Flexure at Subduction Zones
Due Monday March 15, 2004

Introduction

Treating the Earth's lithosphere as a thin elastic plate, and treating its deformation as flexure of a beam can explain a number of geologic phenomenon. This type of behavior is observed at different geologic settings around the globe. For example, surface loads, such as the Hawaiian Islands, bend the surrounding lithosphere. There is also observational evidence of elastic bending of the lithosphere at subduction zones. The flexure equation that describes this type of behavior is:

$$D \frac{d^4 w(x)}{dx^4} + P \frac{d^2 w(x)}{dx^2} + \Delta \rho g_0 w(x) = q_a(x)$$

A derivation of this equation can be found in Roger Phillips on-line course notes located on the course web page. This equation can then be solved for the deflection, $w(x)$, in response to an applied load, q_a , and in in-plane force P . The solution is found by integrating the above equation four times and solving for the four constants of integration. The values of the constants of integration will be determined by the boundary conditions applied to the problem. The boundary conditions are determined by the type of geologic setting. Therefore it is clear that this equation can be used to solve for the bending of the lithosphere for many different types of geologic settings. The flexure equation is arguably the single most important equation for studying the geodynamical deformation of the lithosphere in response to loads and in-plane forces.

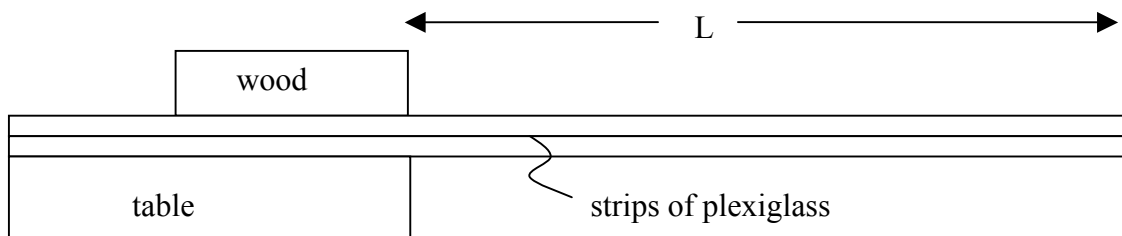
The purpose of this lab is to simulate the flexure of a lithospheric plate at a subduction zone with strips of acrylic plexiglass. We will do this by placing a load q_a at some length L onto a strip of plexiglass and measuring the vertical displacement or deflection $w(x)$ of the strip. In reality, the density contrast between the material over a plate (water) and the material under a plate (mantle or asthenosphere) must be taken into account. However in our case, there is NO contrast; air is the material both above AND below our "plate." Because of this, the usual equation for determining flexure must be modified. The derivation of the modified flexure equation can be found at the end of Roger Phillips' EPSc 353 notes located on the course web page. The modified flexure equation for this lab is:

$$w(x) = \frac{q_a x^2}{2D} \left(L - \frac{x}{3} \right)$$

$w(x)$ = vertical displacement or deflection
 q_a = applied load = mass * g / width of strip
 x = horizontal distance
 L = length of the ruler
 D = flexural rigidity
 g = gravitational acceleration

Procedure

This experiment will be carried out on strips of acrylic plexiglass with different thicknesses in order to determine the dependence of flexural rigidity on the thickness. Pick two strips of plexiglass of equal thickness. Measure the thickness to two significant figures. Place the two strips of acrylic plexiglass back to back and clamp them to the edge of a lab table using a block of wood so that the block of wood is in line with the edge of the table and the hole located at the end of the strip hangs off the edge of the table.



The two strips of plexiglass should now be hanging off of the edge of the table some distance L . Make sure that the deflection between the two strips of plexiglass is initially zero. Tape a flexible measuring tape to the top of the upper strip so that the zero point is at the edge of the block of wood and the table. For the thinner sheets of plexiglass, position the strips so that $35 < L < 40$. For the thicker strips, $40 < L < 45$. Measure this distance L . Now apply a load to the end of the lower strip by filling a metallic bucket with sand and hanging it through the drilled hole at the end of the strip with an S-hook. The deflection at the end of strip should be $\sim 10\text{cm}$ for the thicker strips of plexiglass and $\sim 15\text{cm}$ for the thinner strips. Now, quantify the amount of deflection $w(x)$ by measuring the distance between the bottom of the upper strip and the top of the lower strip at close intervals a distance x away from the edge of the wood block. Do this so you have enough measurements to estimate the flexural rigidity D . After measuring the deflection, weigh and record the mass of the applied load (bucket + sand). Describe what happens to the lower plate after removing the load. Once you have performed the above procedure for one thickness, grab two strips of the other thickness and repeat.

Analysis

1. By way of a dimensional analysis, calculate the units of D , the flexural rigidity. Show your work.
2. Estimate the value of D for both the thin sheet of acrylic plexiglass and the thick sheet.

3. The flexural rigidity, D , of a material is equal to the thickness of the material, h , raised to an integer power times k where k is a constant dependent on the physical properties of the material.

$$D = kh^n \text{ where } k = \frac{E}{12(1-\nu^2)}$$

D = flexural rigidity

h = thickness

E = Young's Modulus

ν = Poisson's ratio

What integer power of h best satisfies this equation? Show your work.

4. What happens to the lower sheet of plexiglass after removing the load q_a from the end of the strip? Does it return to its initial position? What does this say about the assumption used to derive the flexure equation that the material is perfectly elastic? How does this affect the calculations made in problem three? What other sources of error can you think of?

****Notes from 2004 Lab****

- The concept of measuring the deflection between two suspended strips of plexiglass worked excellent because it eliminated the deflection due to gravity so that the measured deflection was a function of the applied load only.
- The assumption that the acrylic plexiglass is perfectly elastic is only really valid for small applied loads q_a (by small applied loads I mean loads that give a deflection of only a few inches at the end of the strips). Larger loads resulted in more of a viscoelastic rheology that introduced some error into the results. However, the load must be sufficient enough for the students to be able to make accurate deflection measurements.
- One group of students tried placing a piece of paper behind the sheets of plexiglass and tracing the deflection of the sheets onto the paper. They then took the paper and measured the deflection with a ruler on a flat table. This seemed to work well and also eliminates some of the error involved with trying to hold a ruler steady next to the plexiglass sheets to measure the deflection.
- Question three needs to be re-worked. One of the objectives here was to show the students that the flexural rigidity is dependent on the thickness (ie rigidity is proportional to the thickness cubed) from the data. Most of the students just ignored their data and used a dimensional analysis to calculate the proportionality between rigidity and thickness, but this is my fault for wording the question the way I did.
- This lab could be significantly improved by adding another thickness or two to the data set.
- Feel free to use this write-up as guide to rewrite with your own modifications and improvements.