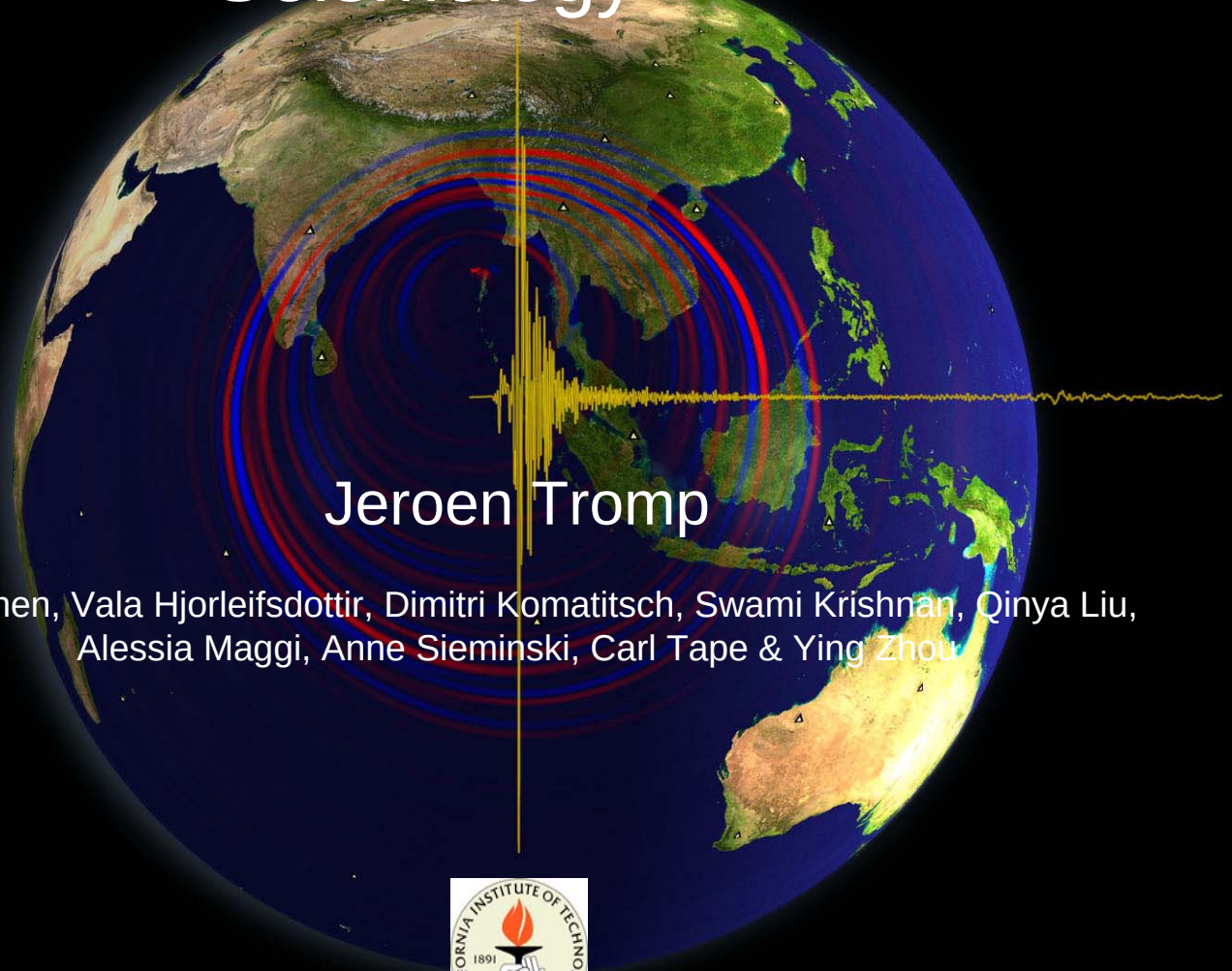
Spectral-Element and Adjoint Methods in Seismology



Jeroen Tromp

Min Chen, Vala Hjorleifsdottir, Dimitri Komatitsch, Swami Krishnan, Qinya Liu, Alessia Maggi, Anne Sieminski, Carl Tape & Ying Zhou

Introduction to the Spectral-Element Method (SEM)

Governing Equations

Equation of motion:

$$\rho\,\partial_t^2 \mathbf{s} - \boldsymbol{\nabla} \cdot \mathbf{T} = \mathbf{f}$$

Boundary condition:

$$\hat{\mathbf{n}} \cdot \mathbf{T} = \mathbf{0}$$

Initial conditions:

$$\mathbf{s}(\mathbf{x}, 0) = \mathbf{0}, \qquad \partial_t \mathbf{s}(\mathbf{x}, 0) = \mathbf{0}$$

Earthquake source:

$$\mathbf{f} = -\mathbf{M} \cdot \boldsymbol{\nabla} \delta(\mathbf{x} - \mathbf{x}_\mathrm{s}) S(t)$$

Weak Form

$$\int_\Omega \rho\, \mathbf{w} \cdot \partial_t^2 \mathbf{s}\, \mathrm{d}^3\mathbf{x} = -\int_\Omega \boldsymbol{\nabla}\mathbf{w} : \mathbf{T}\, \mathrm{d}^3\mathbf{x} + \mathbf{M} : \boldsymbol{\nabla}\mathbf{w}(\mathbf{x}_\mathrm{s}) S(t)$$

- Weak form valid for any test vector
- Boundary conditions automatically included
- Source term explicitly integrated

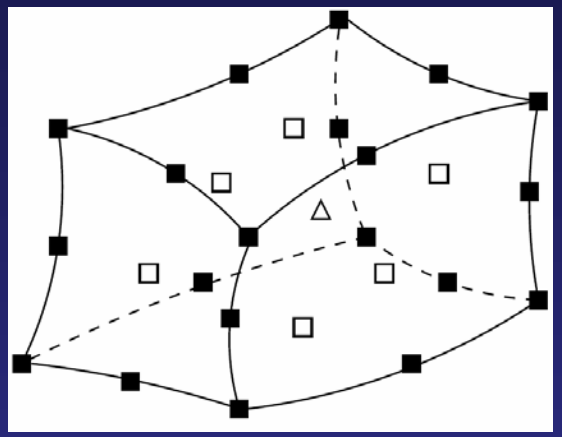
Finite-fault (kinematic) rupture:

$$\mathbf{M} : \boldsymbol{\nabla}\mathbf{w}(\mathbf{x}_\mathrm{s})\, S(t) \rightarrow \int_{S_\mathrm{s}} \mathbf{m}(\mathbf{x}_\mathrm{s},\, t) : \boldsymbol{\nabla}\mathbf{w}(\mathbf{x}_\mathrm{s})\, d^2\mathbf{x}_\mathrm{s}$$

Finite-Elements

Mapping from reference cube to hexahedral elements:

$$\mathbf{x}(\boldsymbol{\xi}) = \sum_{a=1}^{M} \mathbf{x}_a N_a(\boldsymbol{\xi})$$



Volume relationship:

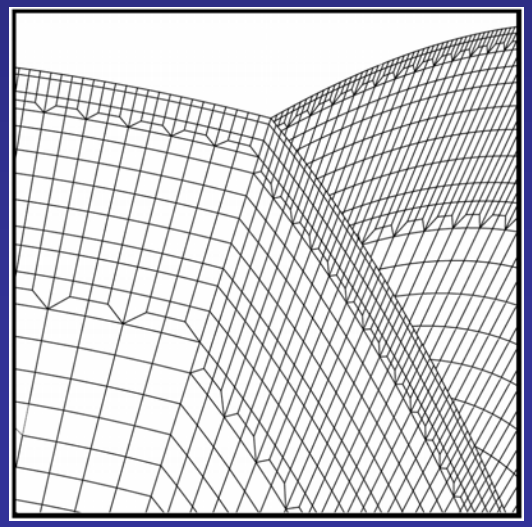
$$\mathrm{d}^3\mathbf{x} = \mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z = J\,\mathrm{d}\xi\,\mathrm{d}\eta\,\mathrm{d}\zeta = J\,\mathrm{d}^3\boldsymbol{\xi}$$

Jacobian of the mapping:

$$J = \left|\frac{\partial(x,y,z)}{\partial(\xi,\eta,\zeta)}\right|$$

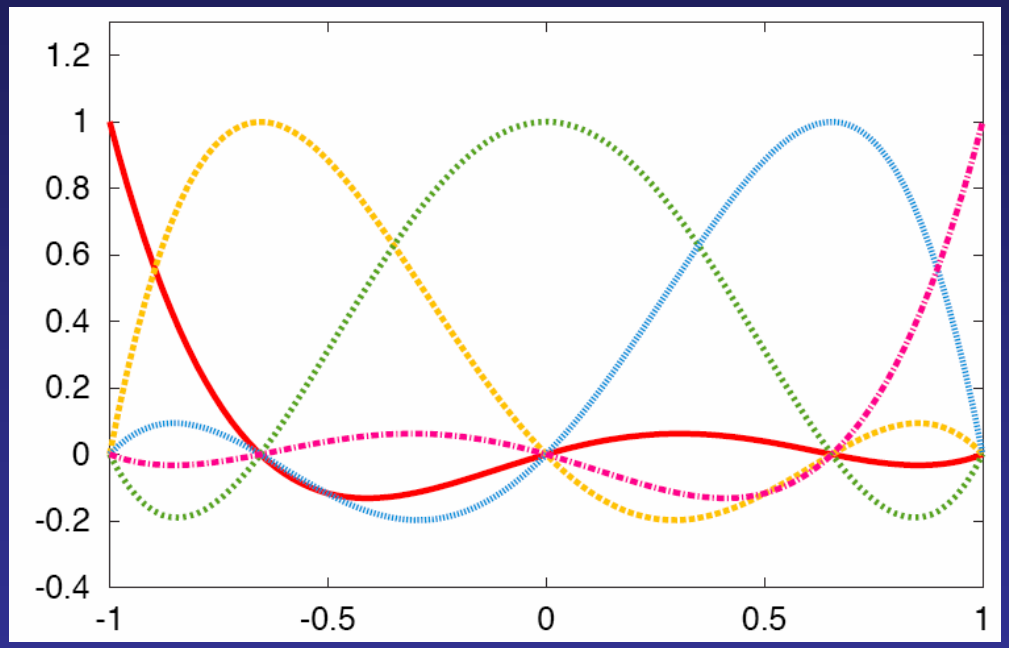
Jacobian matrix:

$$\frac{\partial\mathbf{x}}{\partial\boldsymbol{\xi}} = \sum_{a=1}^{M} \mathbf{x}_a \frac{\partial N_a}{\partial\boldsymbol{\xi}}$$

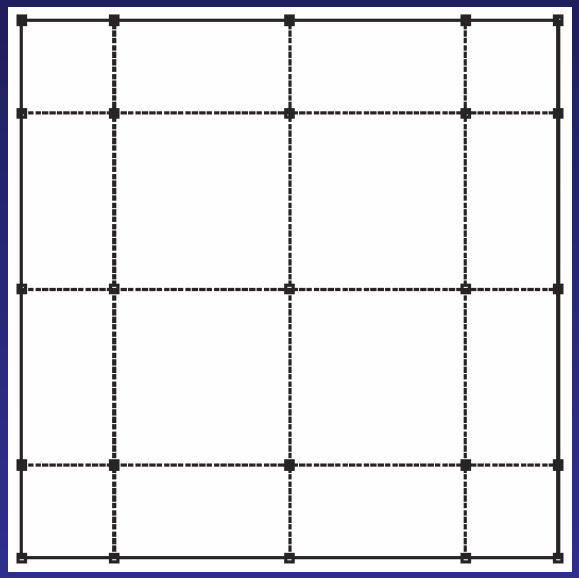


Lagrange Polynomials and Gauss-Lobatto-Legendre (GLL) Points

The 5 degree 4 Lagrange polynomials:



Degree 4 GLL points:



GLL points are n+1 roots of:

$$(1-\xi^2)P_n'(\xi) = 0$$

General definition:

$$h_\alpha(\xi) = \frac{(\xi-\xi_0)\cdots(\xi-\xi_{\alpha-1})(\xi-\xi_{\alpha+1})\cdots(\xi-\xi_n)}{(\xi_\alpha-\xi_0)\cdots(\xi_\alpha-\xi_{\alpha-1})(\xi_\alpha-\xi_{\alpha+1})\cdots(\xi_\alpha-\xi_n)}$$

Note that at a GLL point: $h_\alpha(\xi_\beta) = \delta_{\alpha\beta}$

Interpolation

Representation of functions on an element in terms of Lagrange polynomials:

$$f(\mathbf{x}(\xi,\eta,\zeta)) = \sum_{\alpha=0}^{n}\sum_{\beta=0}^{n}\sum_{\gamma=0}^{n} f^{\alpha\beta\gamma} h_\alpha(\xi) h_\beta(\eta) h_\gamma(\zeta)$$

Gradient on an element:

$$\nabla f(\mathbf{x}(\xi,\eta,\zeta)) = \sum_{i=1}^{3} \hat{\mathbf{x}}_i \sum_{\alpha=0}^{n}\sum_{\beta=0}^{n}\sum_{\gamma=0}^{n} f^{\alpha\beta\gamma} \left[h'_\alpha(\xi) h_\beta(\eta) h_\gamma(\zeta) \partial_i \xi + h_\alpha(\xi) h'_\beta(\eta) h_\gamma(\zeta) \partial_i \eta + h_\alpha(\xi) h_\beta(\eta) h'_\gamma(\zeta) \partial_i \zeta \right]$$

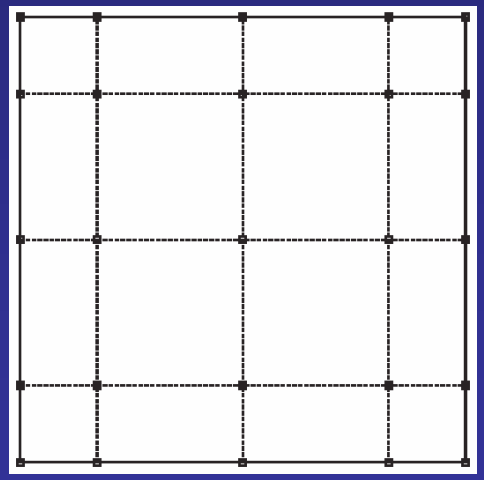
Integration

Integration of functions over an element based upon GLL quadrature:

$$\int_{\Omega_e} f(\mathbf{x})\,\mathrm{d}^3\mathbf{x} = \int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1} f(\mathbf{x}(\xi,\eta,\zeta))\,J(\xi,\eta,\zeta)\,\mathrm{d}\xi\,\mathrm{d}\eta\,\mathrm{d}\zeta = \sum_{\alpha=0}^{n}\sum_{\beta=0}^{n}\sum_{\gamma=0}^{n} \omega_\alpha\omega_\beta\omega_\gamma f^{\alpha\beta\gamma} J^{\alpha\beta\gamma}$$

- Integrations are pulled back to the reference cube
- In the SEM one uses:
  - interpolation on GLL points
  - GLL quadrature

Degree 4 GLL points:



The Diagonal Mass Matrix

Representation of the displacement:

$$\mathbf{s}(\mathbf{x}(\xi,\eta,\zeta),t) = \sum_{i=1}^{3} \hat{\mathbf{x}}_i \sum_{\sigma=0}^{n}\sum_{\tau=0}^{n}\sum_{\nu=0}^{n} s_i^{\sigma\tau\nu}(t) h_\sigma(\xi) h_\tau(\eta) h_\nu(\zeta)$$

Representation of the test vector:

$$\mathbf{w}(\mathbf{x}(\xi,\eta,\zeta)) = \sum_{i=j}^{3} \hat{\mathbf{x}}_j \sum_{\sigma=0}^{n}\sum_{\tau=0}^{n}\sum_{\nu=0}^{n} w_i^{\alpha\beta\gamma} h_\alpha(\xi) h_\beta(\eta) h_\gamma(\zeta)$$

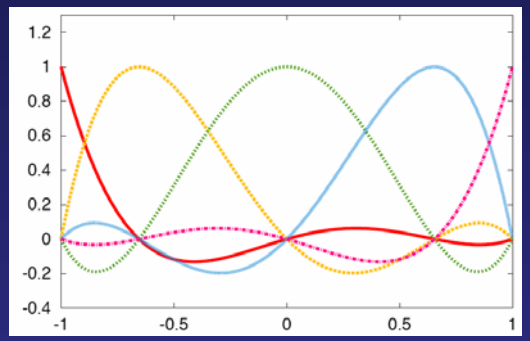
Weak form:

$$\int_\Omega \rho\, \mathbf{w}\cdot\partial_t^2 \mathbf{s}\, \mathrm{d}^3\mathbf{x} = -\int_\Omega \boldsymbol{\nabla}\mathbf{w} : \mathbf{T}\, \mathrm{d}^3\mathbf{x} + \mathbf{M} : \boldsymbol{\nabla}\mathbf{w}(\mathbf{x}_\mathrm{s}) S(t)$$

Diagonal mass matrix:

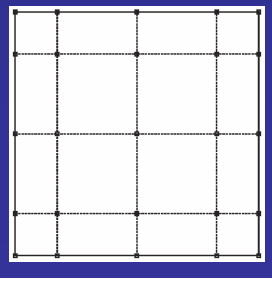
$$\int_{\Omega_e} \rho\, \mathbf{w}\cdot\partial_t^2 \mathbf{s}\, \mathrm{d}^3\mathbf{x} = \int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1} \rho(\mathbf{x}(\boldsymbol{\xi}))\, \mathbf{w}(\mathbf{x}(\boldsymbol{\xi}))\cdot\partial_t^2 \mathbf{s}(\mathbf{x}(\boldsymbol{\xi}),t)\, J(\boldsymbol{\xi})\, \mathrm{d}^3\boldsymbol{\xi} = \sum_{\alpha=0}^{n}\sum_{\beta=0}^{n}\sum_{\gamma=0}^{n} \omega_\alpha\omega_\beta\omega_\gamma J^{\alpha\beta\gamma} \rho^{\alpha\beta\gamma} \sum_{i=1}^{3} w_i^{\alpha\beta\gamma} \ddot{s}_i^{\alpha\beta\gamma}$$

Degree 4 Lagrange polynomials:

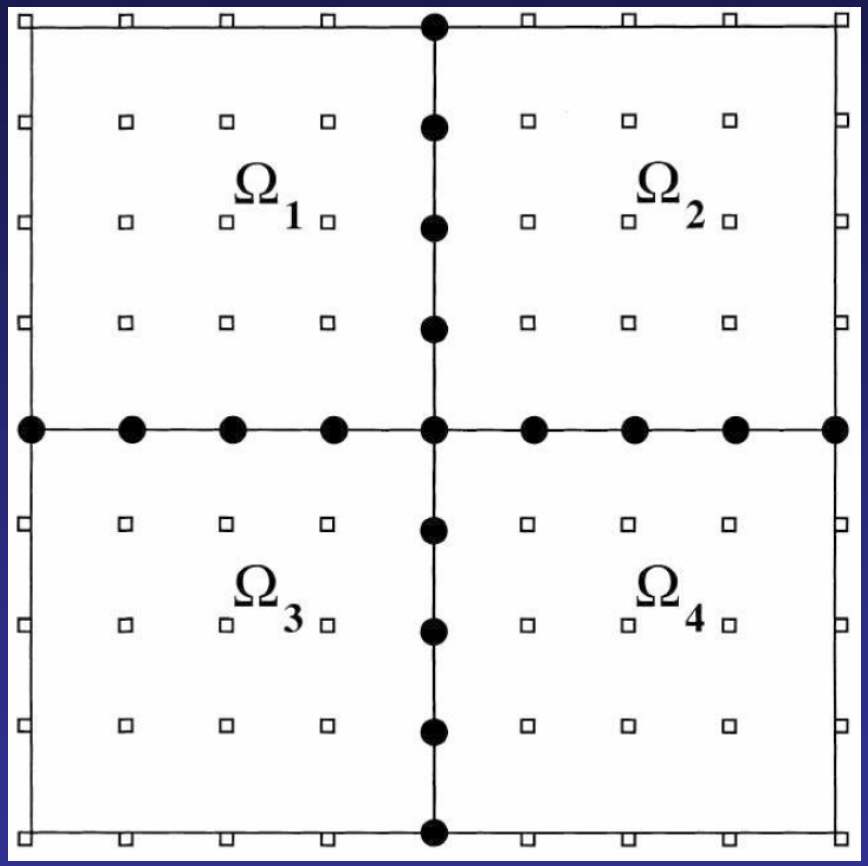


- Integrations are pulled back to the reference cube
- In the SEM one uses:
  - interpolation on GLL points
  - GLL quadrature

Degree 4 GLL points:



Assembly



Need to distinguish:

- local mesh
- global mesh

Efficient routines available from FEM applications

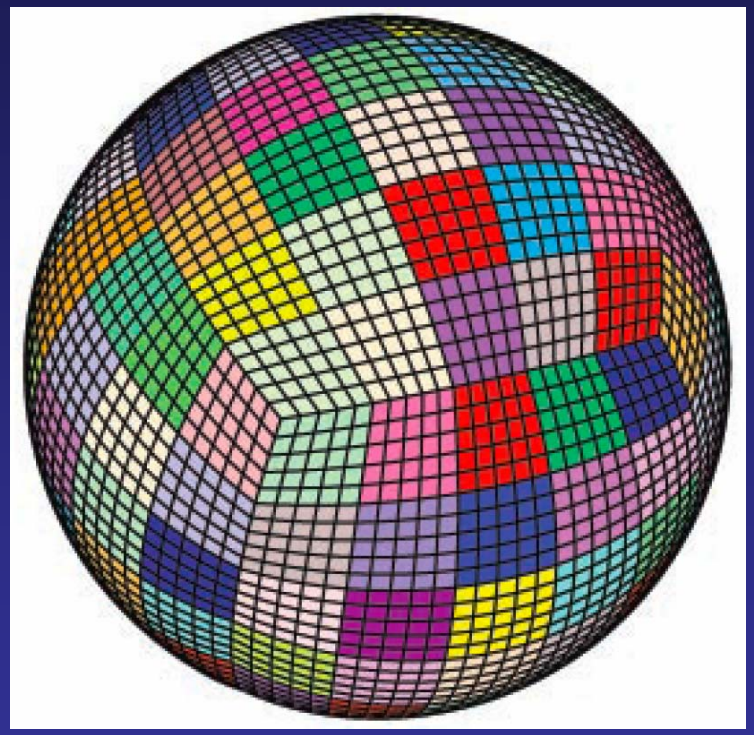
Global equations:

$$M\ddot{U} = -KU + F$$

Global SEM time-marching is based upon an explicit second-order time scheme

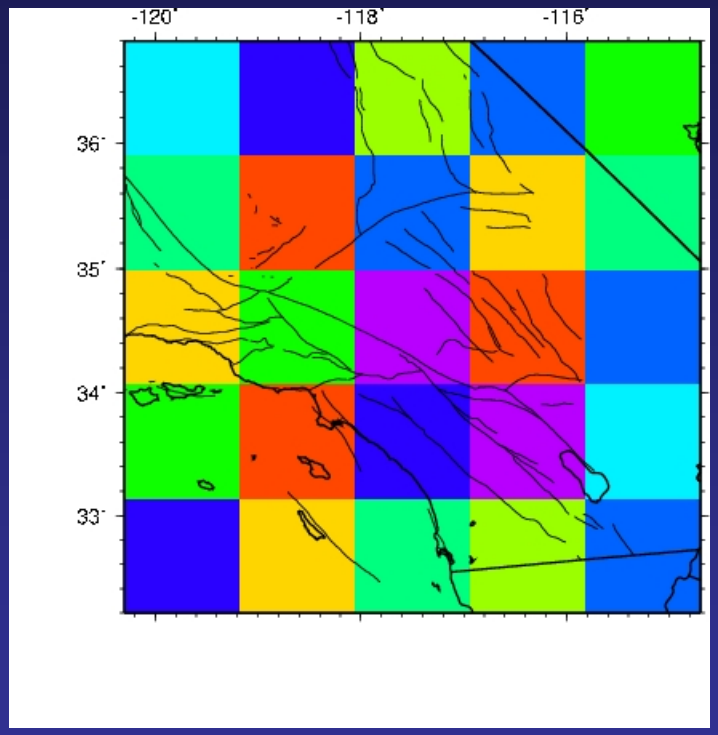
Parallel Implementation

Global mesh partitioning:



Cubed Sphere: 6 n^2 mesh slices

Regional mesh partitioning:

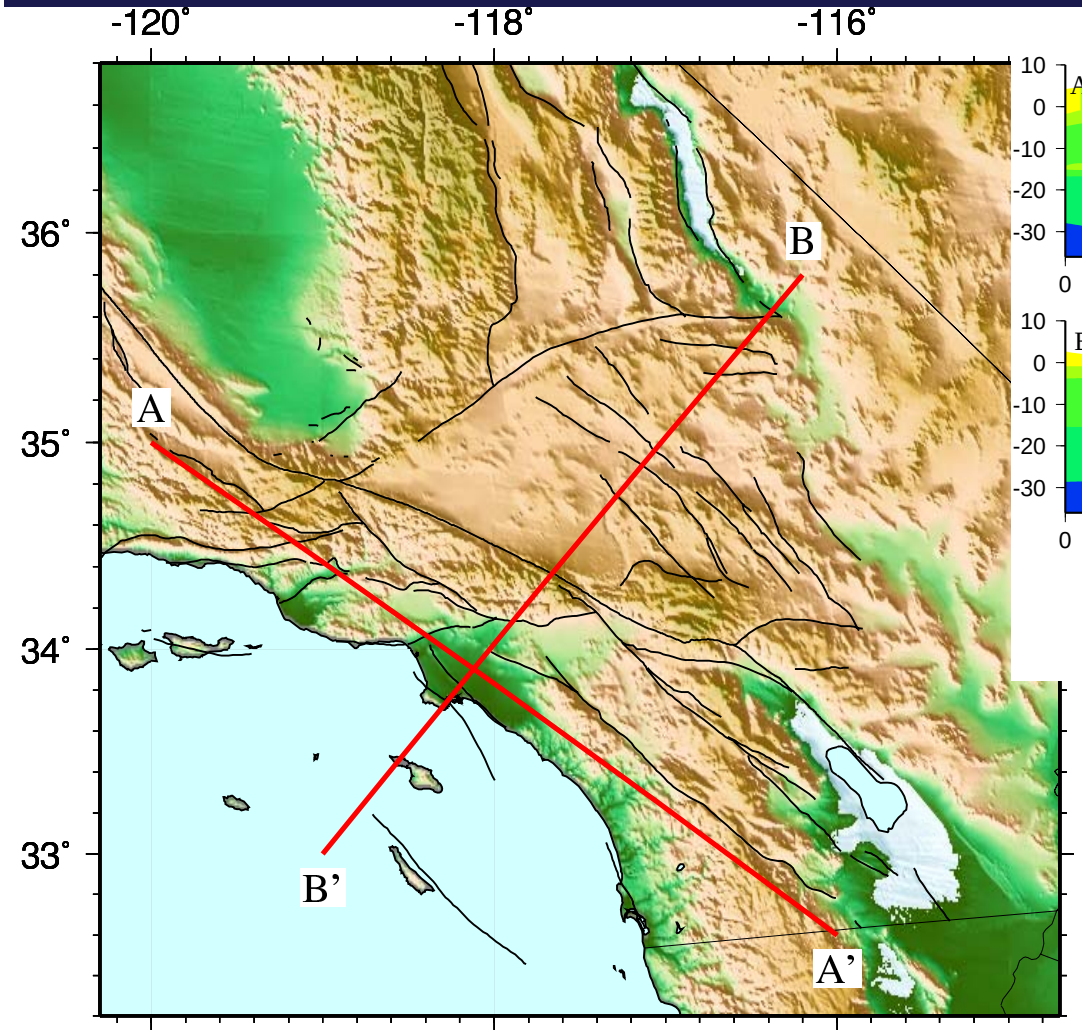


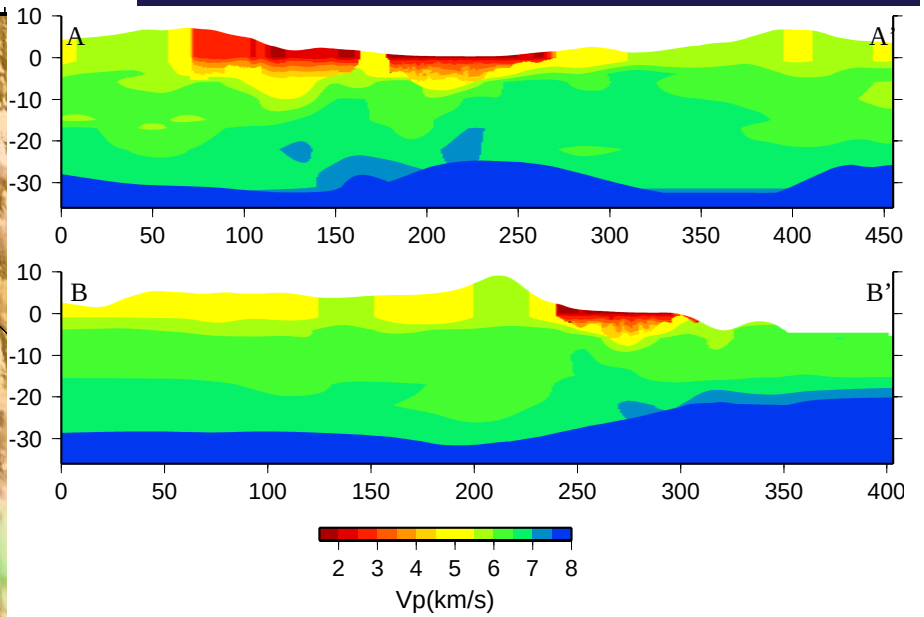
n x m mesh slices

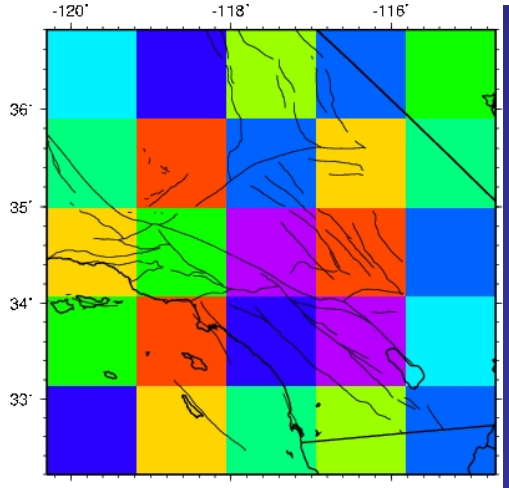
Basin Code: SPECFEM3D_BASIN

- Freely available for non-commercial use from geodynamics.org
- Manual available at geodynamics.org
- 3D Attenuation
- 3D Anisotropy
- Ocean load
- Topography & bathymetry
- Kinematic ruptures
- Movies
- Models:
  - Harvard 3D Southern California model
  - SOCAL 1D model
  - Homogeneous half-space model
- Adjoint capabilities

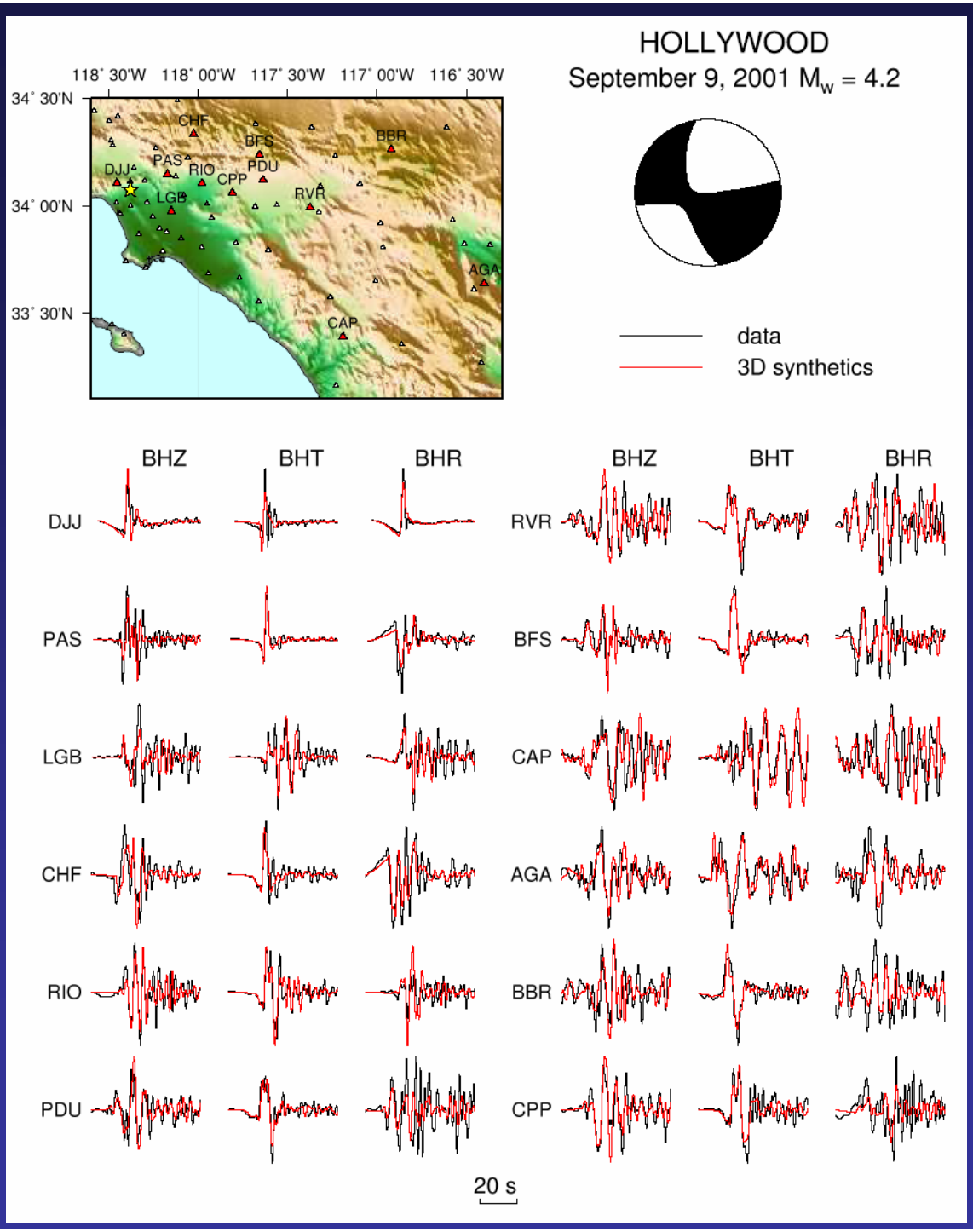
Southern California Simulations







n x *m* mesh slices



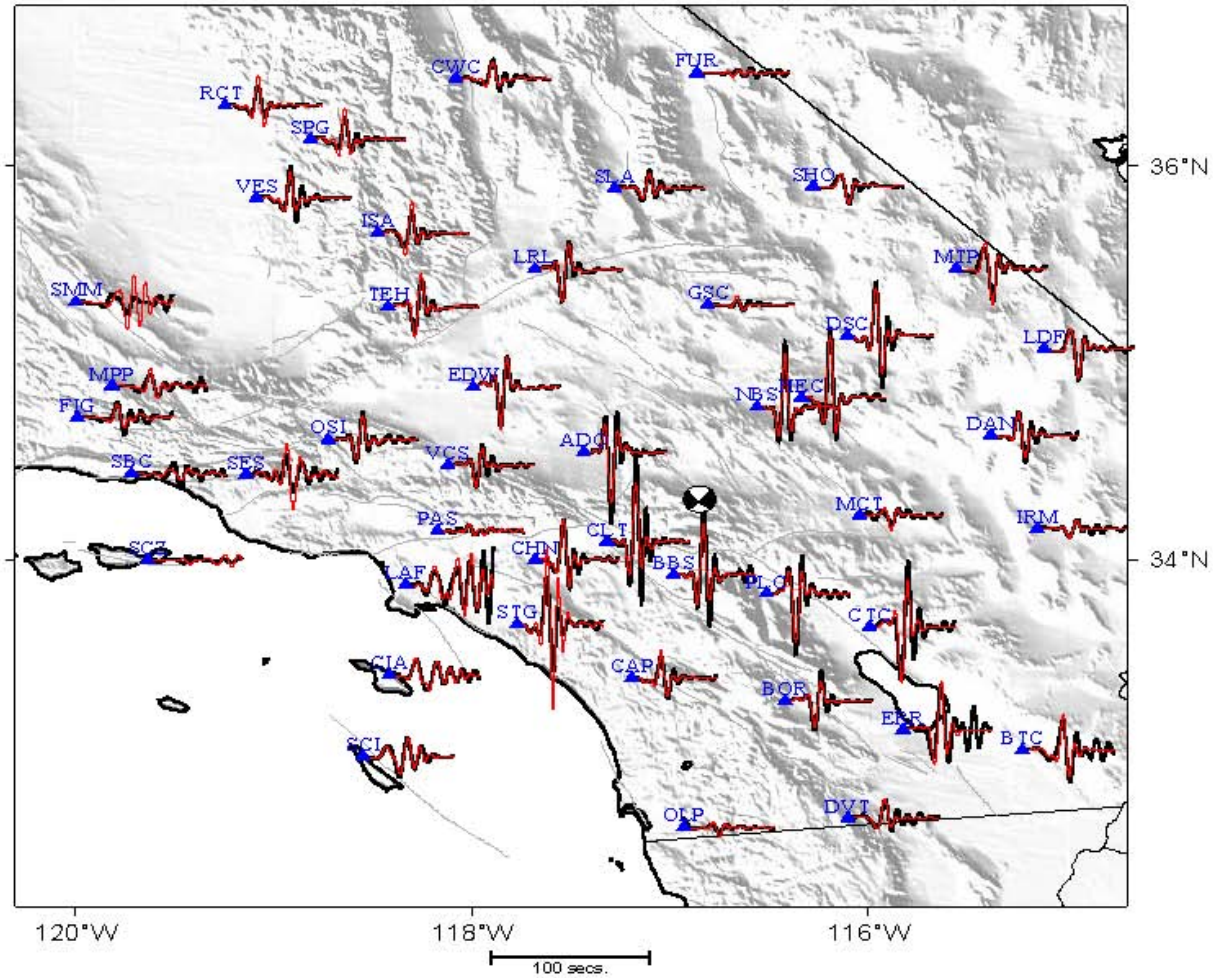
Periods > 2 s

Komatitsch et al. 2004

June 12, 2005, M=5.1 Big Bear

QuickTime™ and a
YUV420 codec decompressor
are needed to see this picture.

3D Regional Forward Simulations



Periods > 6 s

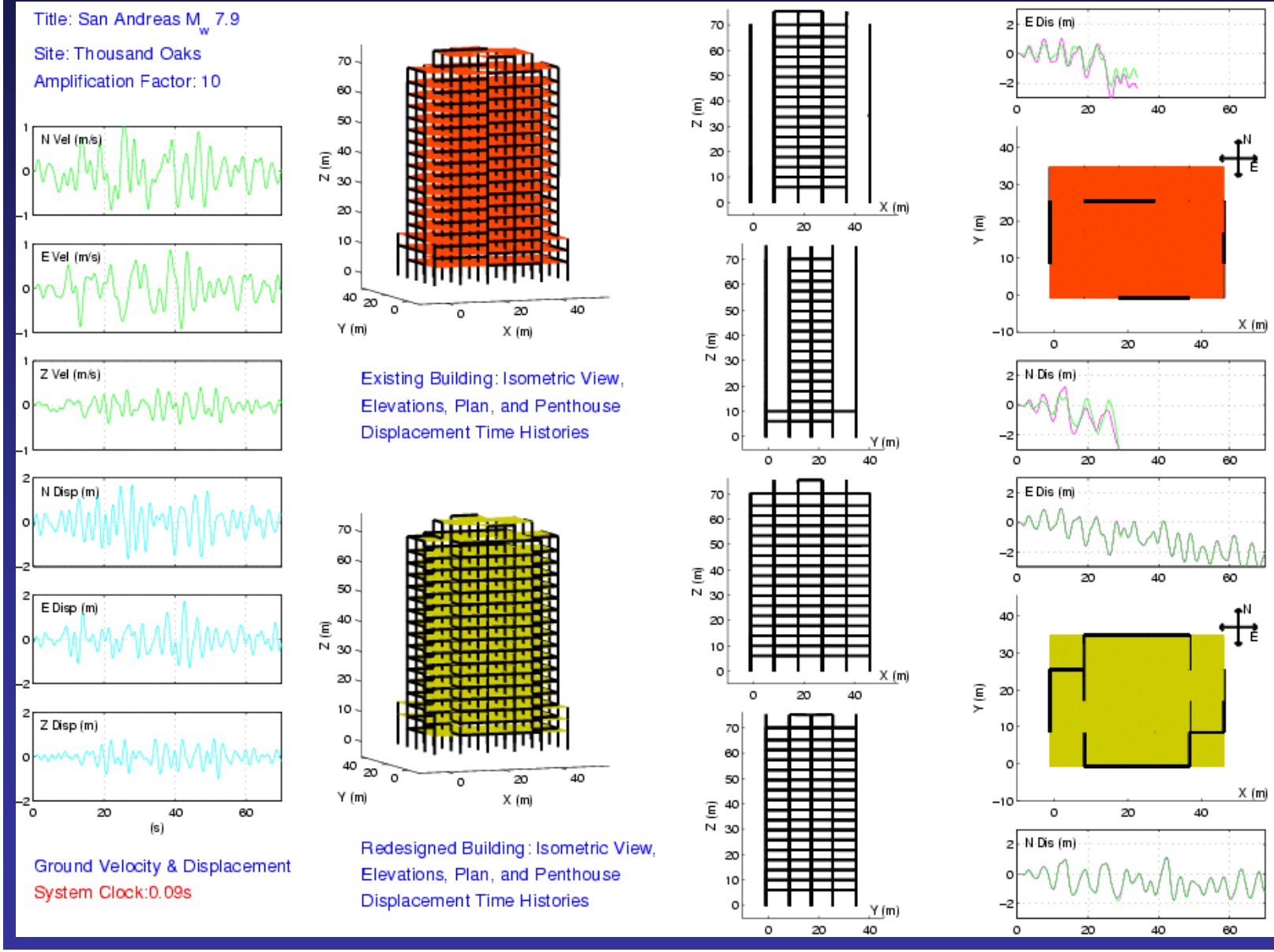
June 12, 2005, M=5.1 Big Bear

Qinya Liu

San Andreas Rupture Scenario

QuickTime™ and a
YUV420 codec decompressor
are needed to see this picture.

San Andreas Rupture Scenario: Quantitative Seismic Hazard Assessment



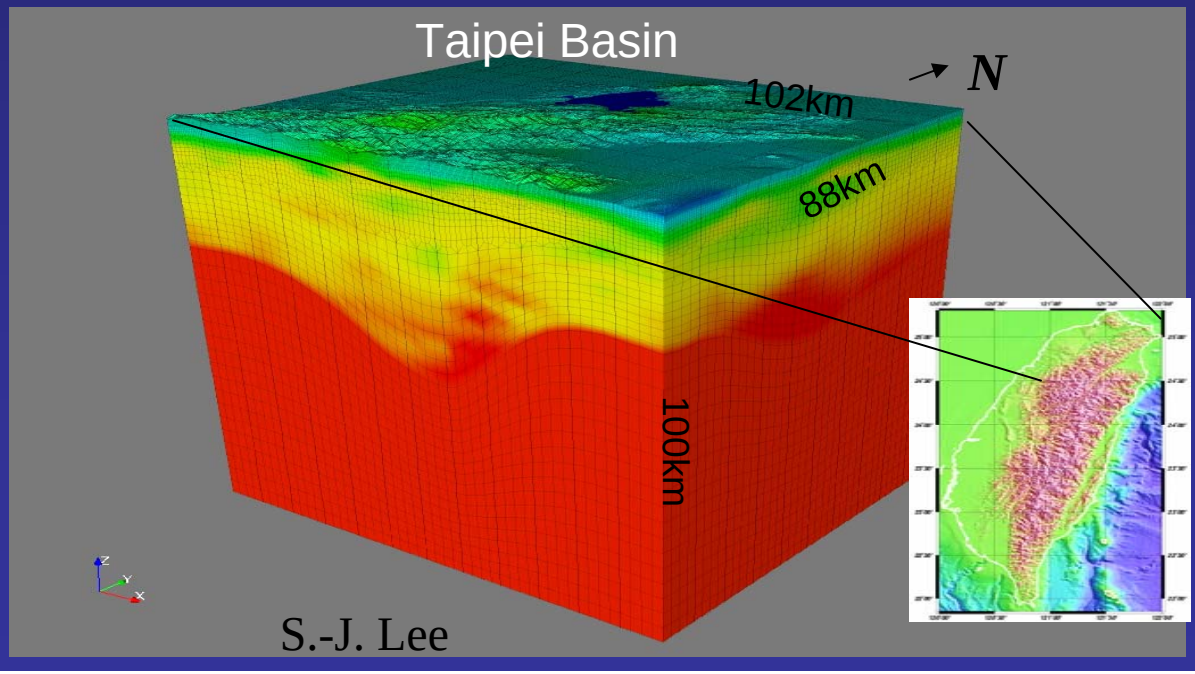
Swami Krishnan

Near Real-Time Applications

- Automated near real-time simulations of all M>3.5 events
- ShakeMovies at http://www.shakemovie.caltech.edu/
- Soon:
  - CMT solutions
  - Synthetic seismograms

SPECFEM3D_BASIN: Future Plans

- Switch to a (parallel) CUBIT hexahedral finite-element mesher (Casarotti, Lee)
  - Topography & bathymetry
  - Major geological interfaces
  - Basins
  - Fault surfaces
- Use ParMETIS or SCOTCH for mesh partitioning & load-balancing
- Retain the SPECFEM3D_BASIN solver (takes ParMETIS meshes; Komatitsch)
- Add dynamic rupture capabilities (Ampuero, Lapusta, Kaneko)

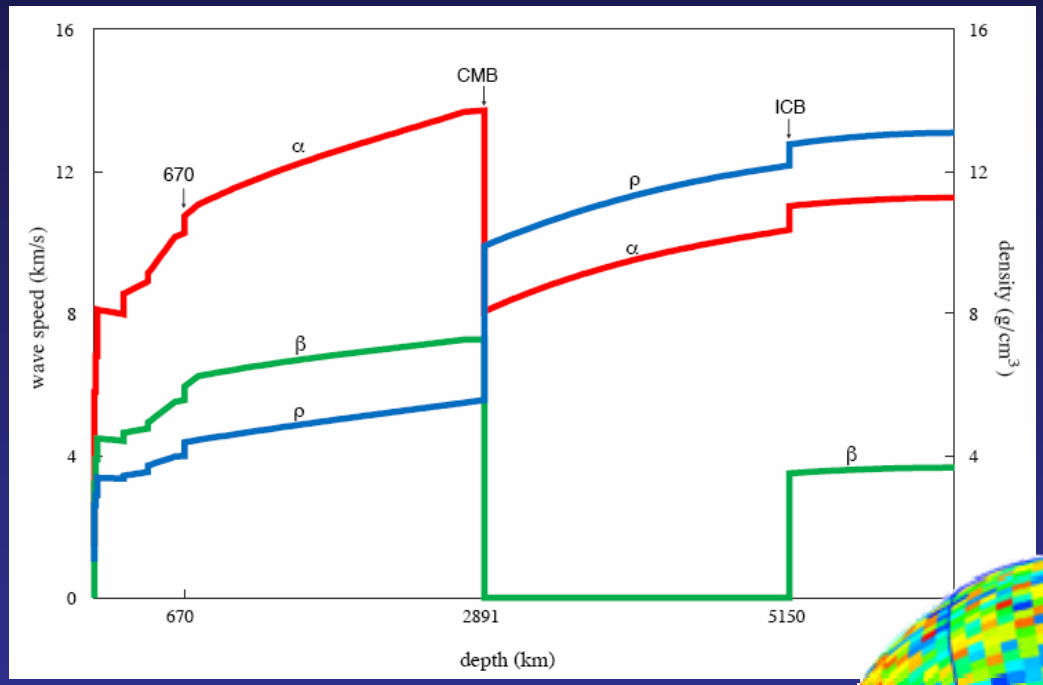


Global Code: SPECFEM3D_GLOBE

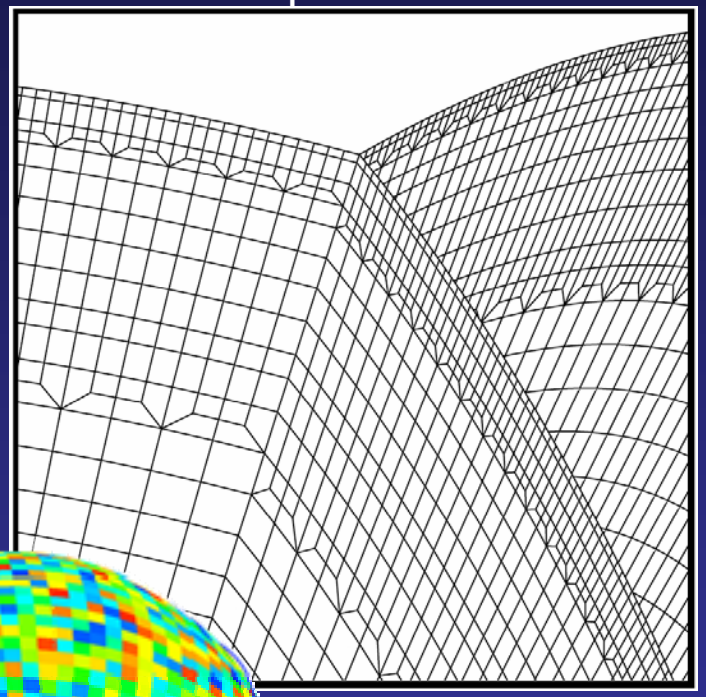
- Freely available for non-commercial use from geodynamics.org
- Manual available at geodynamics.org
- 3D Attenuation
- 3D Anisotropy
- Ocean load
- Topography & bathymetry
- Rotation
- Self-gravitation (Cowling approximation)
- Kinematic ruptures
- Movies
- Models:
  - 1D models: isotropic PREM, transversely isotropic PREM, AK135, IASP91, 1066A
  - S20RTS
  - Crust2.0
- Adjoint capabilities

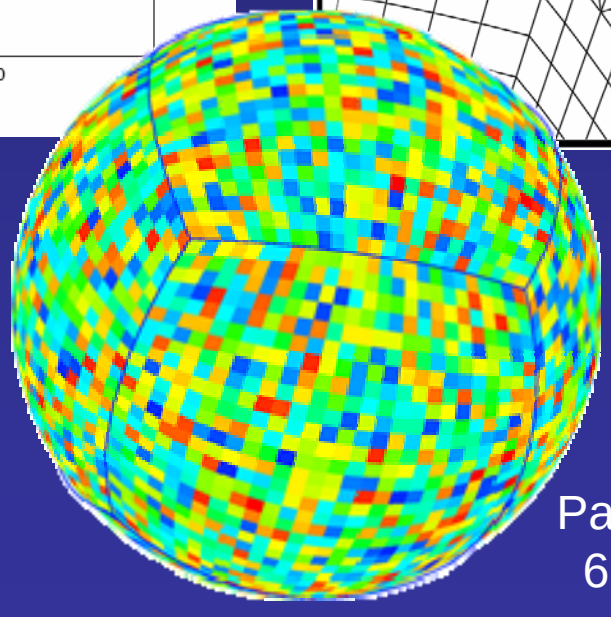
Global Simulations

PREM benchmarks



Cubed sphere mesh





Parallel implementation:
6 n^2 mesh slices

SEM Implementation of Attenuation

Anelastic, anisotropic constitutive relationship:

$$\mathbf{T}(t) = \int_{-\infty}^{\infty} \partial_t \mathbf{c}(t-t') : \nabla \mathbf{s}(t')\, dt'$$

Equivalent Standard Linear Solid (SLS) formulation:

$$\mathbf{T} = \mathbf{c}^U : \nabla \mathbf{s} - \sum_{\ell=1}^{L} \mathbf{R}^\ell$$

Memory variable equation:

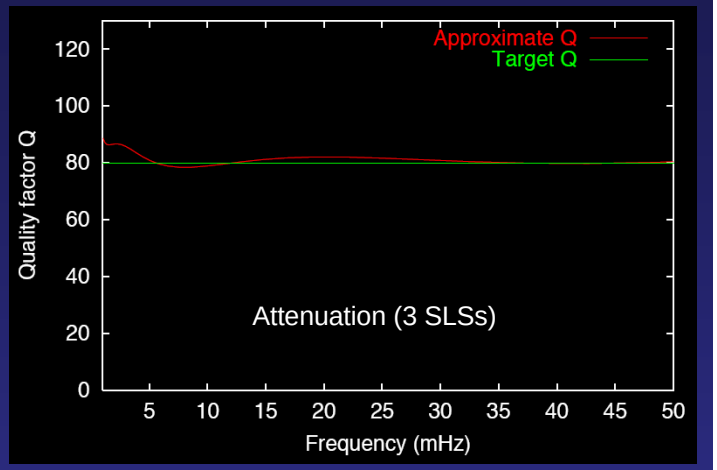
$$\partial_t \mathbf{R}^\ell = -\mathbf{R}^\ell / \tau^{\sigma\ell} + \delta \mathbf{c}^\ell : \nabla \mathbf{s} / \tau^{\sigma\ell}$$

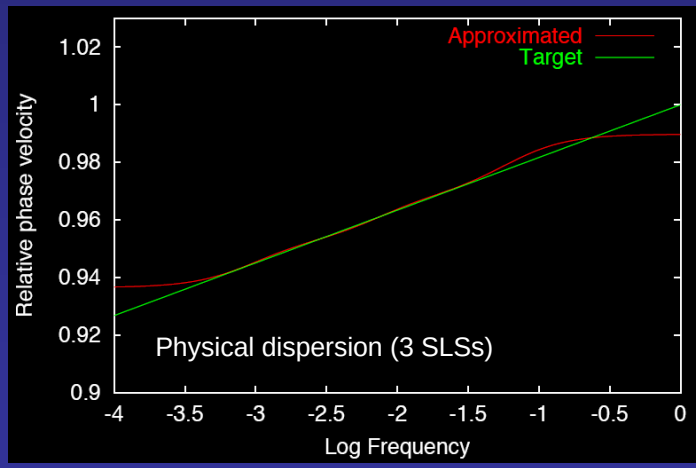
Unrelaxed modulus:

$$c^U_{ijkl} = c^R_{ijkl} \left[1 - \sum_{\ell=1}^{L} (1 - \tau^{\epsilon\ell}_{ijkl} / \tau^{\sigma\ell}) \right]$$

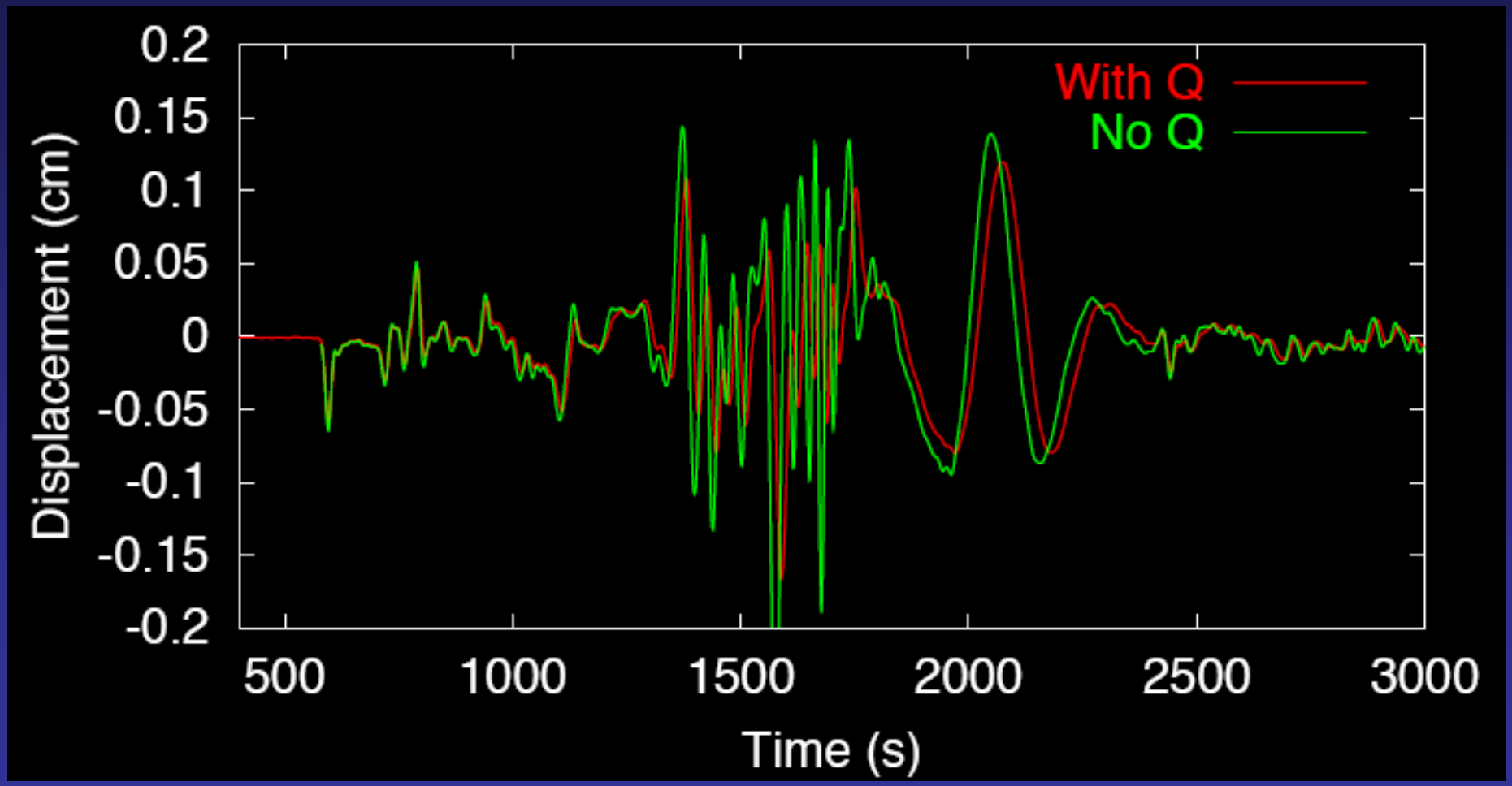
Modulus defect:

$$\delta c^\ell_{ijkl} = -c^R_{ijkl} (1 - \tau^{\epsilon\ell}_{ijkl} / \tau^{\sigma\ell})$$

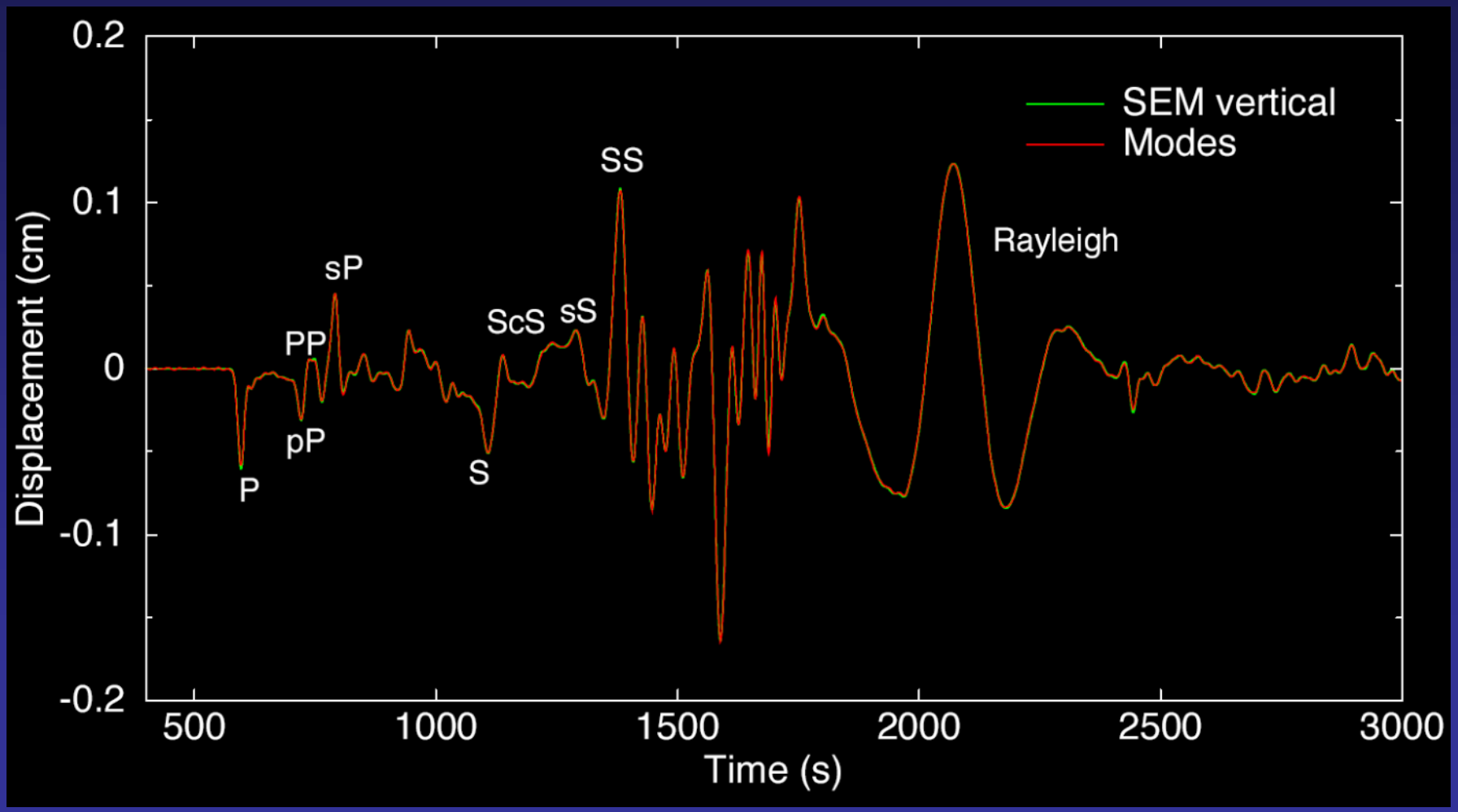




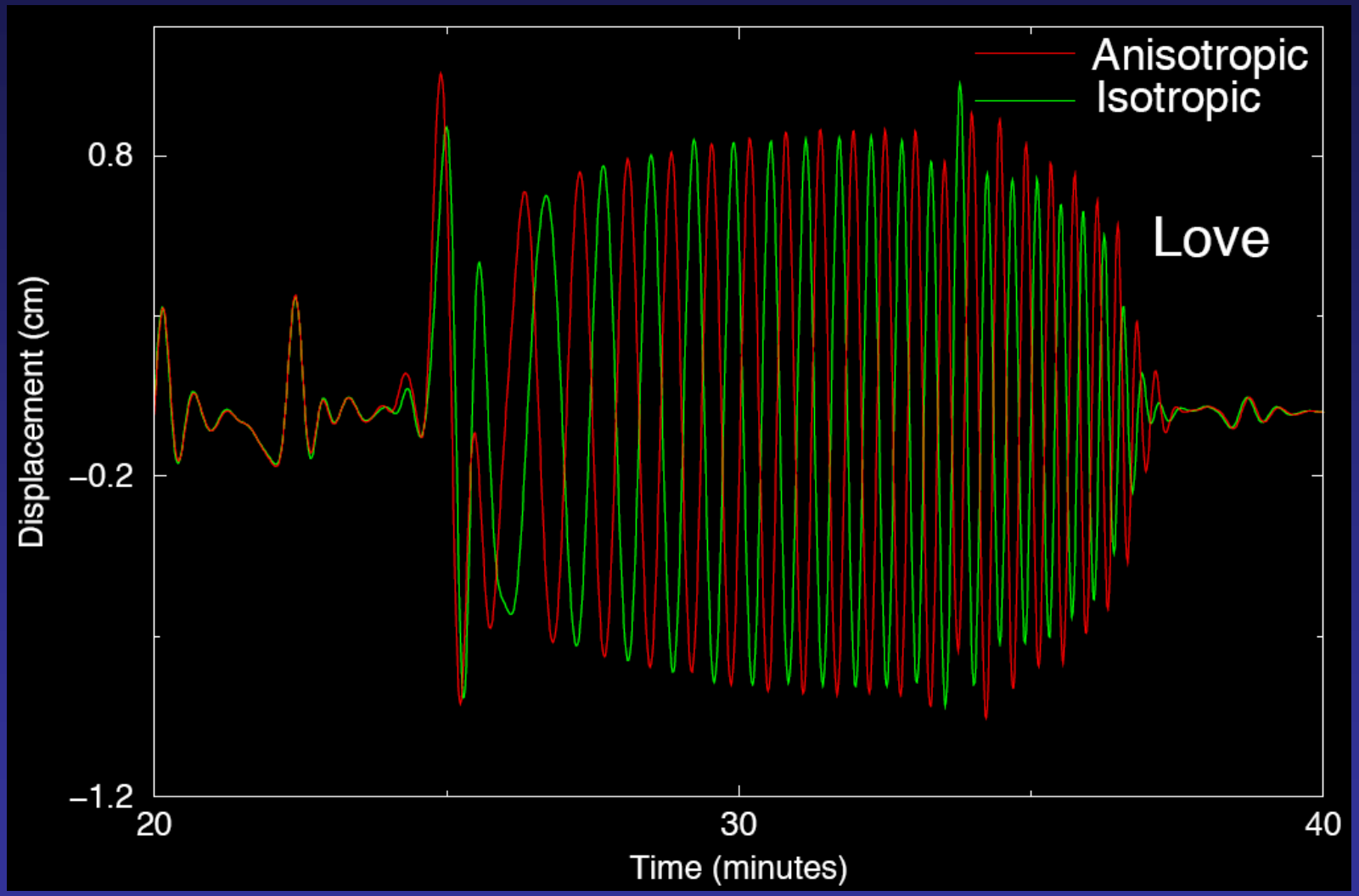
Effect of Attenuation



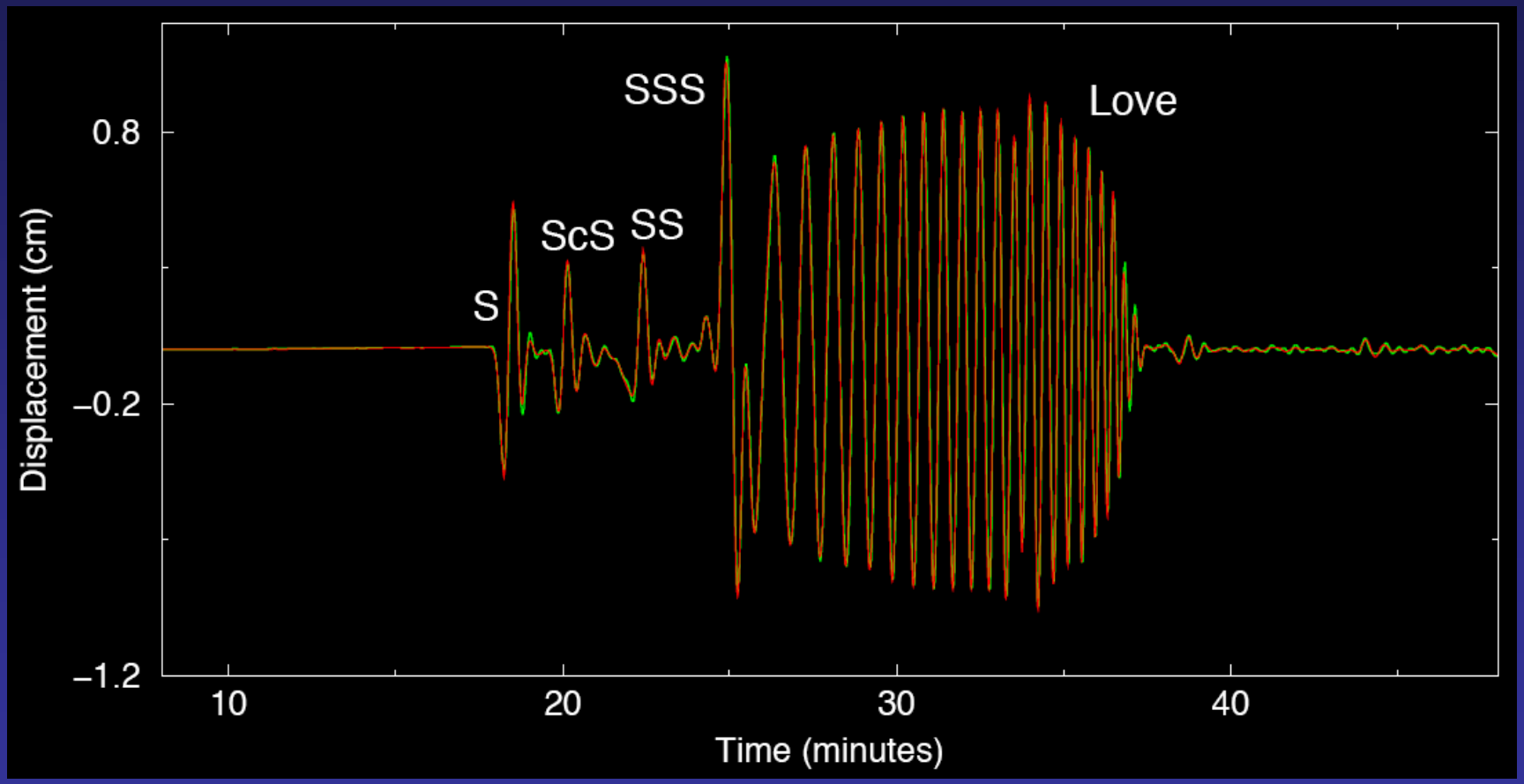
Attenuation



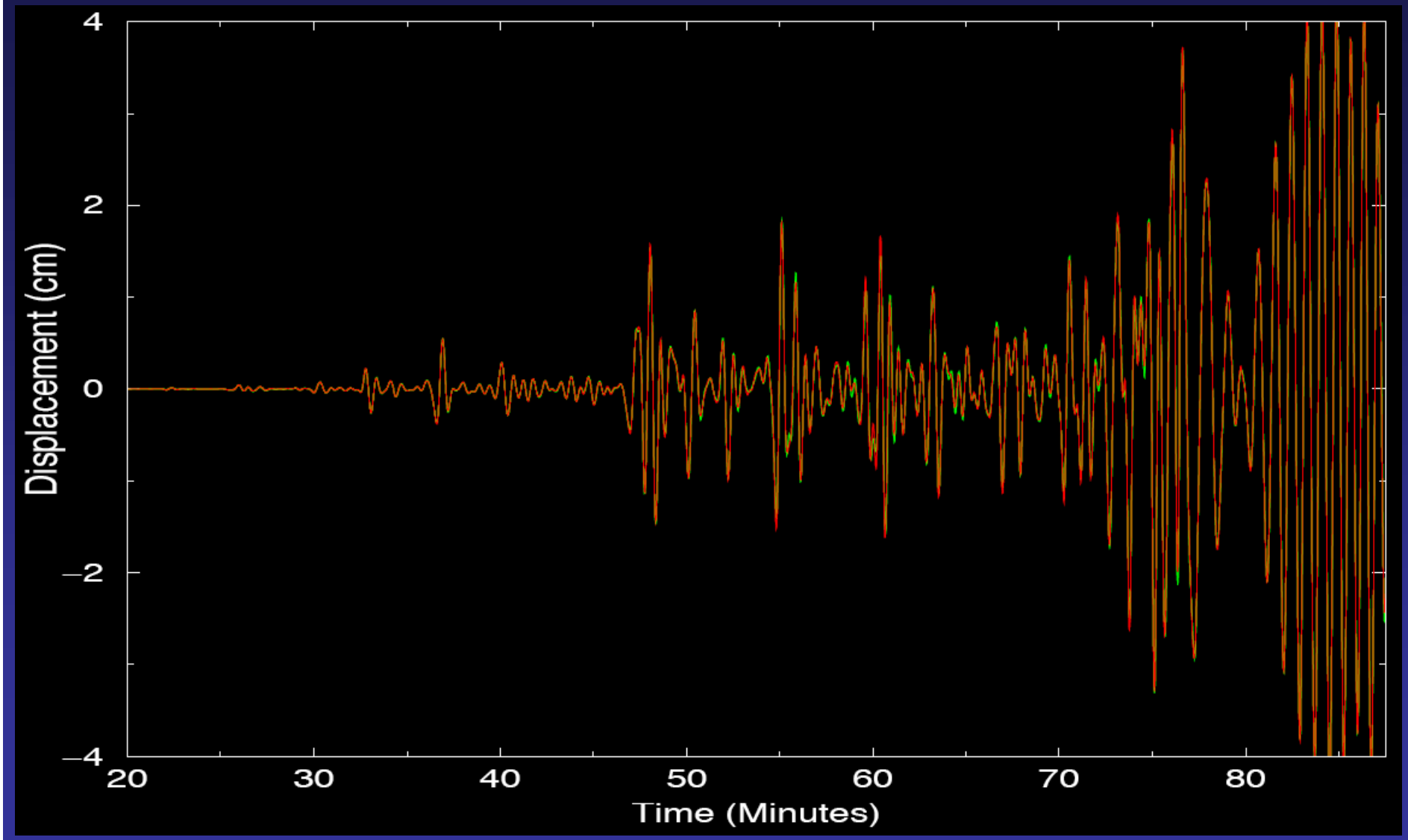
Effect of Anisotropy



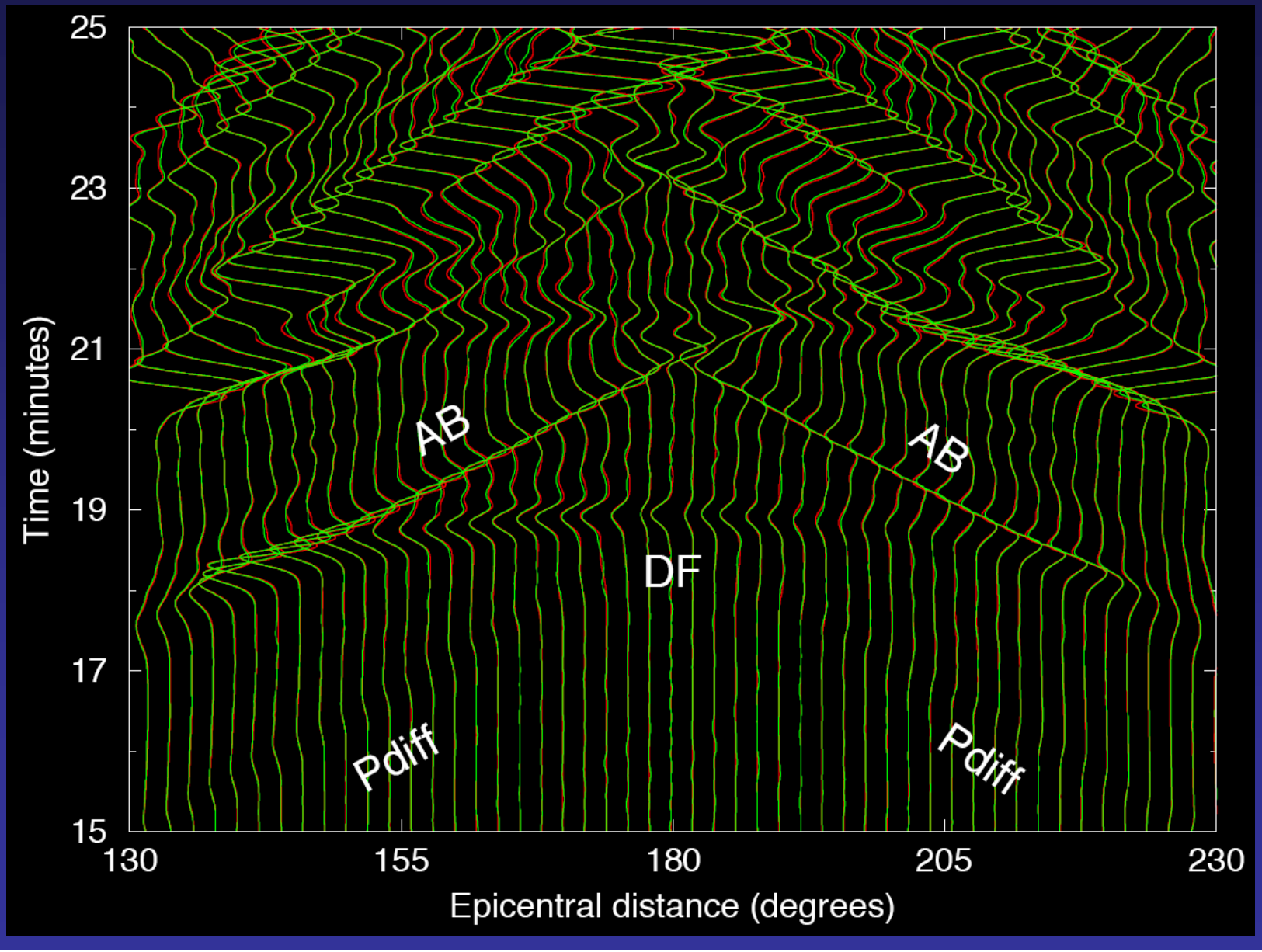
SEM Implementation of Anisotropy



Antipodal Transverse Record



Vertical PKP



Rotation & Self-Gravitation

Wave equation solved by SPECFEM3D_GLOBE in crust, mantle and inner core:

$$\rho\left(\partial_t^2 \mathbf{s} + 2\mathbf{\Omega} \times \partial_t \mathbf{s}\right) = \nabla \cdot \mathbf{T} + \nabla(\rho \mathbf{s} \cdot \mathbf{g}) - \nabla \cdot (\rho \mathbf{s})\mathbf{g} + \mathbf{f}$$

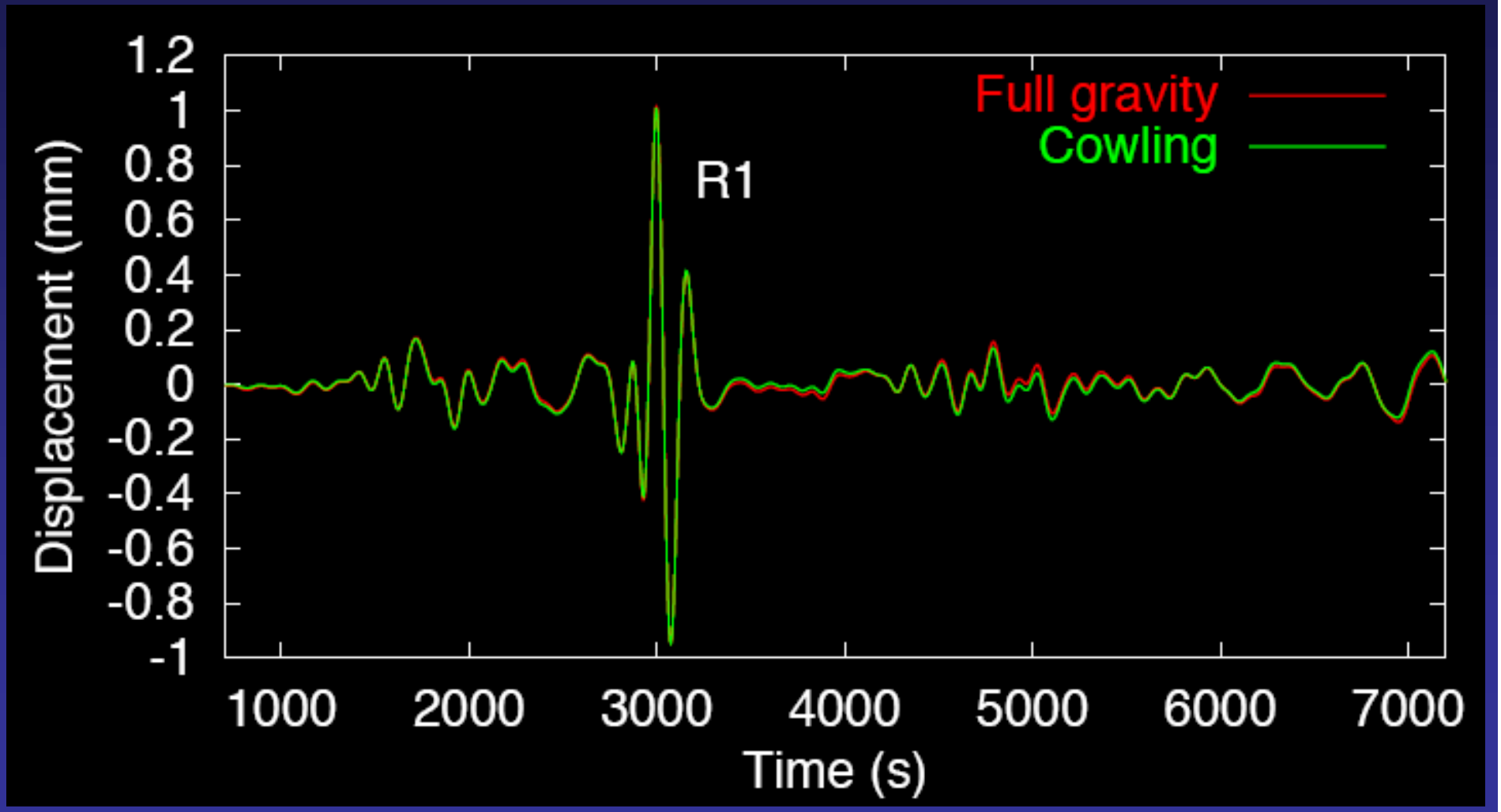
Wave equation solved by SPECFEM3D_GLOBE in fluid outer core:

$$\partial_t^2 \mathbf{s} + 2\mathbf{\Omega} \times \partial_t \mathbf{s} = \nabla(\rho^{-1}\kappa \nabla \cdot \mathbf{s} + \mathbf{s} \cdot \mathbf{g}).$$

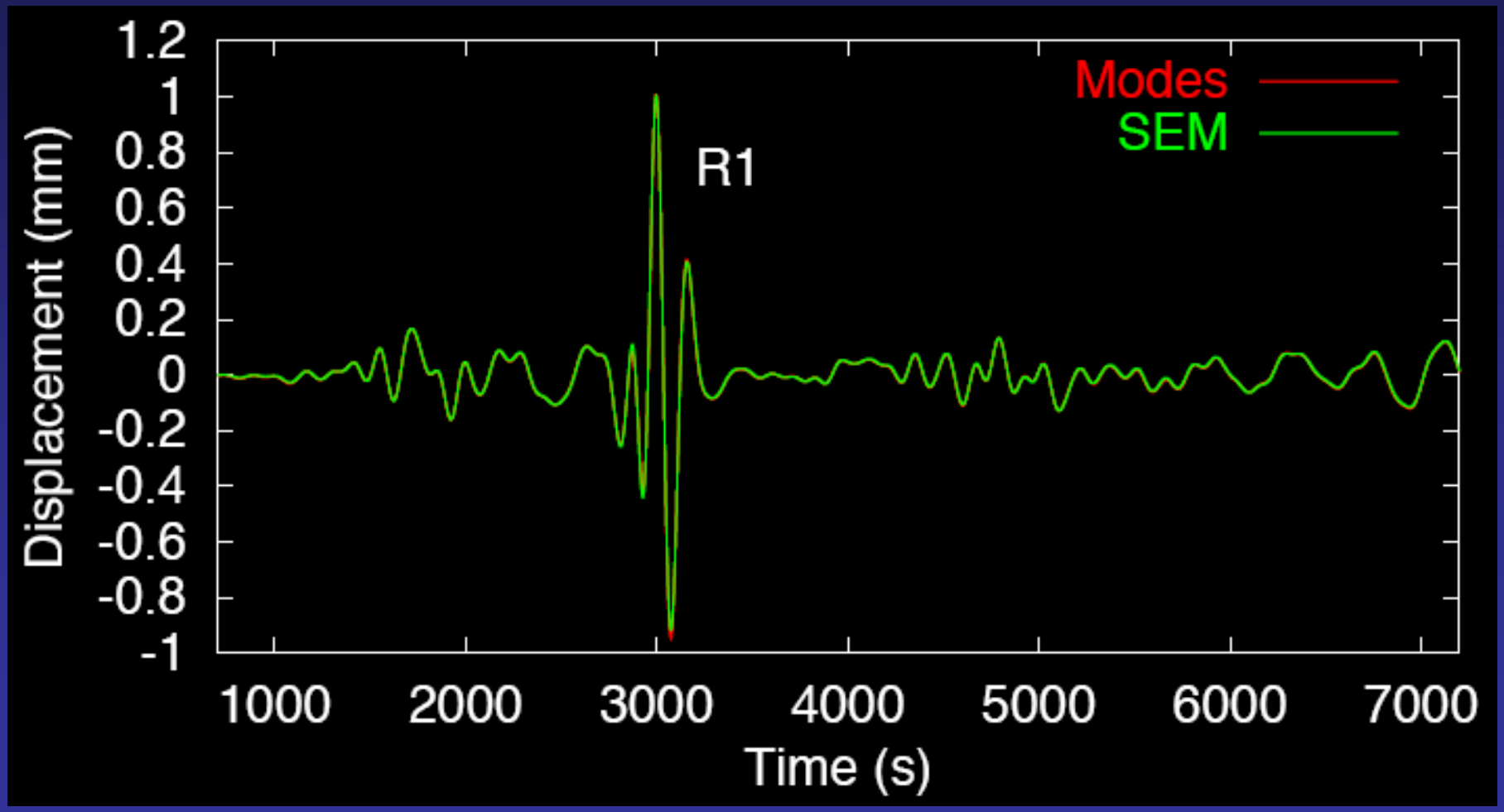
SPECFEM3D_GLOBE uses domain decomposition between the fluid outer core and the solid inner core and mantle matching exactly:

- continuity of traction
- continuity of the normal component of displacement

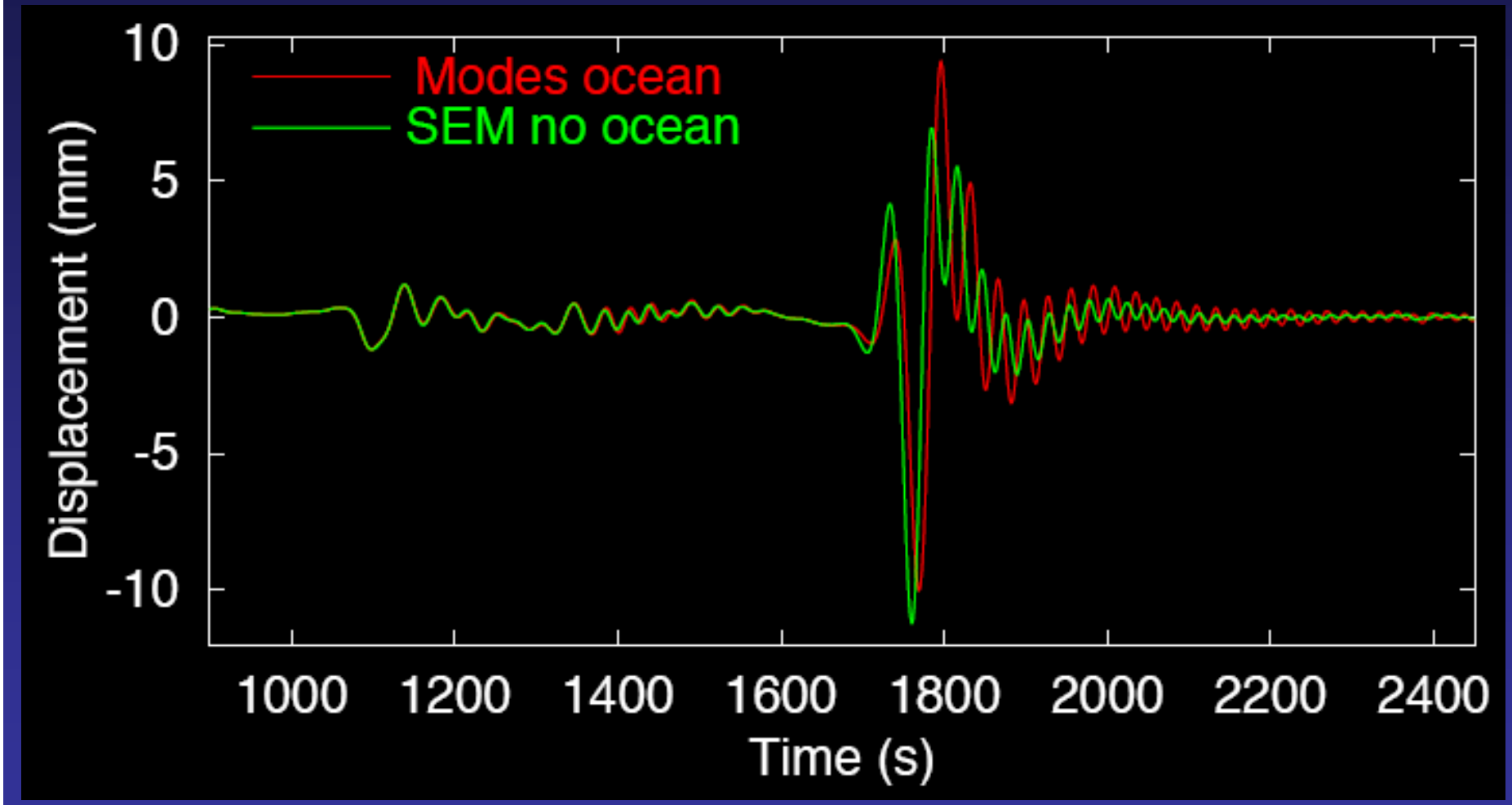
Full Gravity Versus Cowling



SEM Implementation of Gravity

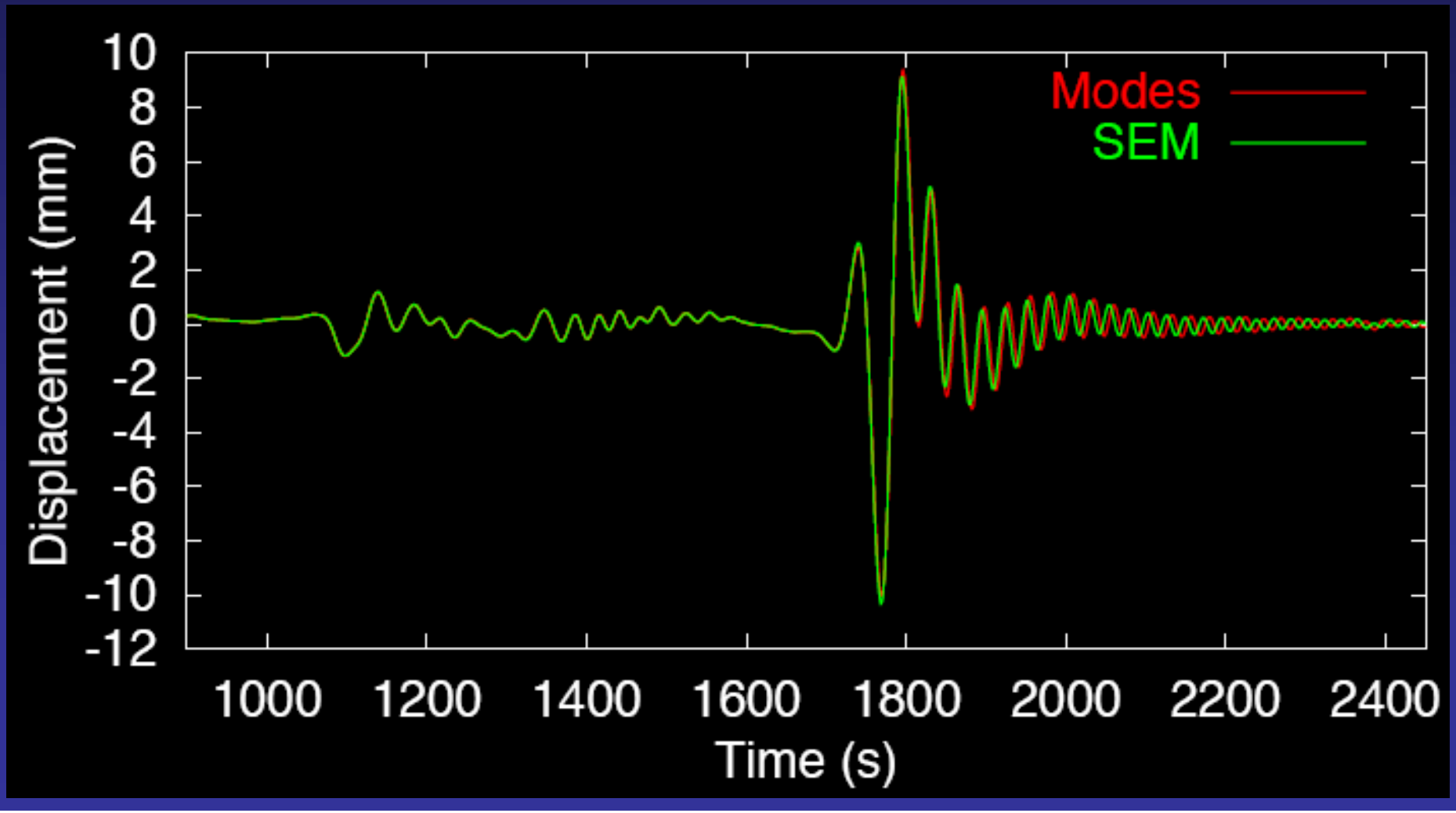


Effect of Ocean



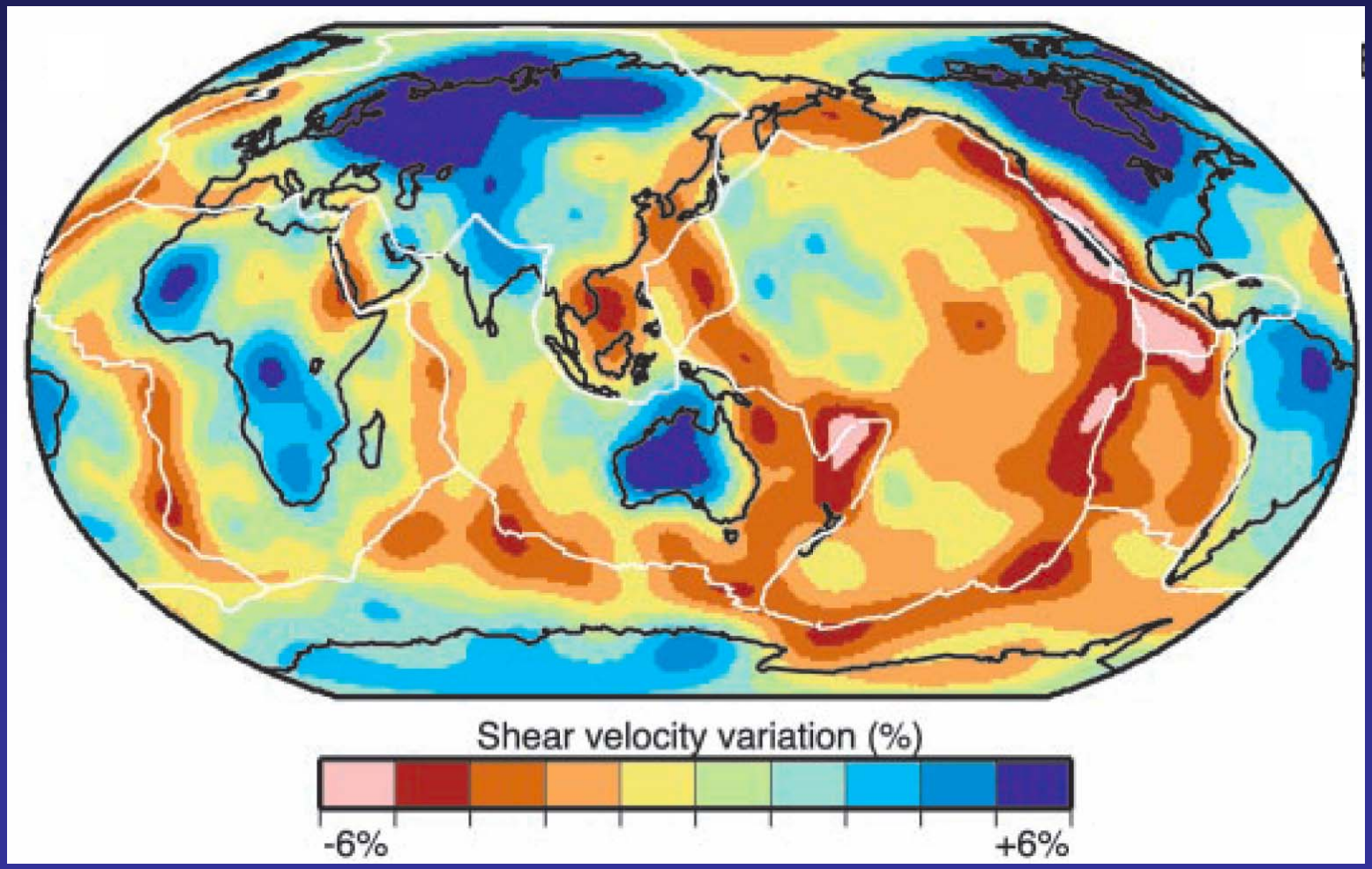
SEM Implementation of Ocean

Modified boundary condition: $p = \rho_w h \hat{\mathbf{n}} \cdot \partial_t^2 \mathbf{s}$



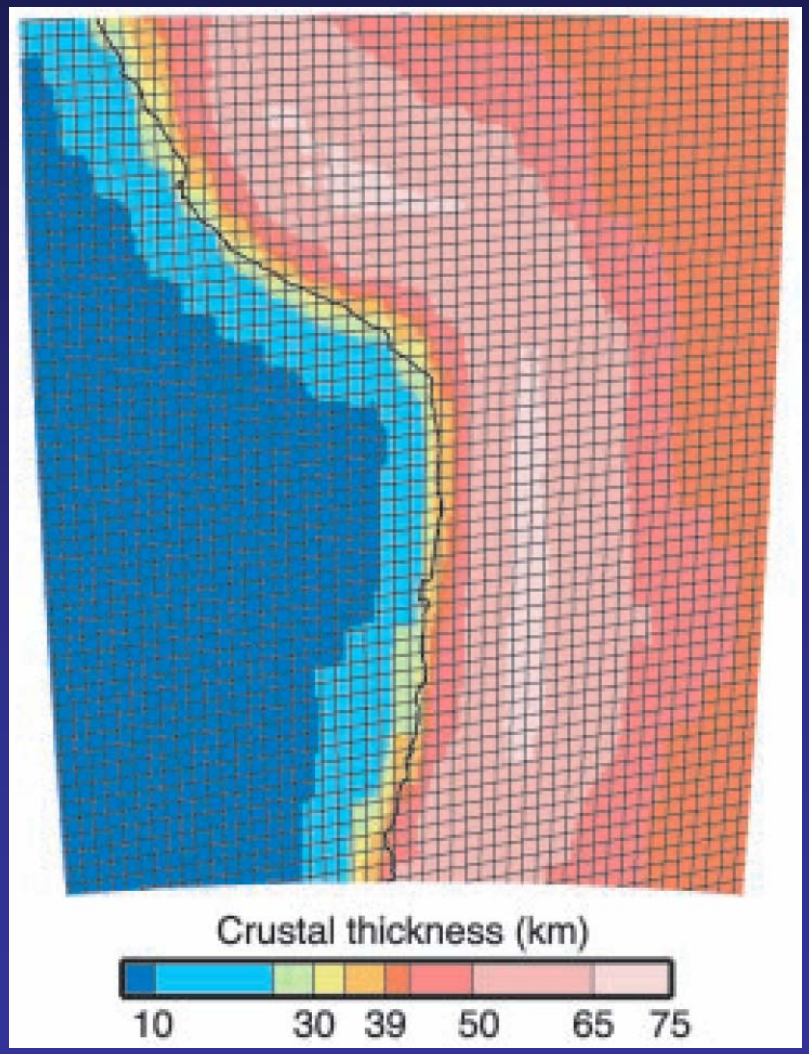
3D Mantle Models

S20RTS (Ritsema et al. 1999)

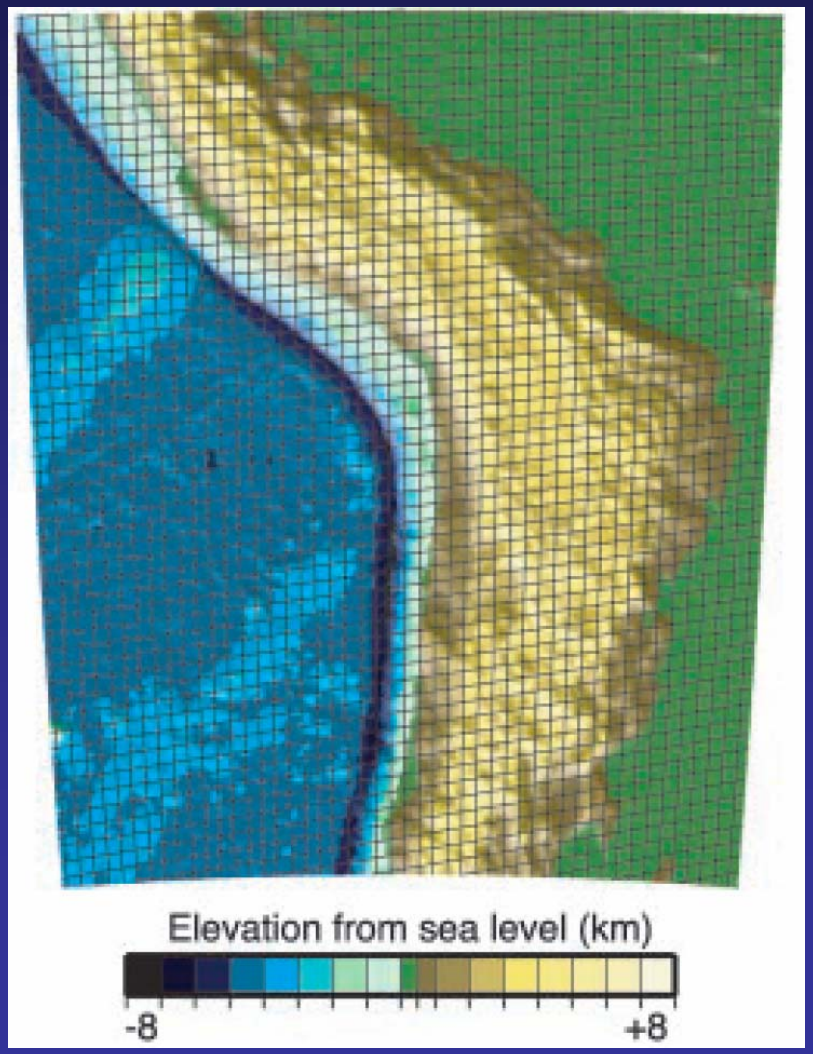


Crustal and Topographic Models

Crust 2.0 (Bassin et al. 2000)

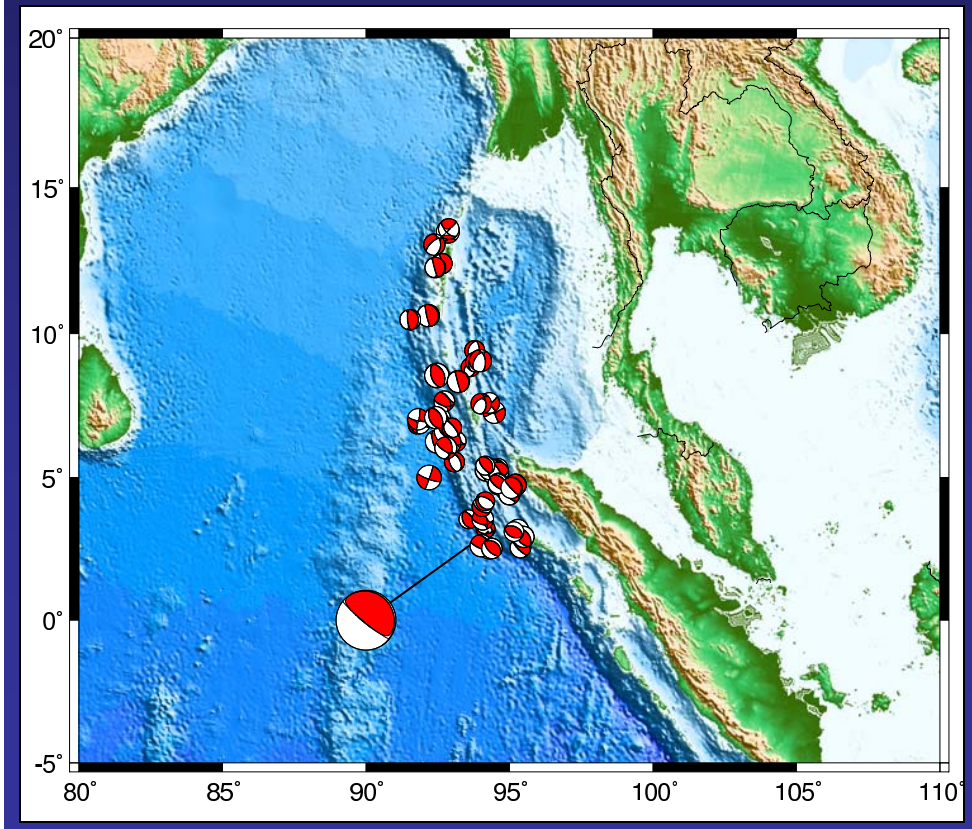


ETOPO5

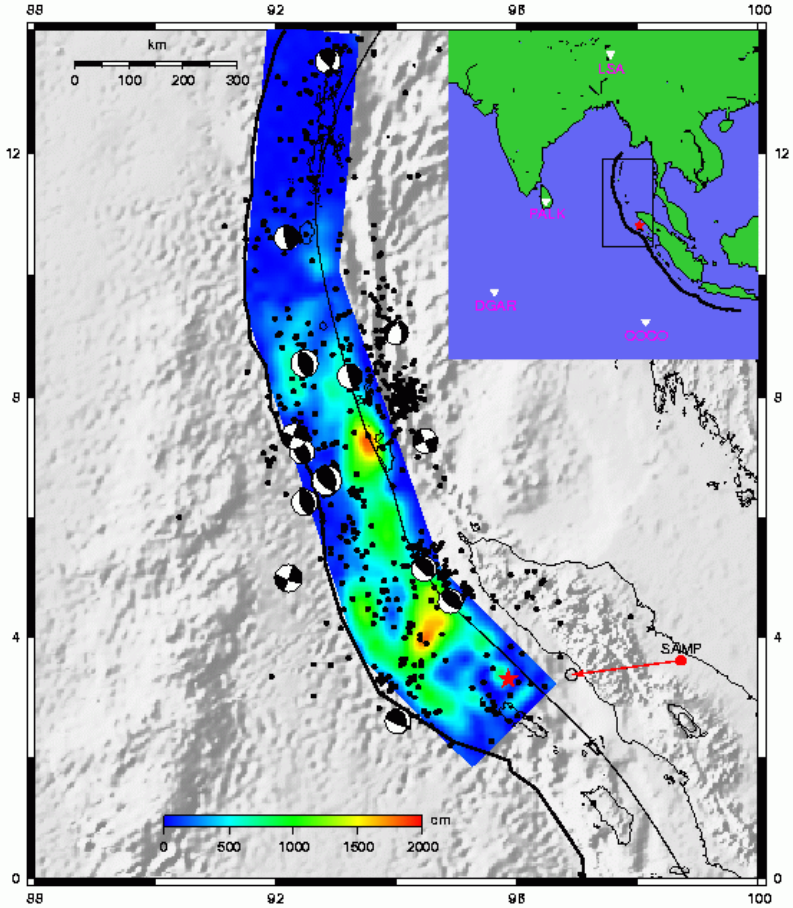


Great 2004 Sumatra-Andaman Earthquake

Main shock & aftershocks (Harvard)



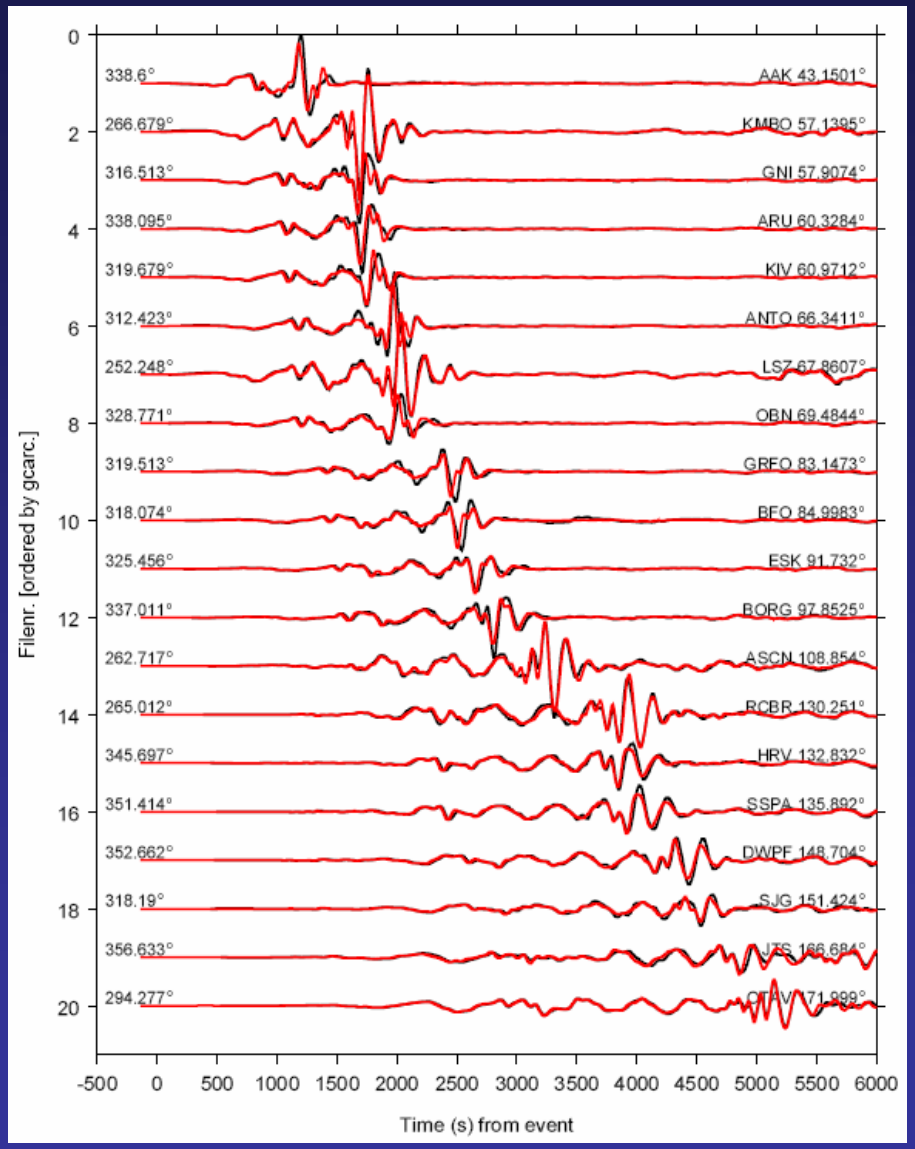
Finite slip model (Chen et al., 2005)

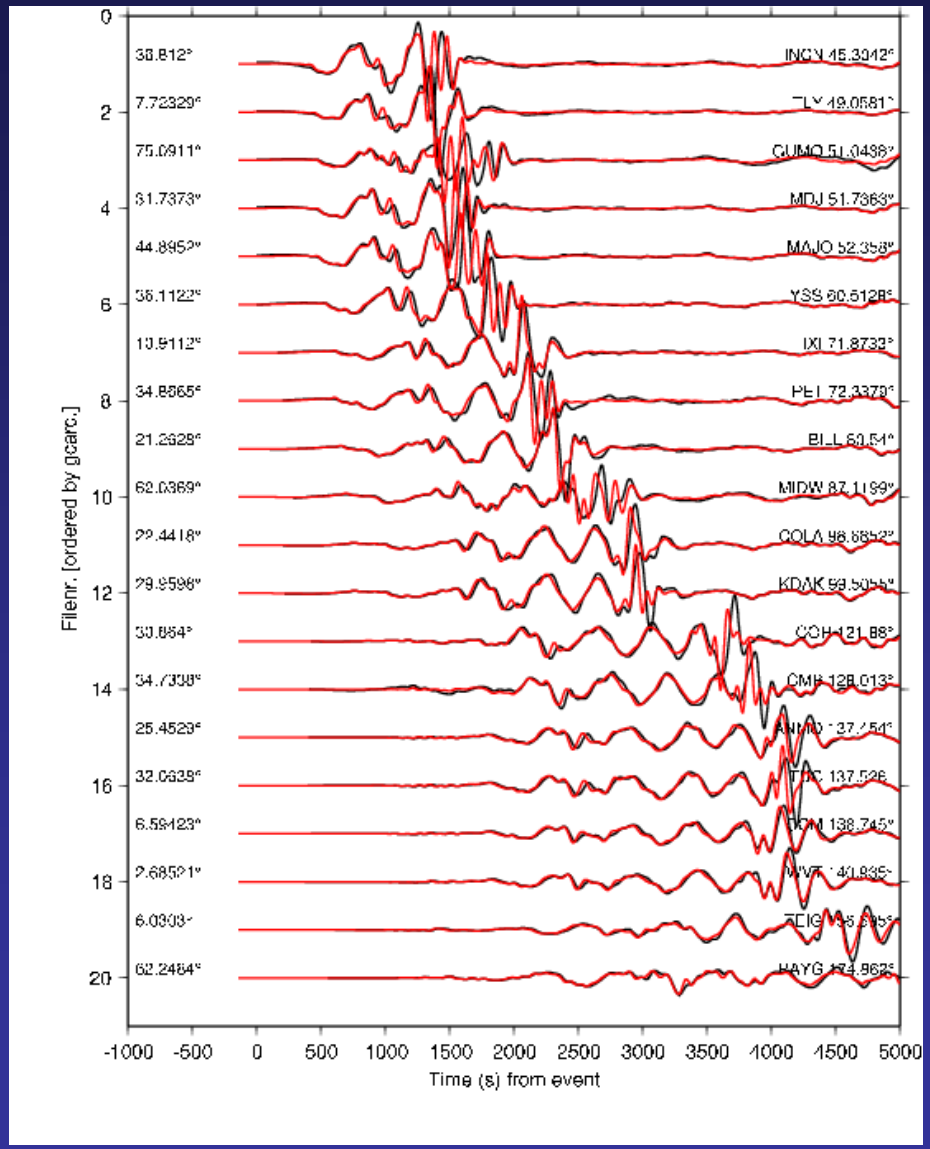


Sumatra Surface Waves

QuickTime™ and a
YUV420 codec decompressor
are needed to see this picture.

Surface-Wave Fits





Vala Hjorleifsdottir

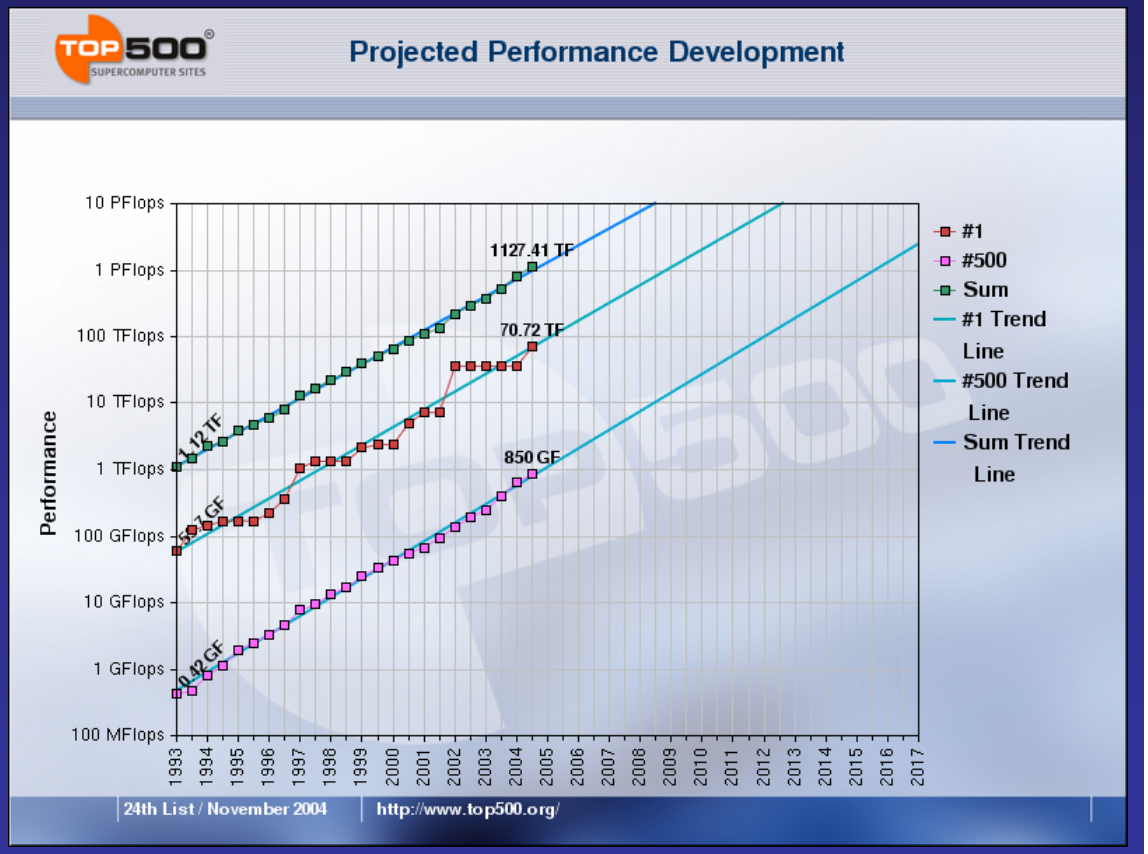
SPECFEM3D_GLOBE: Future Plans

On-demand TeraGrid applications:

- Automated, near real-time simulations of all M>6 earthquakes
- Analysis of past events (more than 20,000 events)
- Seismology Web Portal (prototype available at this meeting)

Petascale simulations:

- Global simulations at 1-2 Hz
- New doubling brick
  (perfect load-balancing)



Adjoint Spectral-Element Simulations (ASEM)

Adjoint Tomography

PDE-constrained waveform tomography:

$$\chi = \tfrac{1}{2}\sum_r \int_0^T [\mathbf{s}(\mathbf{x}_r,t) - \mathbf{d}(\mathbf{x}_r,t)]^2 \,\mathrm{d}t - \int_0^T \int_\Omega \boldsymbol{\lambda}\cdot(\rho\,\partial_t^2\mathbf{s} - \boldsymbol{\nabla}\cdot\mathbf{T} - \mathbf{f})\,\mathrm{d}^3\mathbf{x}\,\mathrm{d}t$$

Change in the waveform misfit function:

$$\begin{aligned}\delta\chi = &\int_0^T \int_\Omega \sum_r [\mathbf{s}(\mathbf{x}_r,t) - \mathbf{d}(\mathbf{x}_r,t)]\delta(\mathbf{x}-\mathbf{x}_r)\cdot\delta\mathbf{s}(\mathbf{x},t)\,\mathrm{d}^3\mathbf{x}\,\mathrm{d}t \\ &- \int_0^T \int_\Omega (\delta\rho\boldsymbol{\lambda}\cdot\partial_t^2\mathbf{s} + \boldsymbol{\nabla}\boldsymbol{\lambda}\colon\delta\mathbf{c}\colon\boldsymbol{\nabla}\mathbf{s} - \boldsymbol{\lambda}\cdot\delta\mathbf{f})\,\mathrm{d}^3\mathbf{x}\,\mathrm{d}t - \int_0^T \int_\Omega [\rho\partial_t^2\boldsymbol{\lambda} - \boldsymbol{\nabla}\cdot(\mathbf{c}\colon\boldsymbol{\nabla}\boldsymbol{\lambda})]\cdot\delta\mathbf{s}\,\mathrm{d}^3\mathbf{x}\,\mathrm{d}t \\ &- \int_\Omega [\rho(\boldsymbol{\lambda}\cdot\partial_t\delta\mathbf{s} - \partial_t\boldsymbol{\lambda}\cdot\delta\mathbf{s})]_T\,\mathrm{d}^3\mathbf{x} - \int_0^T \int_{\partial\Omega} \hat{\mathbf{n}}\cdot(\mathbf{c}\colon\boldsymbol{\nabla}\boldsymbol{\lambda})\cdot\delta\mathbf{s}\,\mathrm{d}^2\mathbf{x}\,\mathrm{d}t,\end{aligned}$$

Adjoint Equations

Adjoint wavefield: $\mathbf{s}^\dagger(\mathbf{x},t) \equiv \boldsymbol{\lambda}(\mathbf{x},T-t)$

Adjoint equation of motion: $\rho\,\partial_t^2\mathbf{s}^\dagger = \boldsymbol{\nabla}\cdot\mathbf{T}^\dagger + \mathbf{f}^\dagger$

Adjoint boundary conditions: $\hat{\mathbf{n}}\cdot\mathbf{T}^\dagger = \mathbf{0}$

Adjoint initial conditions: $\mathbf{s}^\dagger(\mathbf{x},0) = \mathbf{0}, \qquad \partial_t\mathbf{s}^\dagger(\mathbf{x},0) = \mathbf{0}$

Adjoint source: $\mathbf{f}^\dagger(\mathbf{x},t) = \sum_{r=1}^{N}[\mathbf{s}(\mathbf{x}_r,T-t) - \mathbf{d}(\mathbf{x}_r,T-t)]\delta(\mathbf{x}-\mathbf{x}_r)$

Frechet derivative

The Frechet derivative may be expressed as:

$$\delta\chi = \int_\Omega (\delta\rho K_\rho + \delta\mathbf{c} :: \mathbf{K_c})\, \mathrm{d}^3\mathbf{x} + \int_0^T \int_\Omega \mathbf{s}^\dagger \cdot \delta\mathbf{f}\, \mathrm{d}^3\mathbf{x}\, \mathrm{d}t$$

Density and elastic tensor kernels:

$$K_\rho(\mathbf{x}) = -\int_0^T \mathbf{s}^\dagger(\mathbf{x}, T-t) \cdot \partial_t^2 \mathbf{s}(\mathbf{x}, t)\, \mathrm{d}t$$

$$\mathbf{K_c}(\mathbf{x}) = -\int_0^T \boldsymbol{\nabla}\mathbf{s}^\dagger(\mathbf{x}, T-t)\, \boldsymbol{\nabla}\mathbf{s}(\mathbf{x}, t)\, \mathrm{d}t$$

Isotropic Kernels

For isotropic perturbations we have:

$$\delta \mathbf{c} :: \mathbf{K_c} = \delta \ln \mu \, K_\mu + \delta \ln \kappa \, K_\kappa$$

where

$$K_\mu(\mathbf{x}) = -\int_0^T 2\mu(\mathbf{x}) \mathbf{D}^\dagger(\mathbf{x}, T-t) : \mathbf{D}(\mathbf{x}, t) \, dt$$

$$K_\kappa(\mathbf{x}) = -\int_0^T \kappa(\mathbf{x}) [\boldsymbol{\nabla} \cdot \mathbf{s}^\dagger(\mathbf{x}, T-t)][\boldsymbol{\nabla} \cdot \mathbf{s}(\mathbf{x}, t)] \, dt$$

and we have defined the strain deviators

$$\mathbf{D} = \tfrac{1}{2}[\boldsymbol{\nabla}\mathbf{s} + (\boldsymbol{\nabla}\mathbf{s})^T] - \tfrac{1}{3}(\boldsymbol{\nabla} \cdot \mathbf{s})\mathbf{I},$$

$$\mathbf{D}^\dagger = \tfrac{1}{2}[\boldsymbol{\nabla}\mathbf{s}^\dagger + (\boldsymbol{\nabla}\mathbf{s}^\dagger)^T] - \tfrac{1}{3}(\boldsymbol{\nabla} \cdot \mathbf{s}^\dagger)\mathbf{I}$$

In terms of wave speeds:

$$\delta \ln \rho \, K_\rho + \delta \ln \mu \, K_\mu + \delta \ln \kappa \, K_\kappa = \delta \ln \rho \, K_{\rho'} + \delta \ln \beta \, K_\beta + \delta \ln \alpha \, K_\alpha$$

where

$$K_{\rho'} = K_\rho + K_\kappa + K_\mu \qquad K_\beta = 2\left(K_\mu - \frac{4}{3}\frac{\mu}{\kappa} K_\kappa\right) \qquad K_\alpha = 2\left(\frac{\kappa + \frac{4}{3}\mu}{\kappa}\right) K_\kappa$$

Traveltime Frechet Derivatives

Traveltime tomography:

$$\chi(m) = \frac{1}{2}\sum_{r=1}^{N}[T_r(m) - T_r^{\mathrm{obs}}]^2$$

Change in the misfit function:

$$\delta\chi = \sum_{r=1}^{N}[T_r(m) - T_r^{\mathrm{obs}}]\delta T_r = \int K(\mathbf{x})\,\delta\ln m(\mathbf{x})\,\mathrm{d}^3\mathbf{x}$$

Traveltime anomaly in terms of *banana-donut* kernel (Dahlen):

$$\delta T_r = \int K_r(\mathbf{x})\,\delta\ln m(\mathbf{x})\,\mathrm{d}^3\mathbf{x}$$

The kernel K is a weighted sum of banana-donut kernels K_r :

$$K(\mathbf{x}) = \sum_{r=1}^{N}[T_r(m) - T_r^{\mathrm{obs}}]\,K_r(\mathbf{x})$$

Adjoint Wavefield

Kernel in terms of the *adjoint* wavefield:

$$K(\mathbf{x}) = \sum_{r=1}^{N} [T_r(m) - T_r^{\mathrm{obs}}]\, K_r(\mathbf{x}) = \int_0^T \mathbf{D}(\mathbf{x},t) : \mathbf{D}^\dagger(\mathbf{x}, T-t)\, \mathrm{d}t$$

The adjoint wavefield $\mathbf{s}^\dagger$ is generated by the adjoint source

$$\mathbf{f}^\dagger(\mathbf{x},t) = \sum_{r=1}^{N} [T_r(m) - T_r^{\mathrm{obs}}]\, \partial_t \mathbf{s}(\mathbf{x}_r, T-t)\delta(\mathbf{x}-\mathbf{x}_r)$$

Notes:

- Need simultaneous access to the regular wavefield at time *t* and the adjoint wavefield at time *T – t*
- Use of the time-reversed velocity as the source for the adjoint wavefield

Numerical Implementation

$$K_\rho(\mathbf{x}) = -\int_0^T \mathbf{s}^\dagger(\mathbf{x}, T-t) \cdot \partial_t^2 \mathbf{s}(\mathbf{x}, t)\, \mathrm{d}t$$

Need simultaneous access to $\mathbf{s}^\dagger(\mathbf{x}, T-t)$ and $\mathbf{s}(\mathbf{x}, t)$

- During calculation of adjoint field $\mathbf{s}^\dagger$, reconstruct $\mathbf{s}$ by solving the `backward' wave equation

  Need to store from a previous forward simulation:

  – Last snapshot $\mathbf{s}(\mathbf{x}, T)$

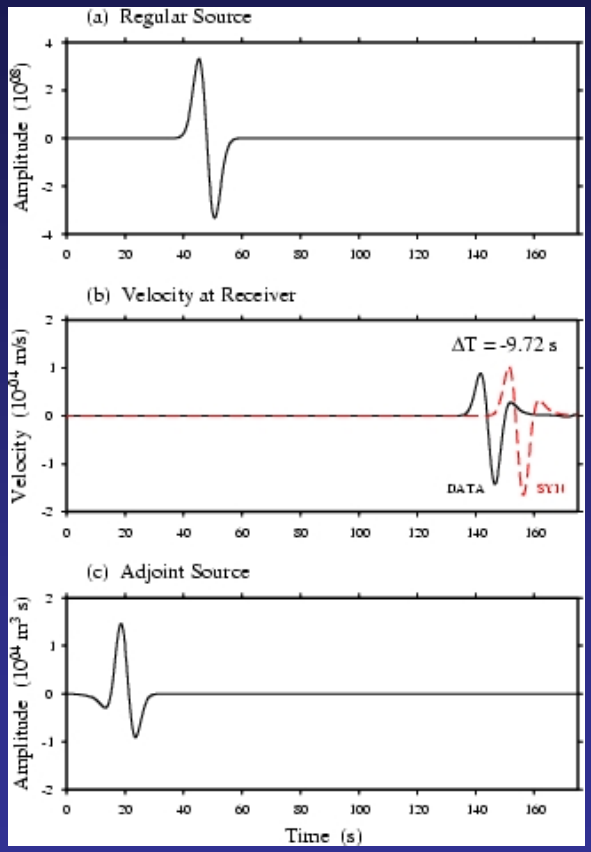
  – Wavefield absorb on artificial boundaries

  Challenge:

  – `Undoing' attenuation

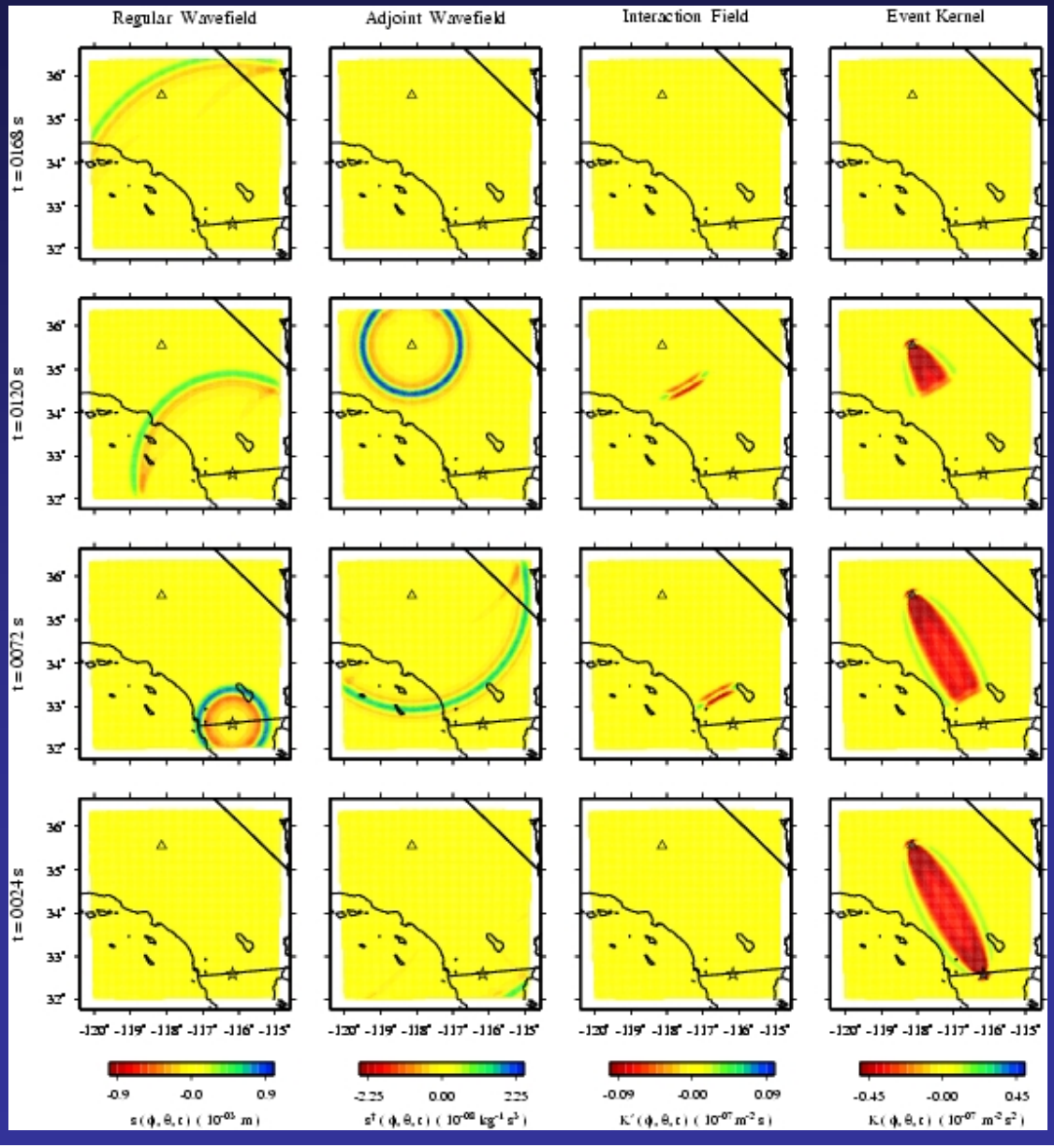
2D Adjoint Tomography

Construction of a Banana-Donut Kernel

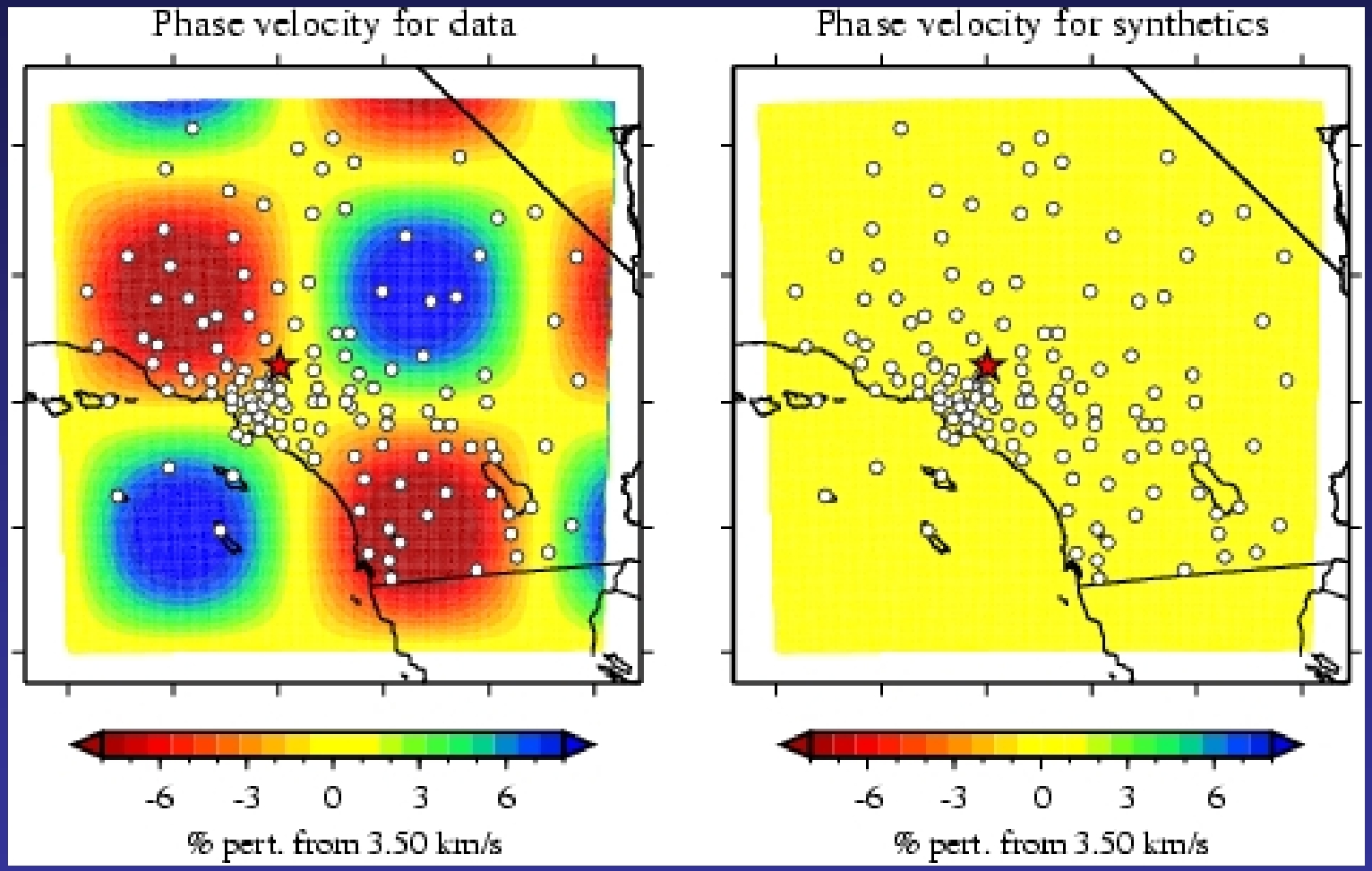


$$\delta T_r = \int K_r(\mathbf{x})\, \delta \ln m(\mathbf{x})\, \mathrm{d}^3\mathbf{x}$$

Tape et al. 2006

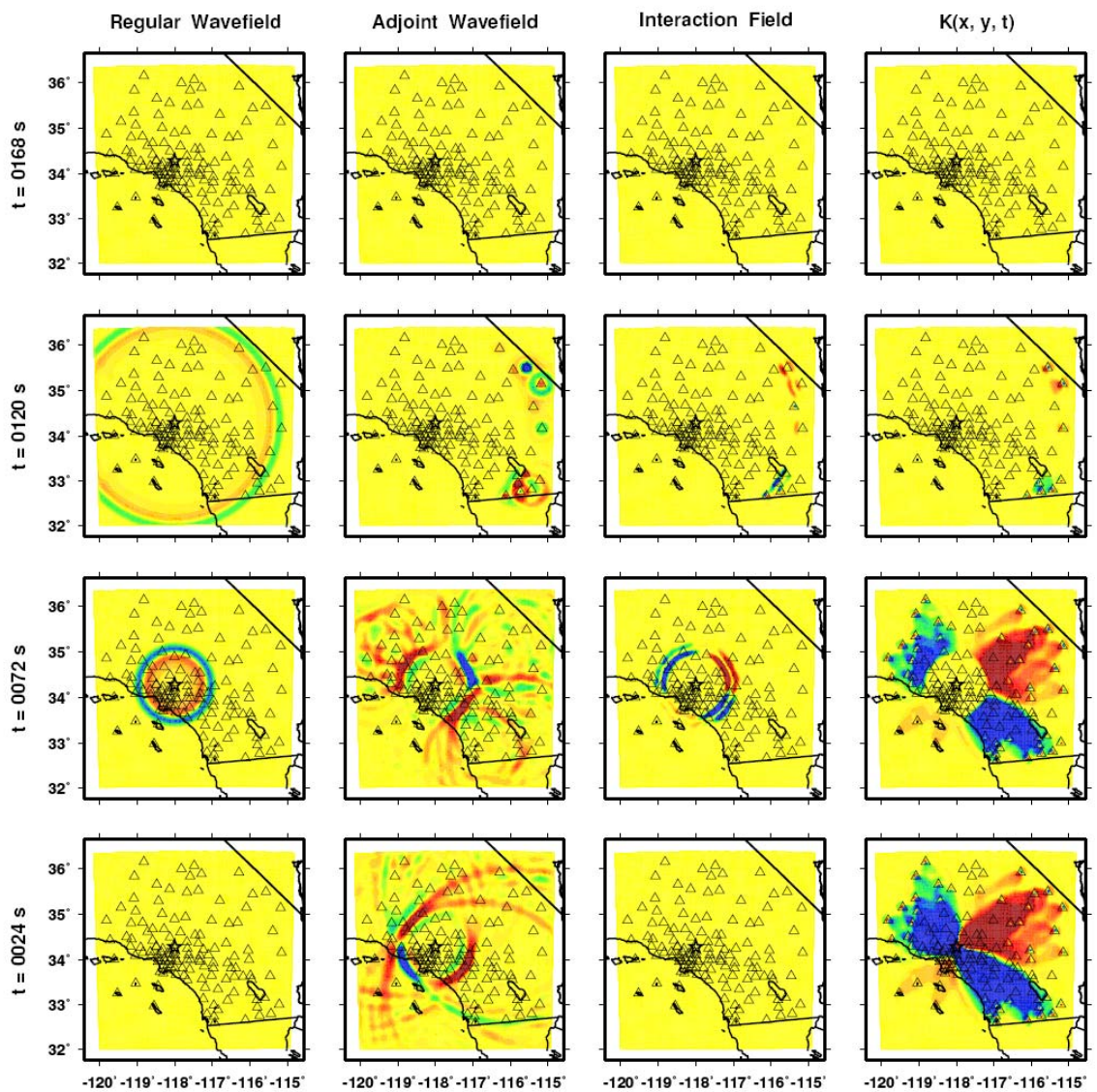


Adjoint Tomography



Construction of an Event Kernel

$$K(\mathbf{x}) = \int_0^T \mathbf{D}(\mathbf{x}, t) : \mathbf{D}^\dagger(\mathbf{x}, T-t)\, dt$$

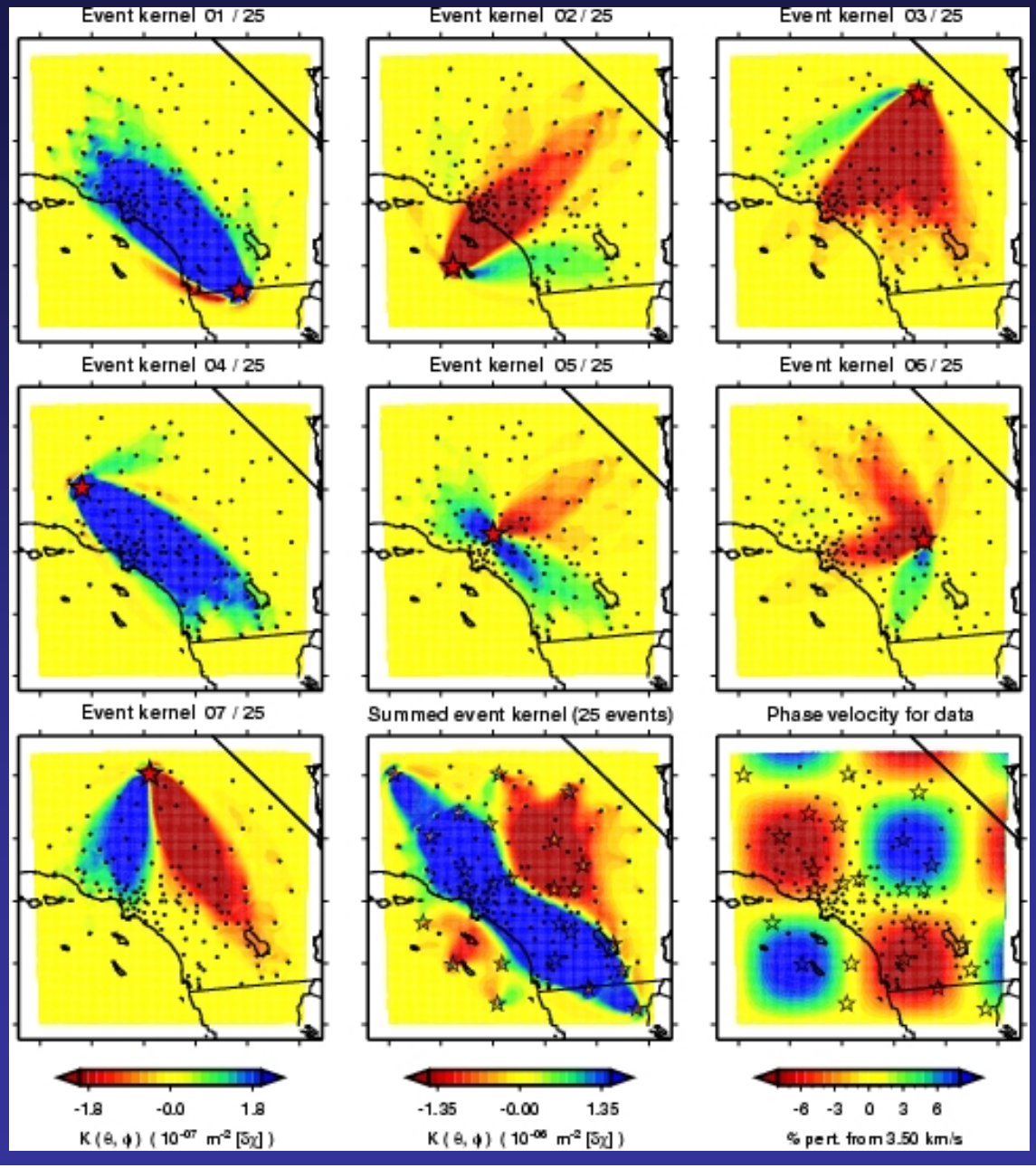


Event Kernel:

- Sum of weighted banana-donut kernels
- Two simulations per event

$$K(\mathbf{x}) = \sum_{r=1}^{N} [T_r(m) - T_r^{\mathrm{obs}}]\, K_r(\mathbf{x})$$

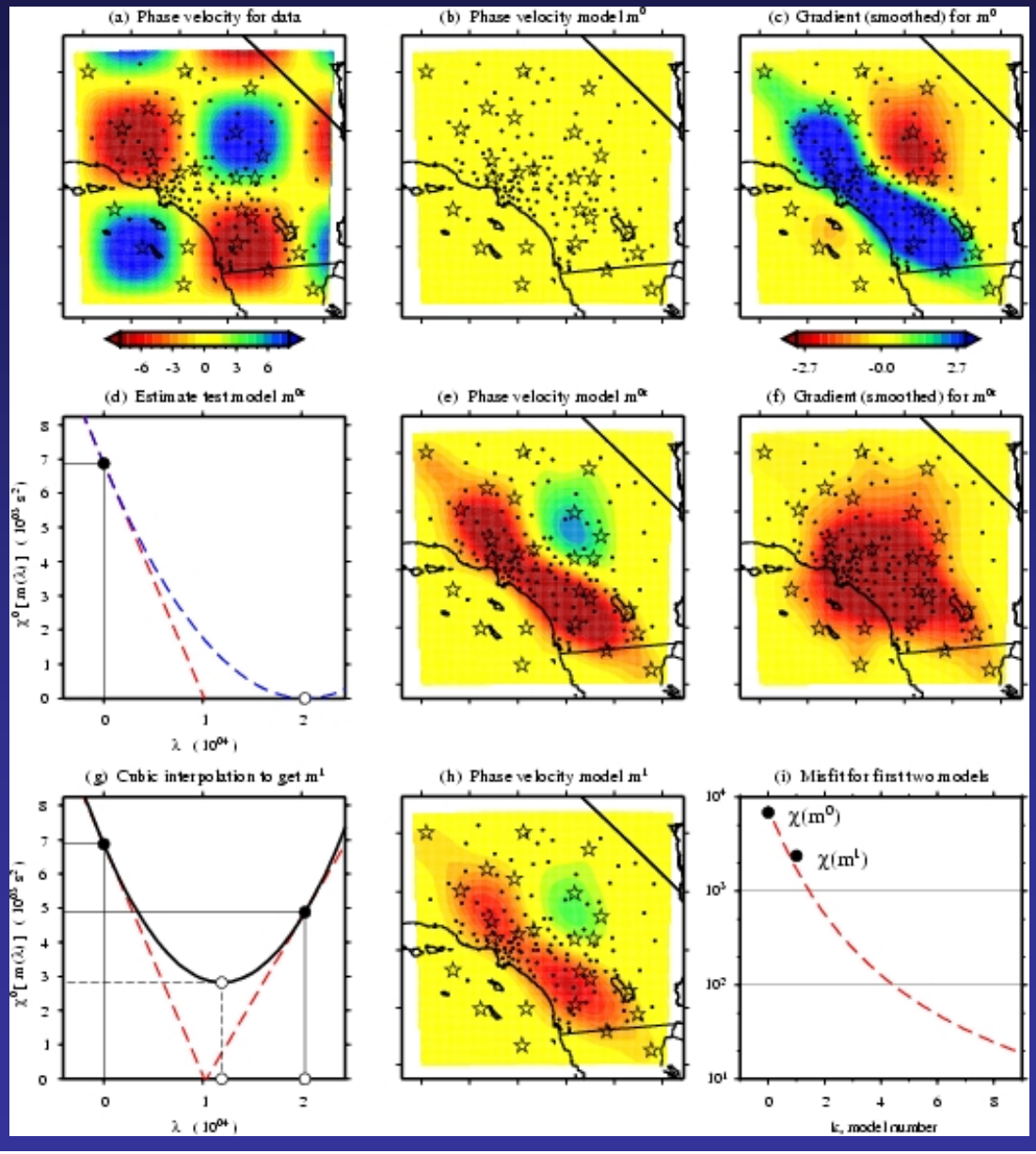
Construction of the Misfit Kernel



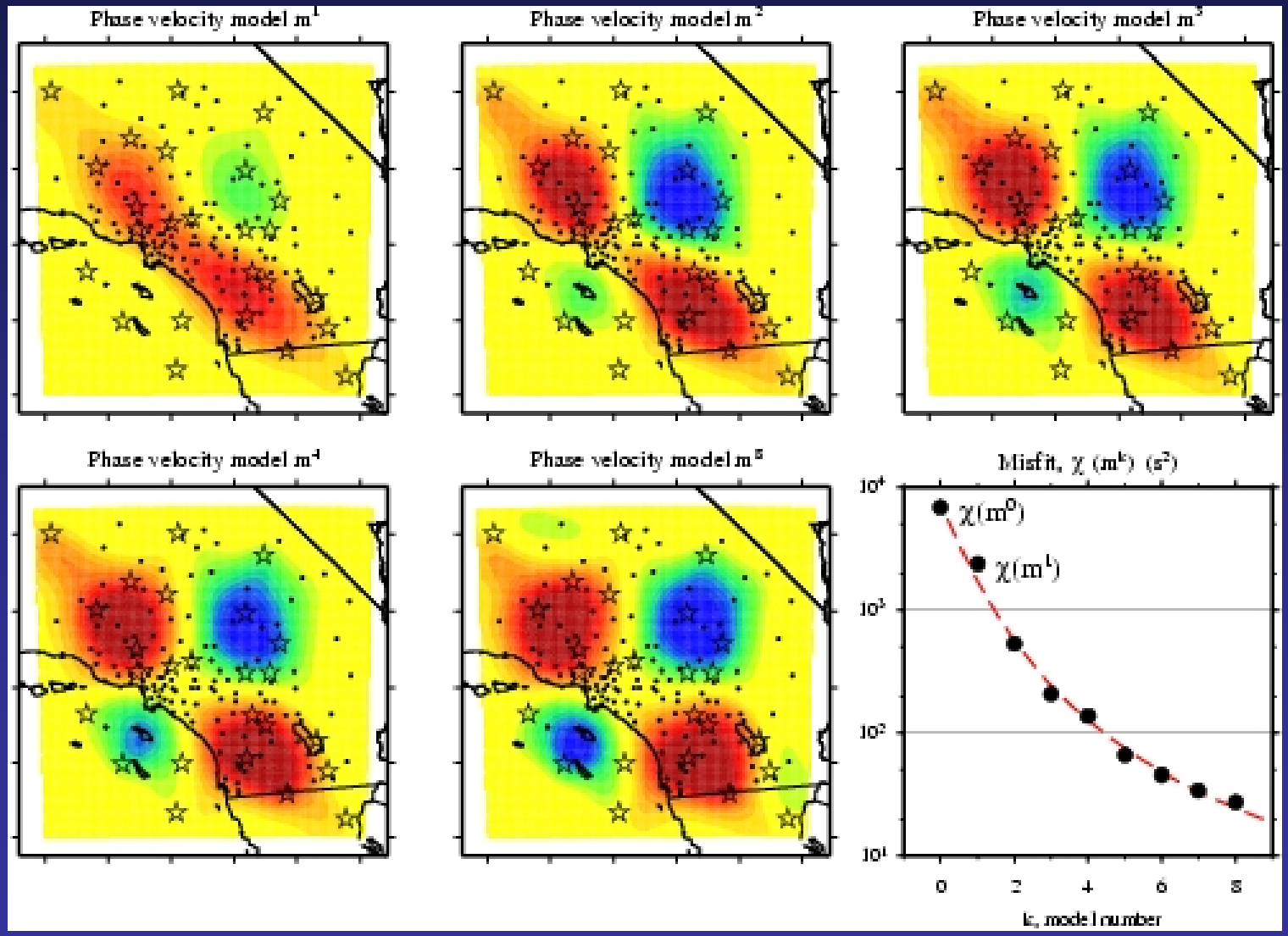
Misfit kernel:

- Sum of all the event kernels
- Two simulations per event
- Gradient of misfit function

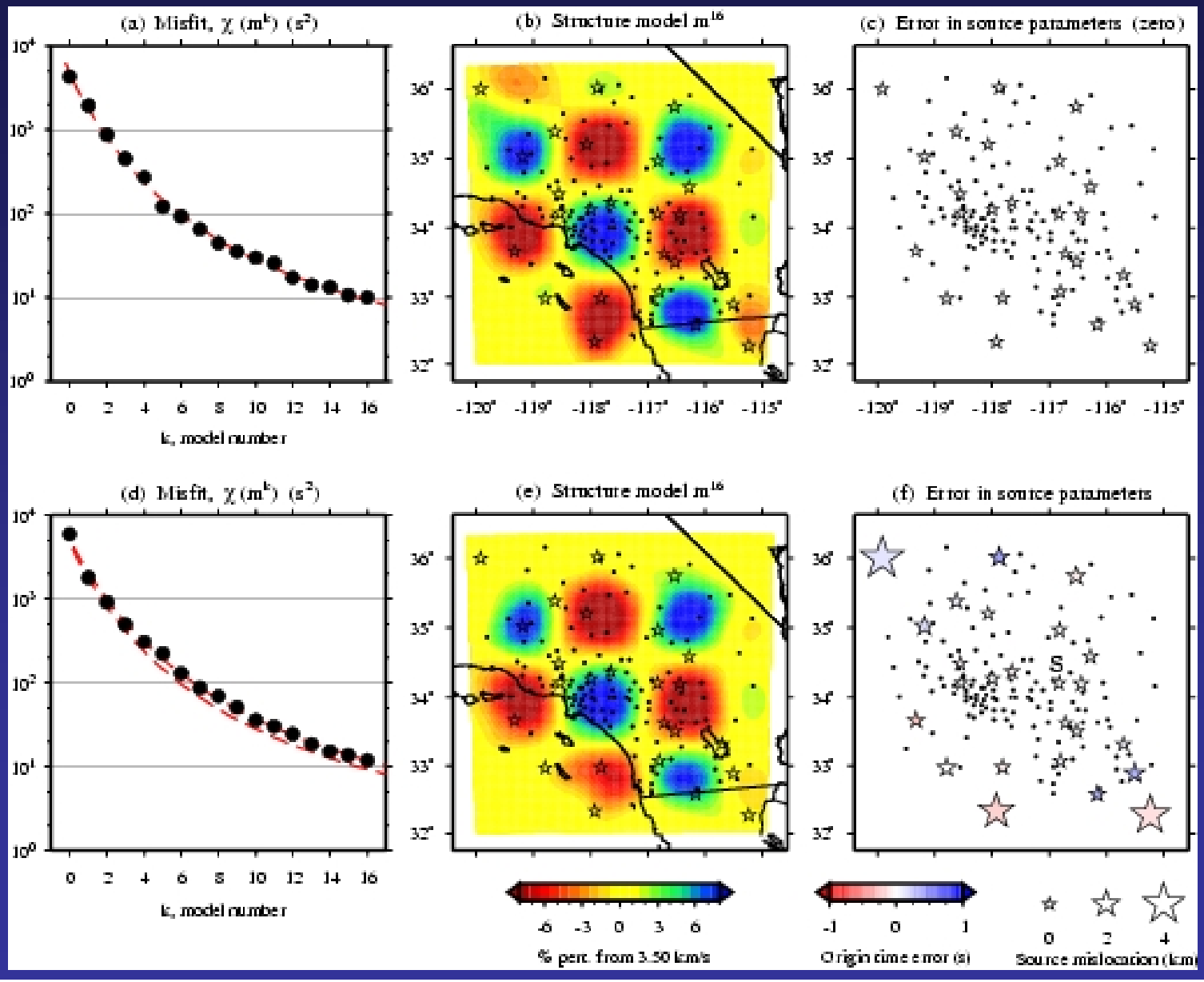
Conjugate Gradient Algorithm



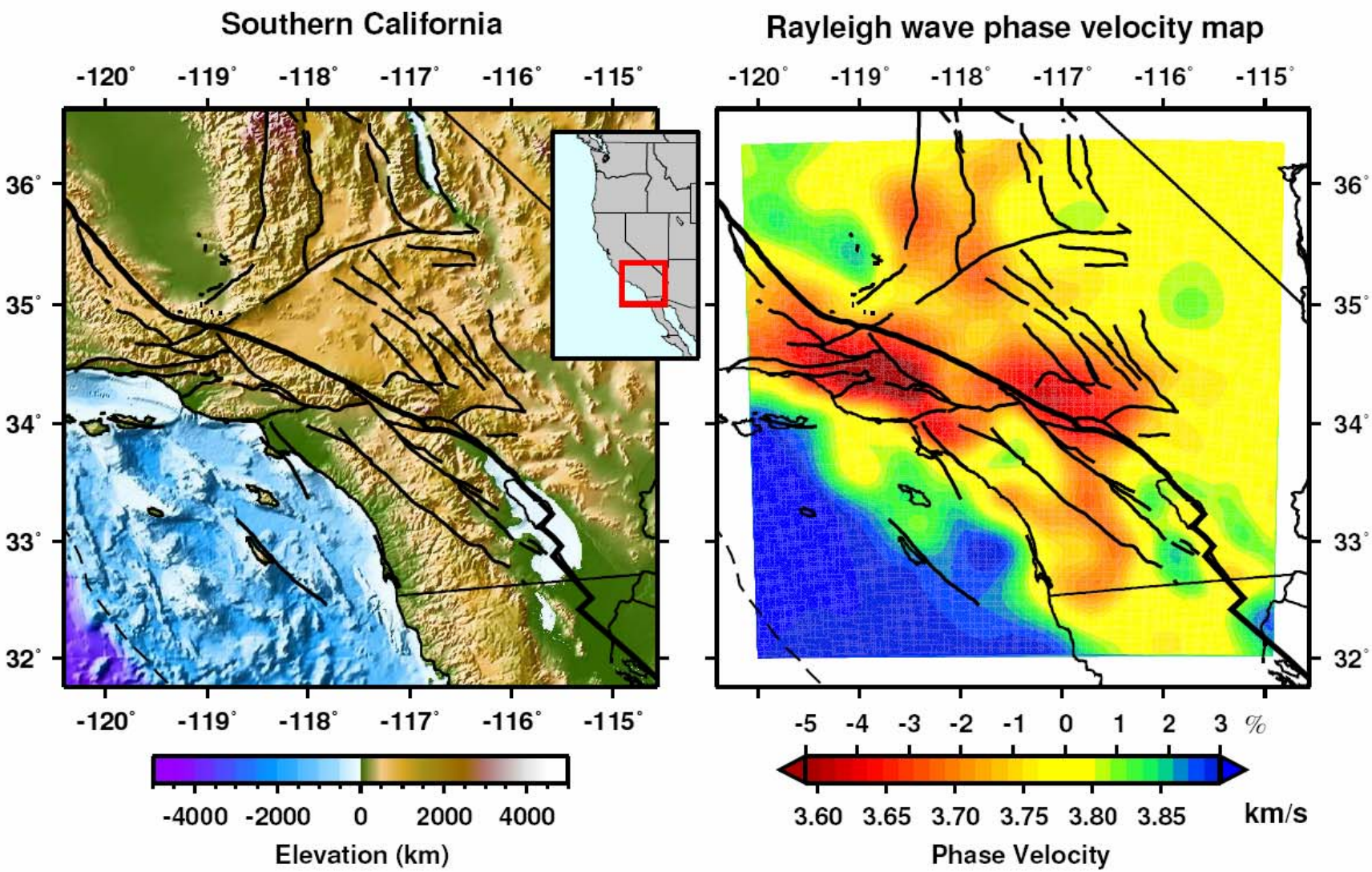
Conjugate Gradient Algorithm



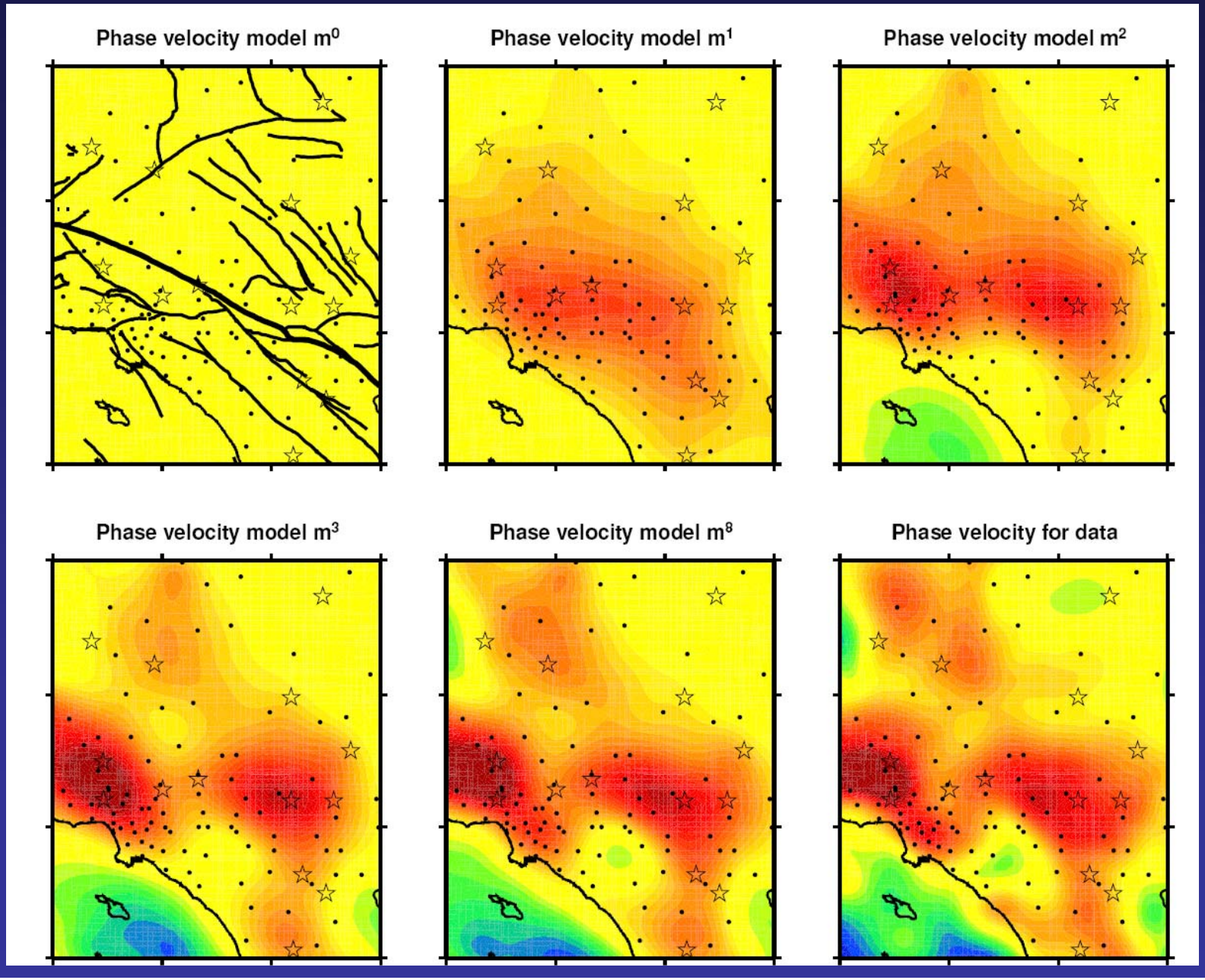
Joint Structure-Source Inversion



A Southern California example



Southern California Rayleigh Waves

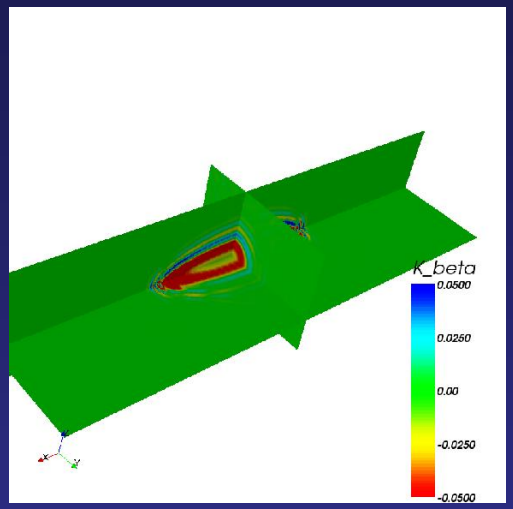


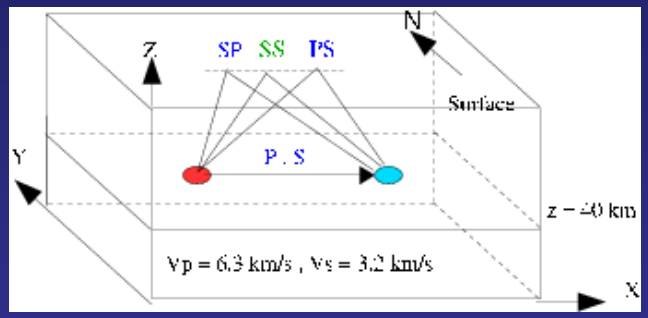
25 events

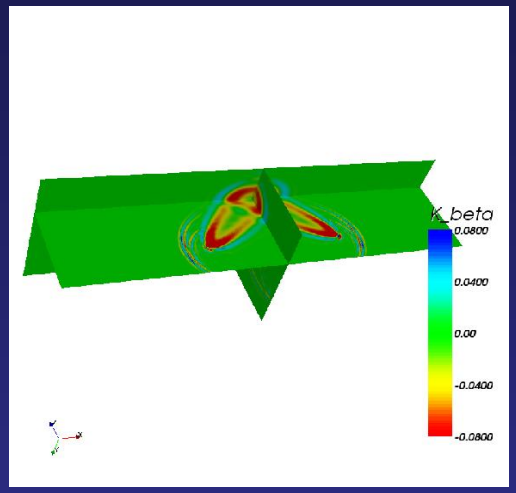
Toward 3D Tomography: SPECFEM3D Adjoint Capabilities

3D Sensitivity Kernels

S

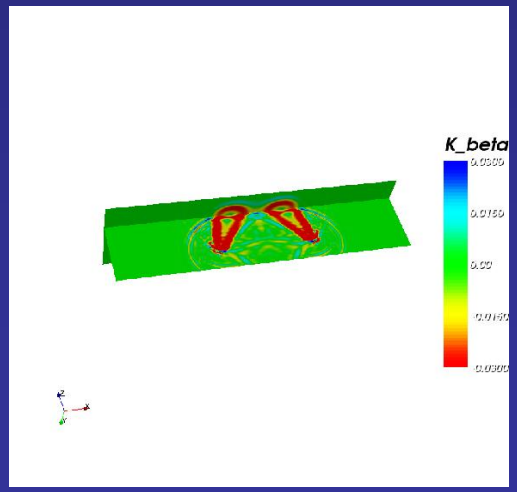


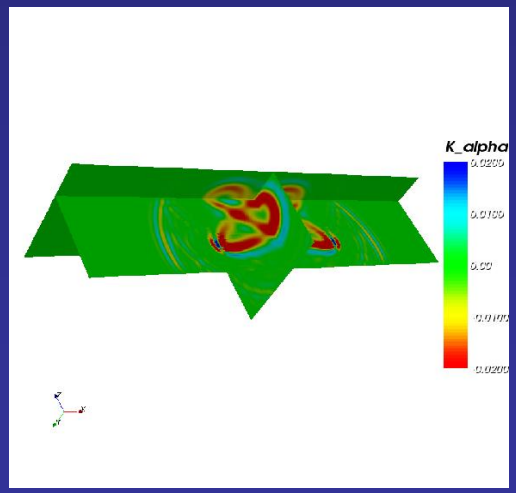




SS

PS



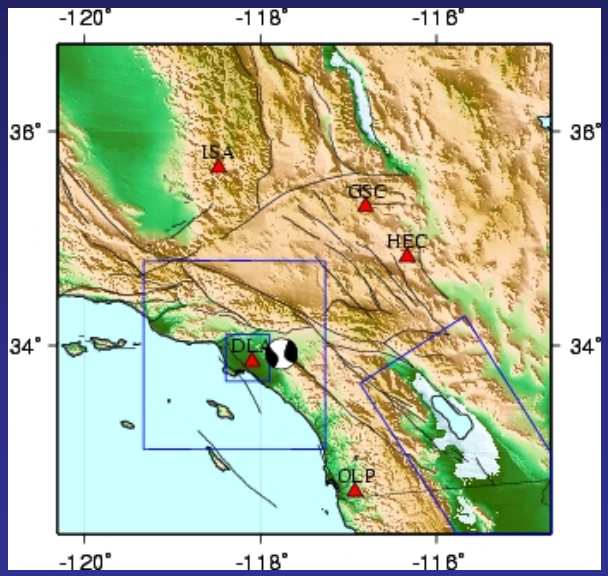


PS

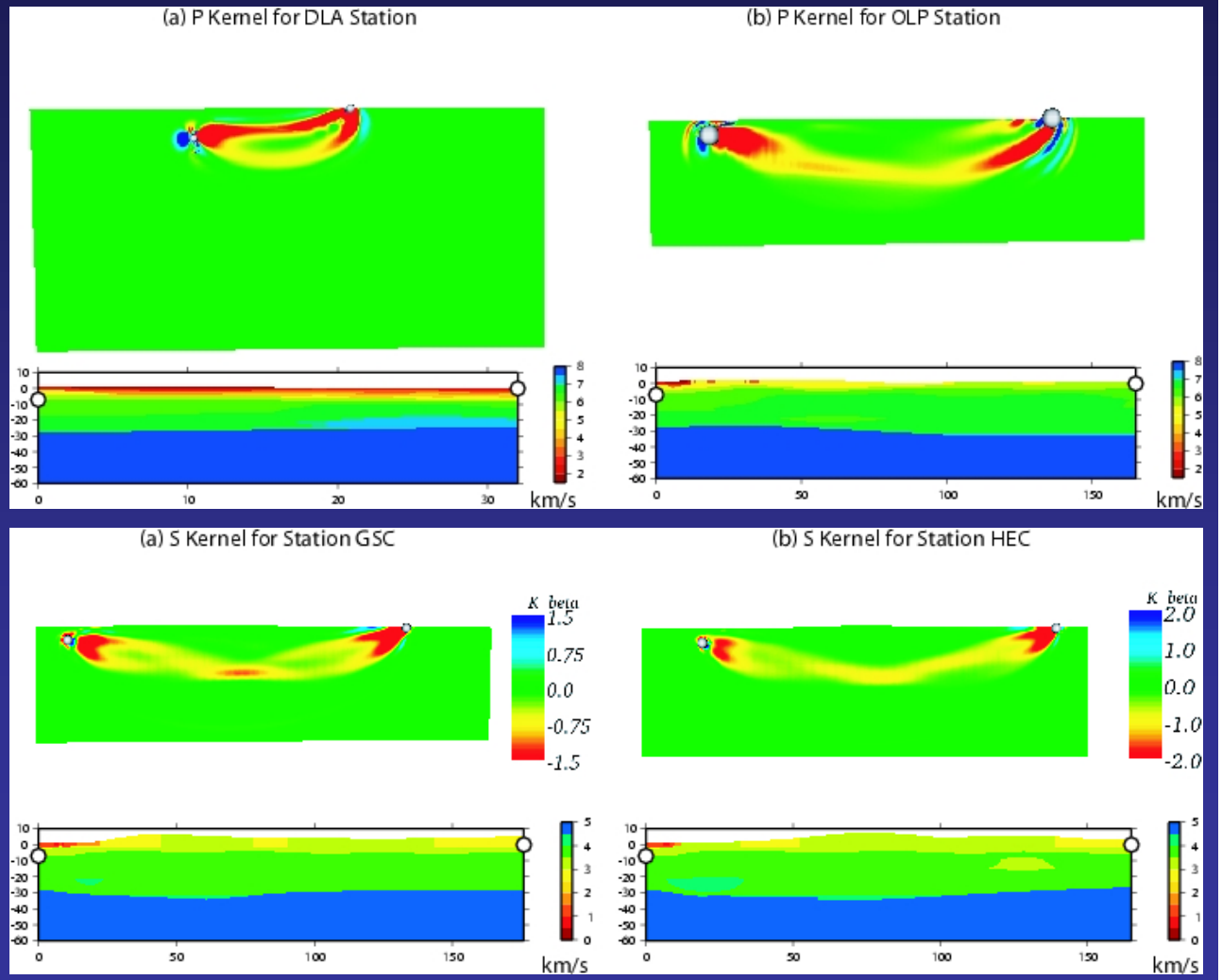
SPECFEM3D_BASIN

Liu & Tromp 2006

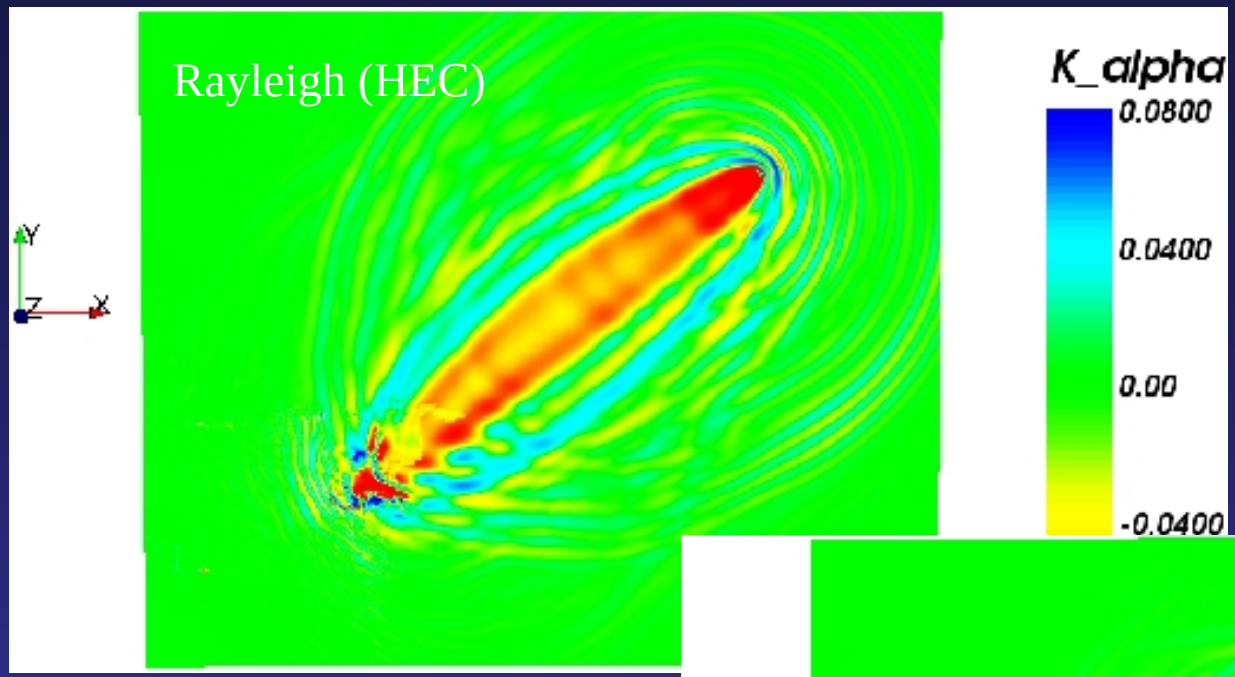
3D Body-Wave Sensitivity Kernels

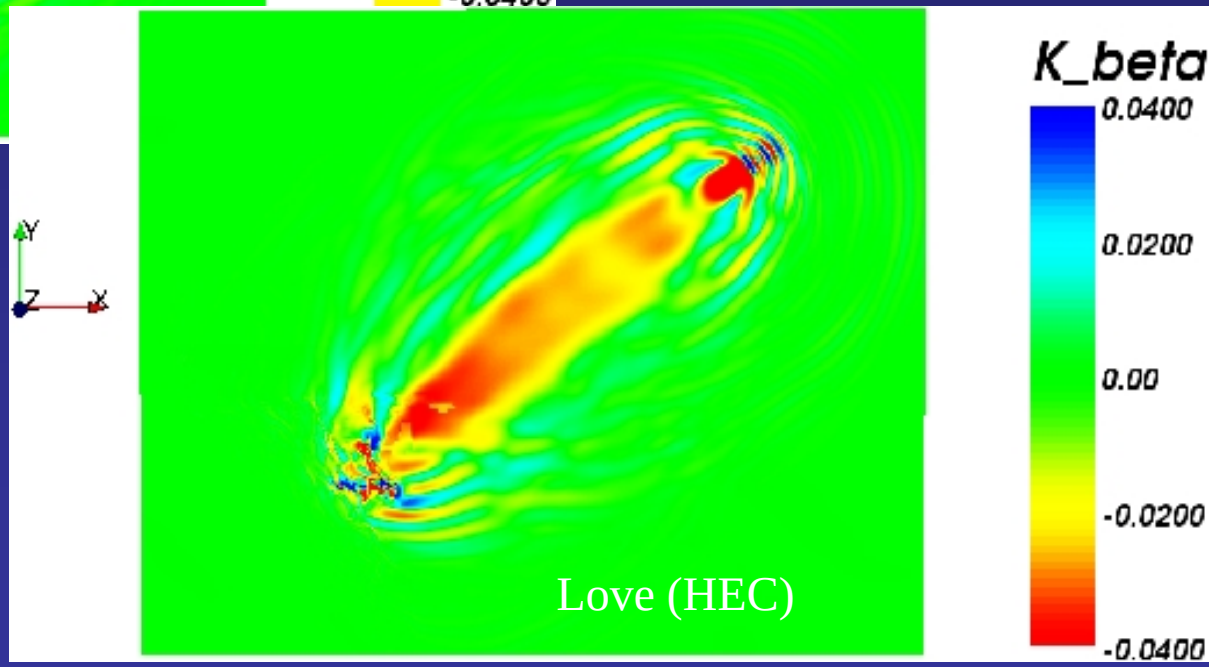


September 3, 2002,
M=4.2 Yorba Linda

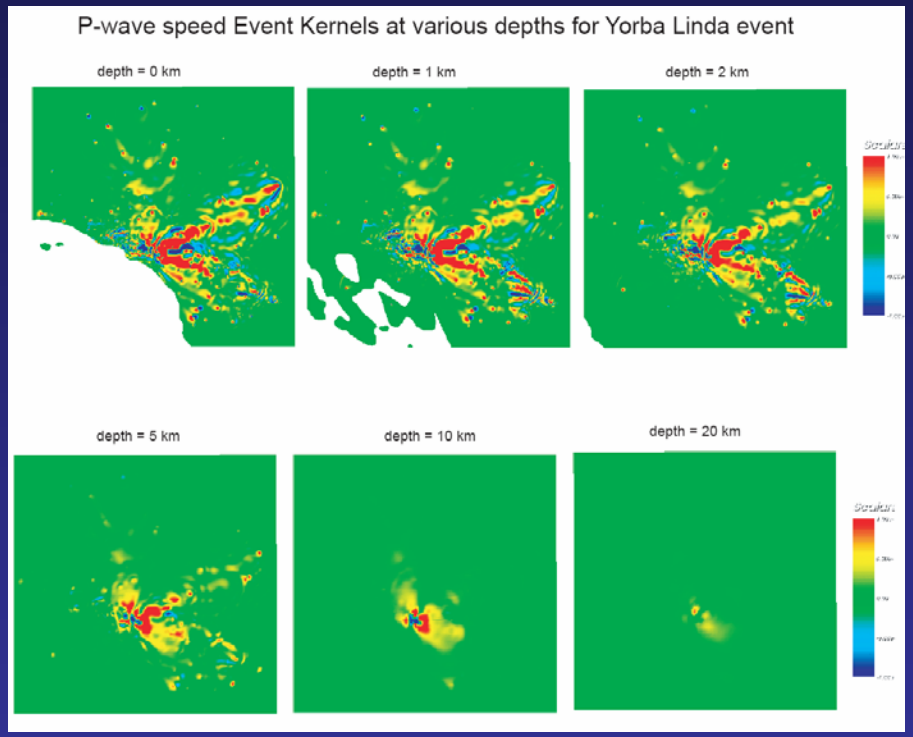


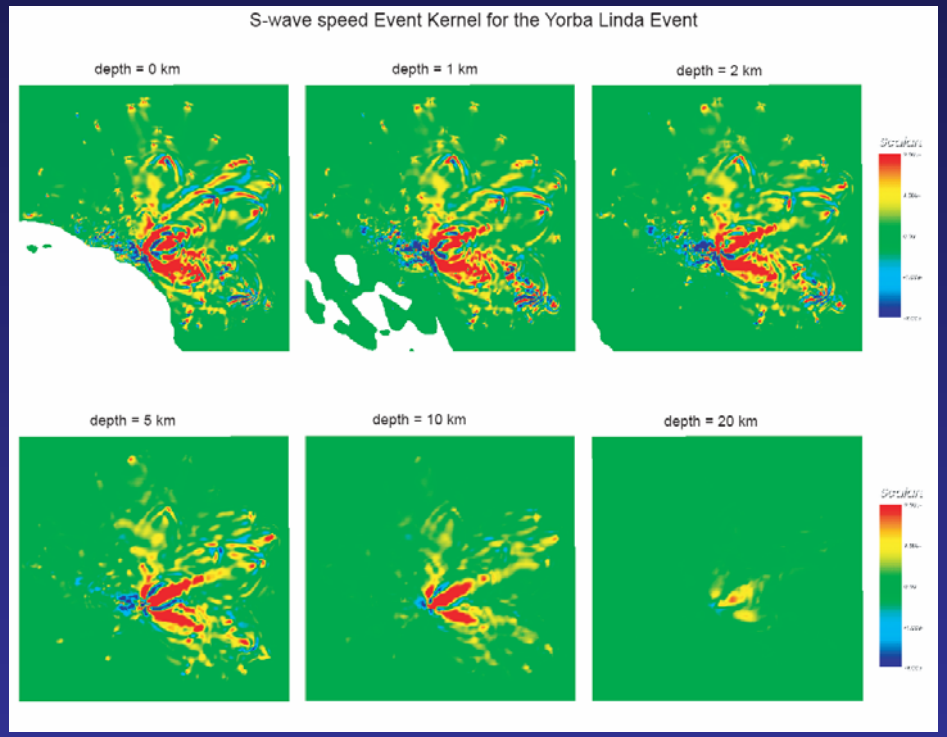
3D Surface-Wave Sensitivity Kernels





Our First 3D Event Kernels!

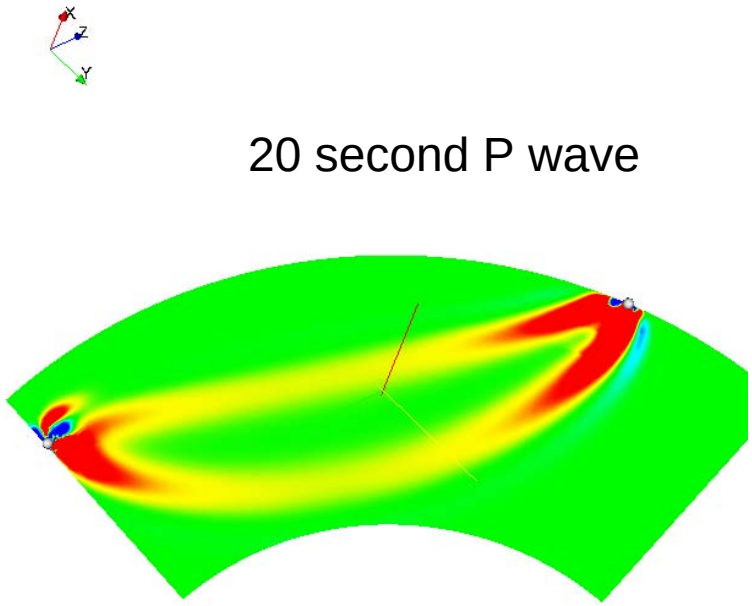




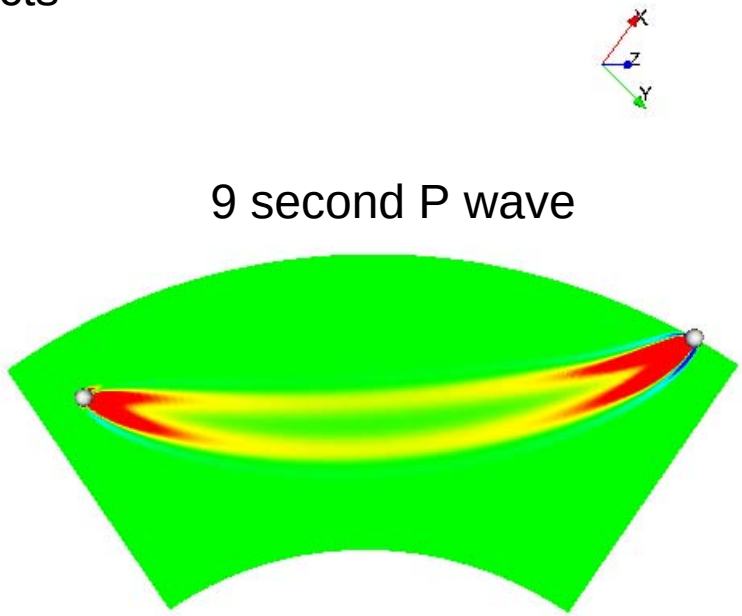
Global 3D Body-Wave Sensitivity Kernels

Finite-Frequency effects

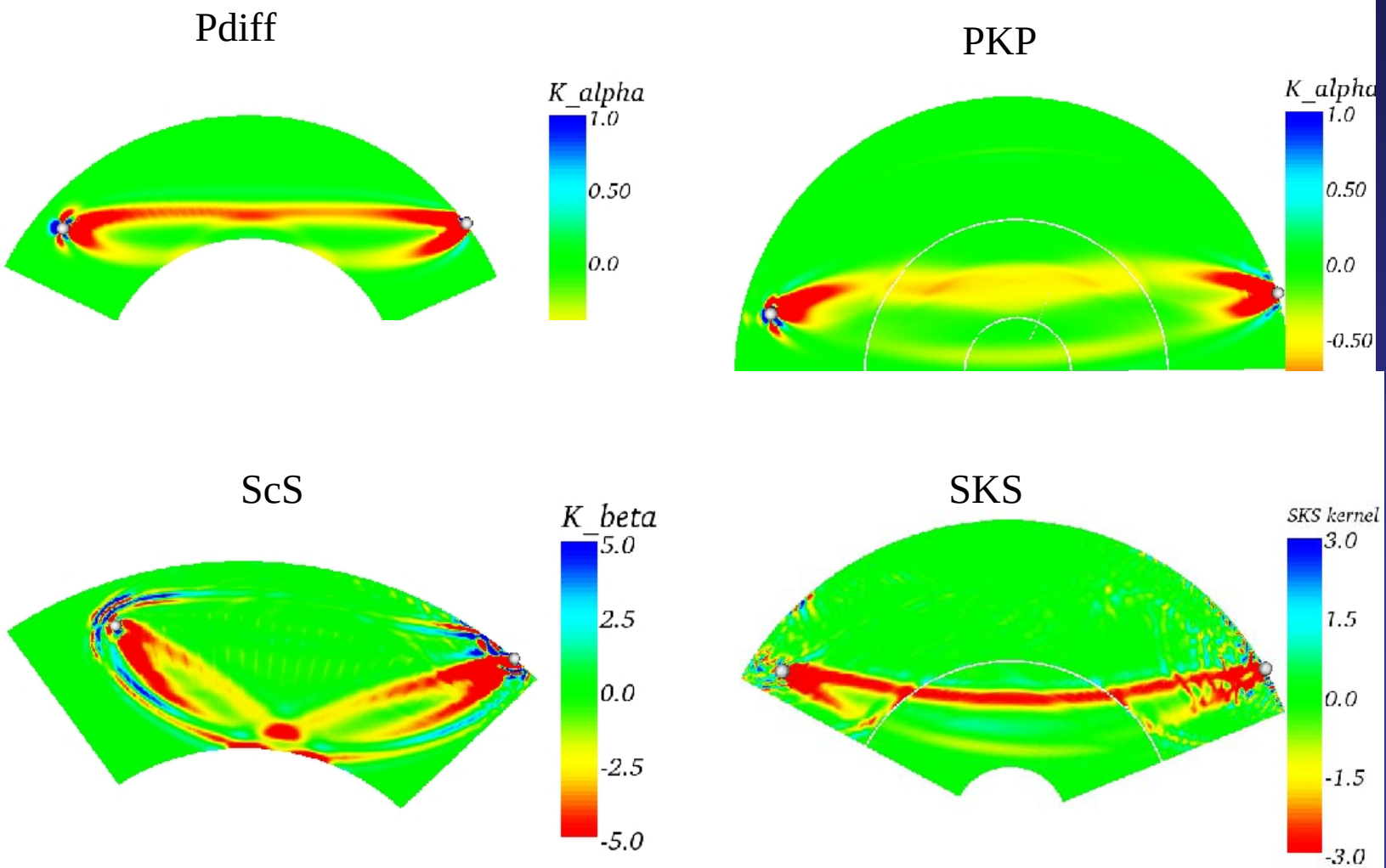
20 second P wave



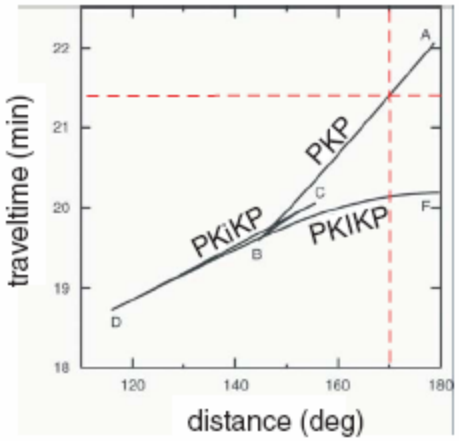
9 second P wave

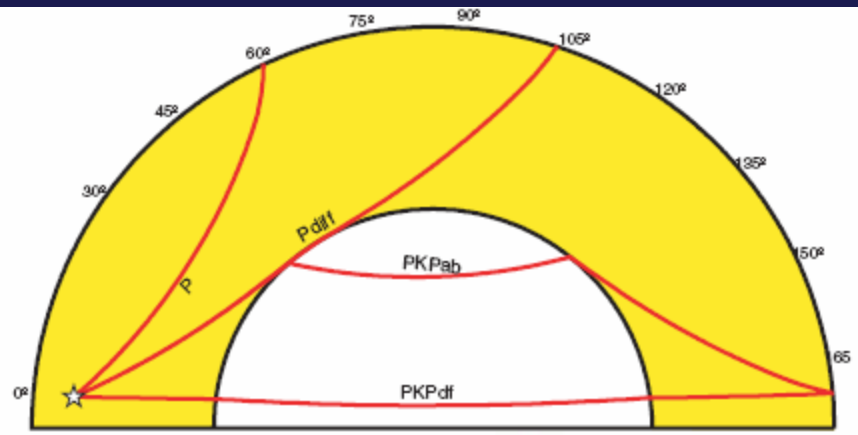


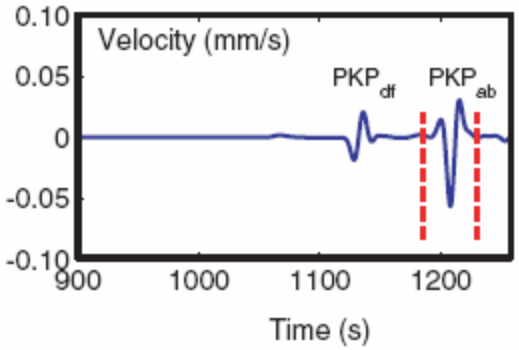
Global 3D Body-Wave Sensitivity Kernels

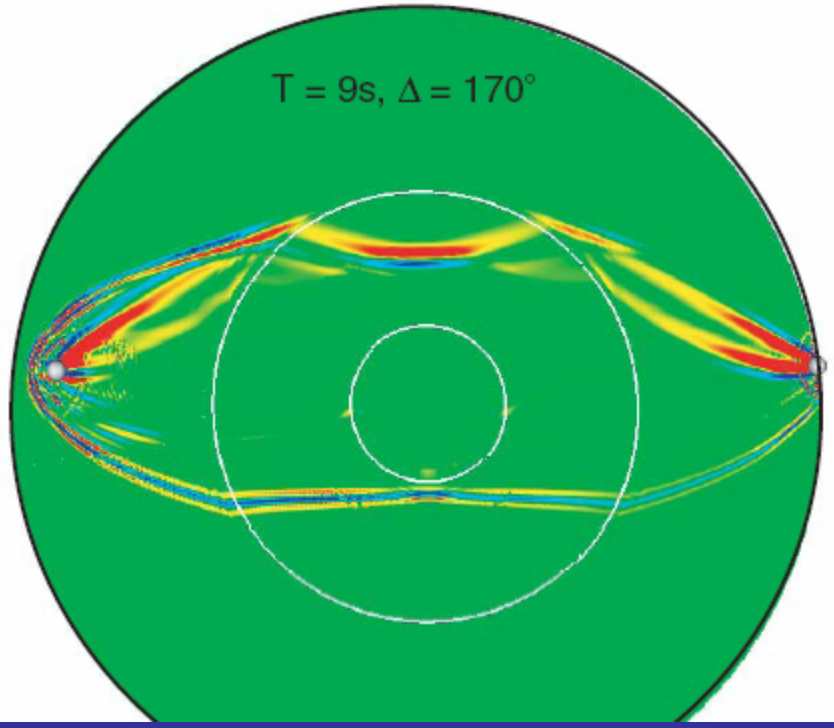


PKP Kernel

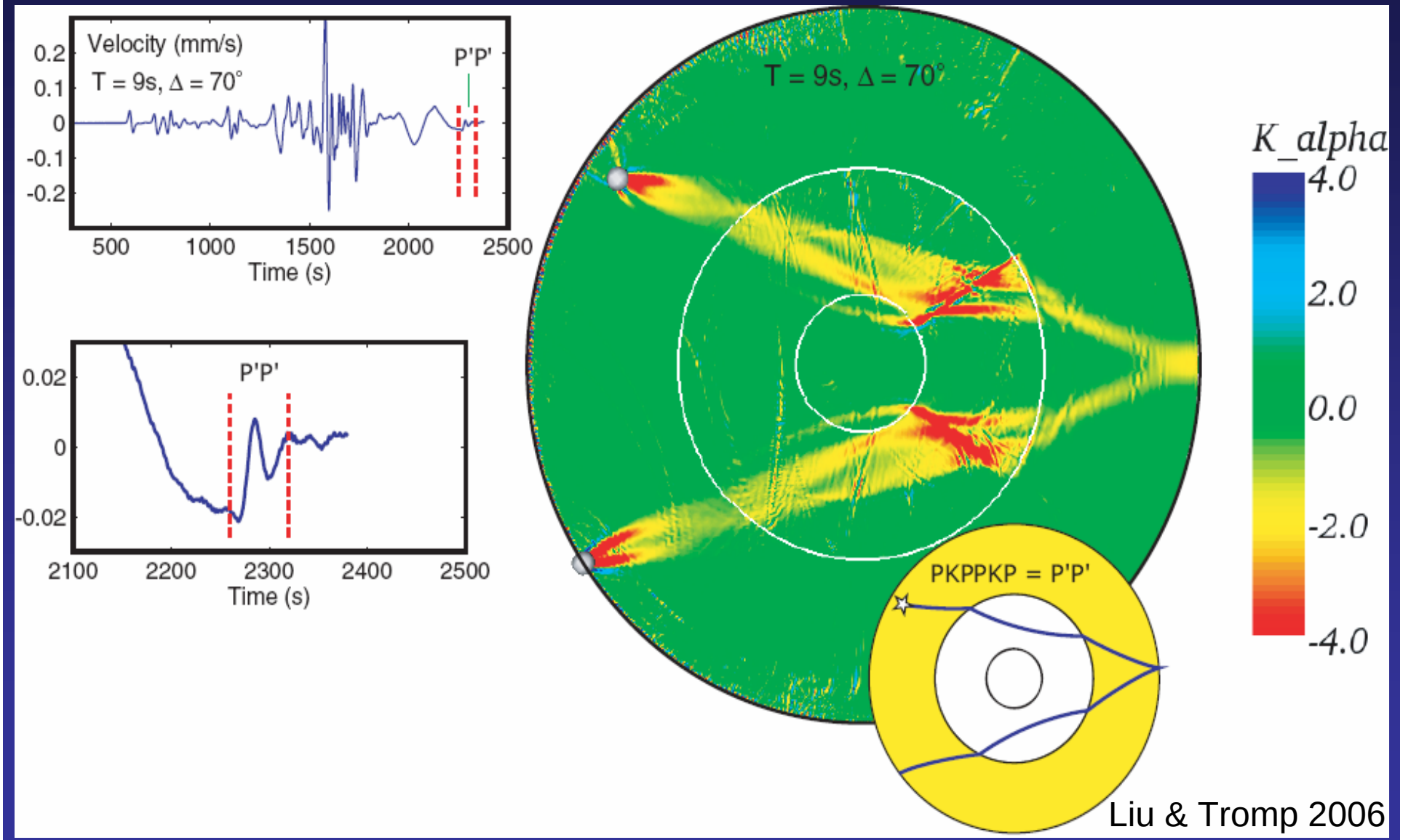

traveltime (min)
distance (deg)
PKP
PKiKP
PKIKP
A
B
C
D
F


PKPab
PKPdf
Pdiff
P


Velocity (mm/s)
PKP df
PKP ab
Time (s)

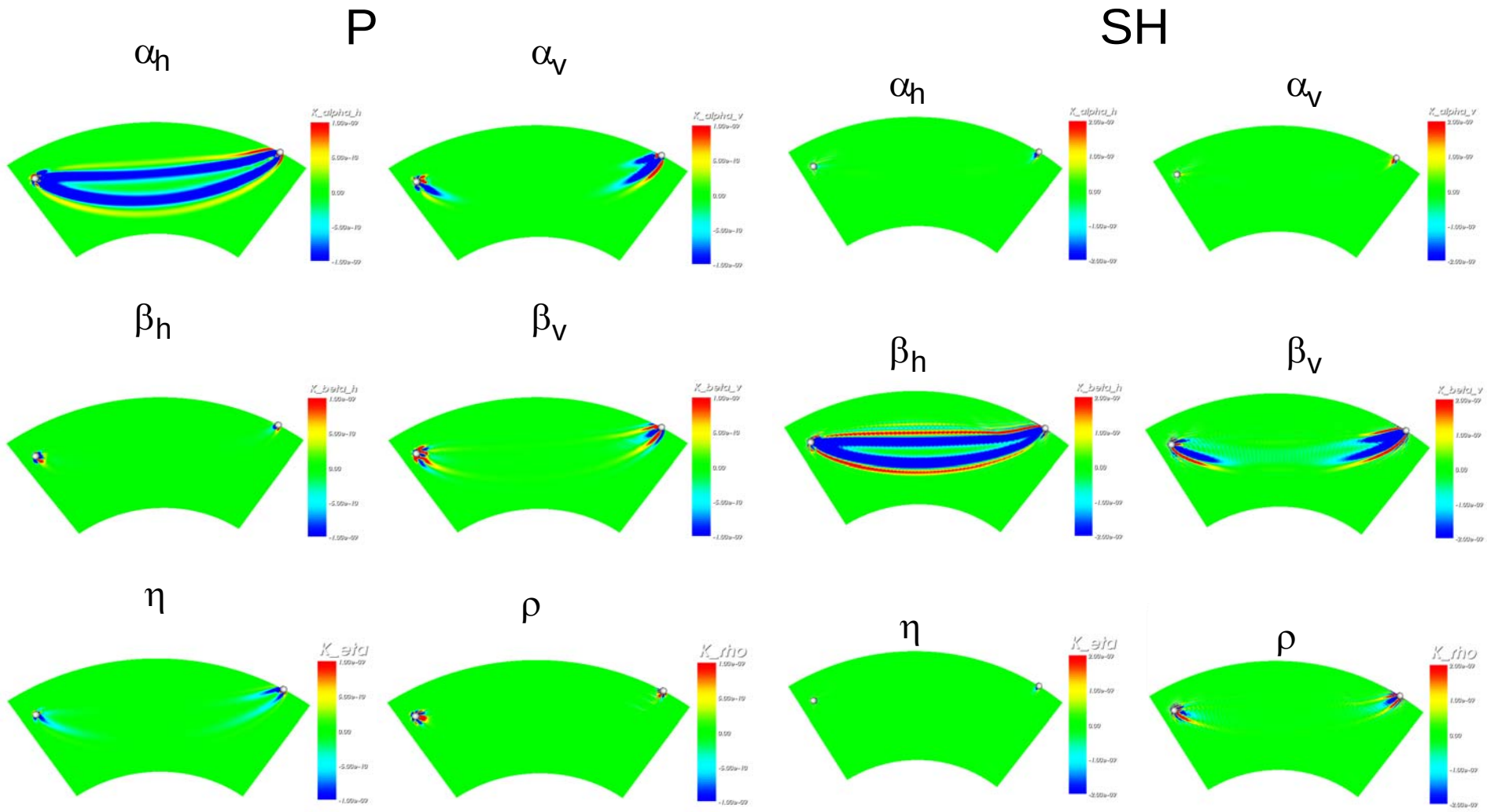

T = 9s, Δ = 170°

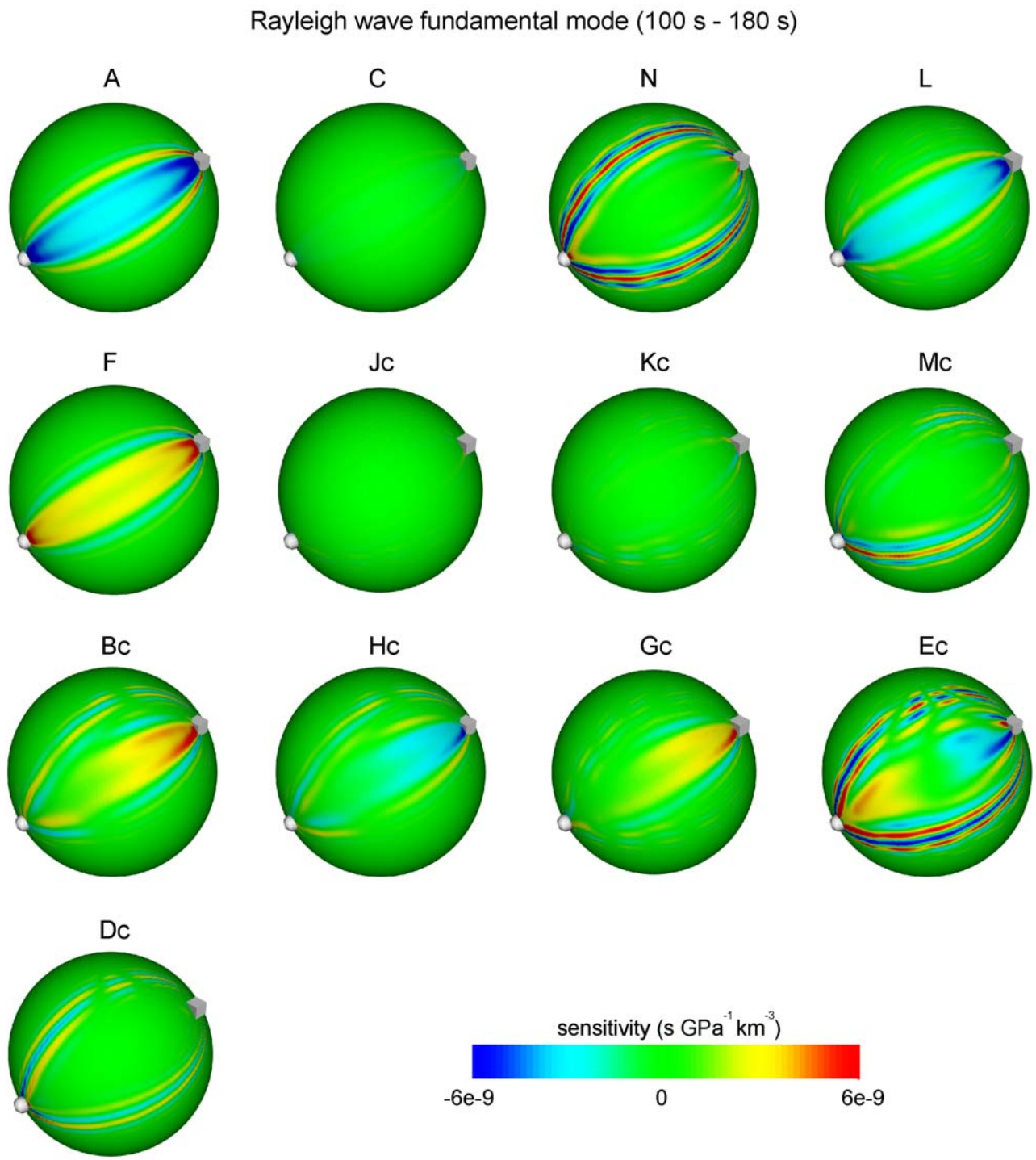
P'P' Kernel



Liu & Tromp 2006

Global 3D Transversely Isotropic Kernels





Transversely Isotropic Parameters: A, C, L, N, F

1 ζ: J, K, M

2 ζ: G, B, H

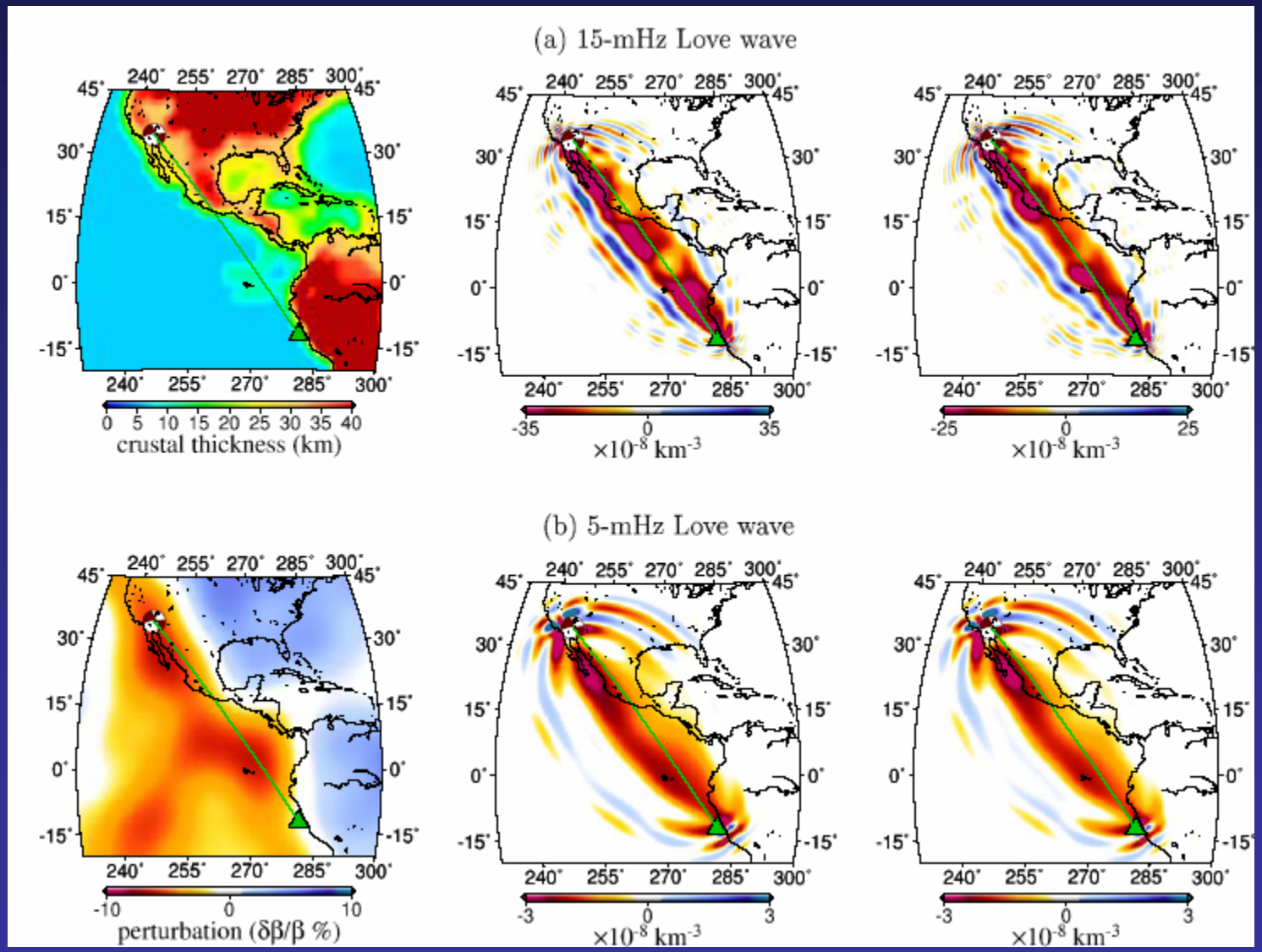
3 ζ: D

4 ζ: E

Anne Sieminski

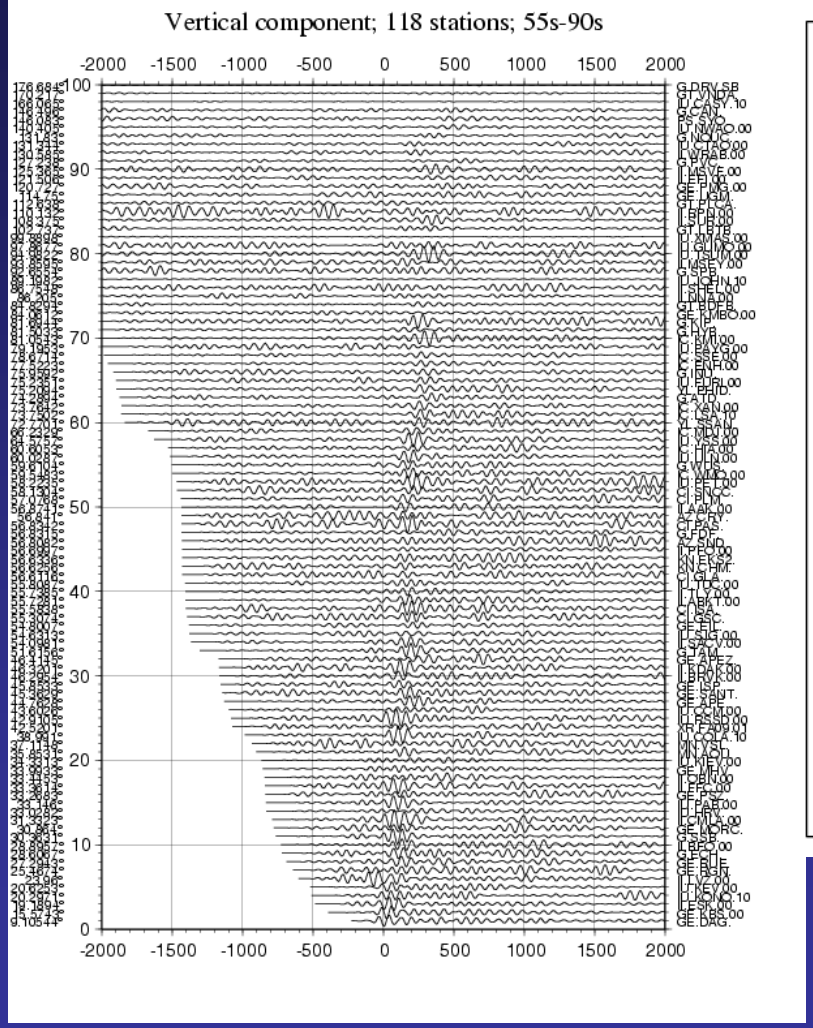
1D Versus 3D Kernels

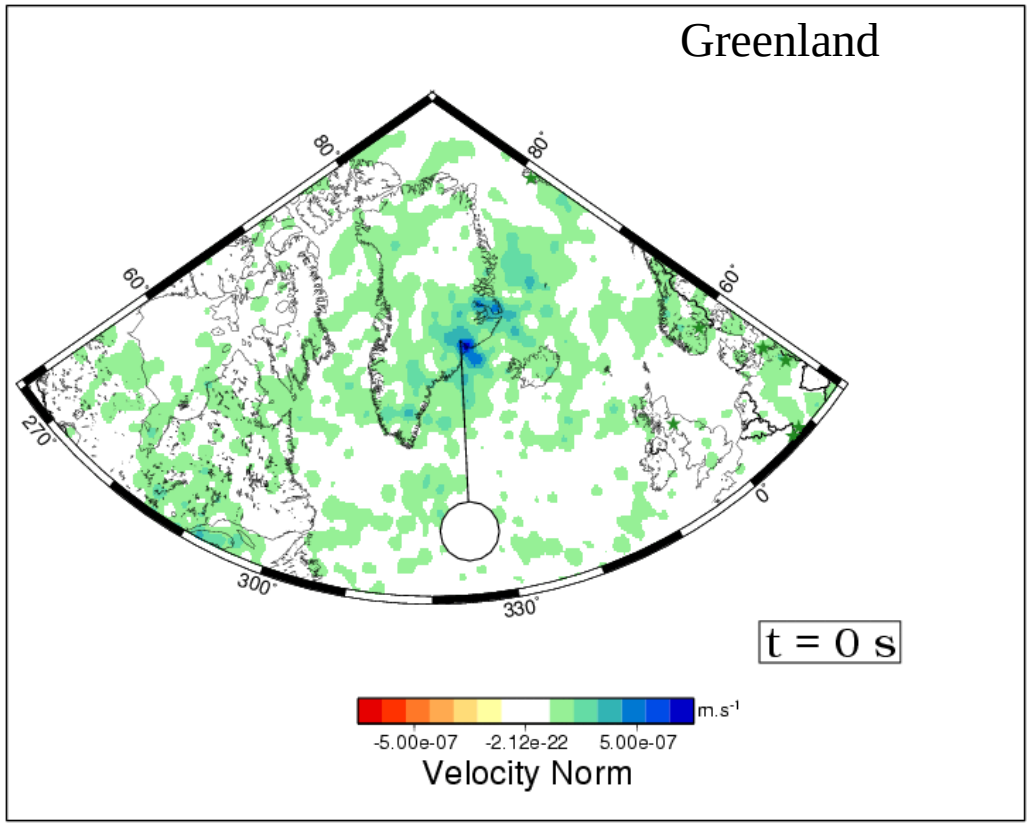
1D 3D



Ying Zhou

Time-Reversal Imaging: Glacial Earthquakes

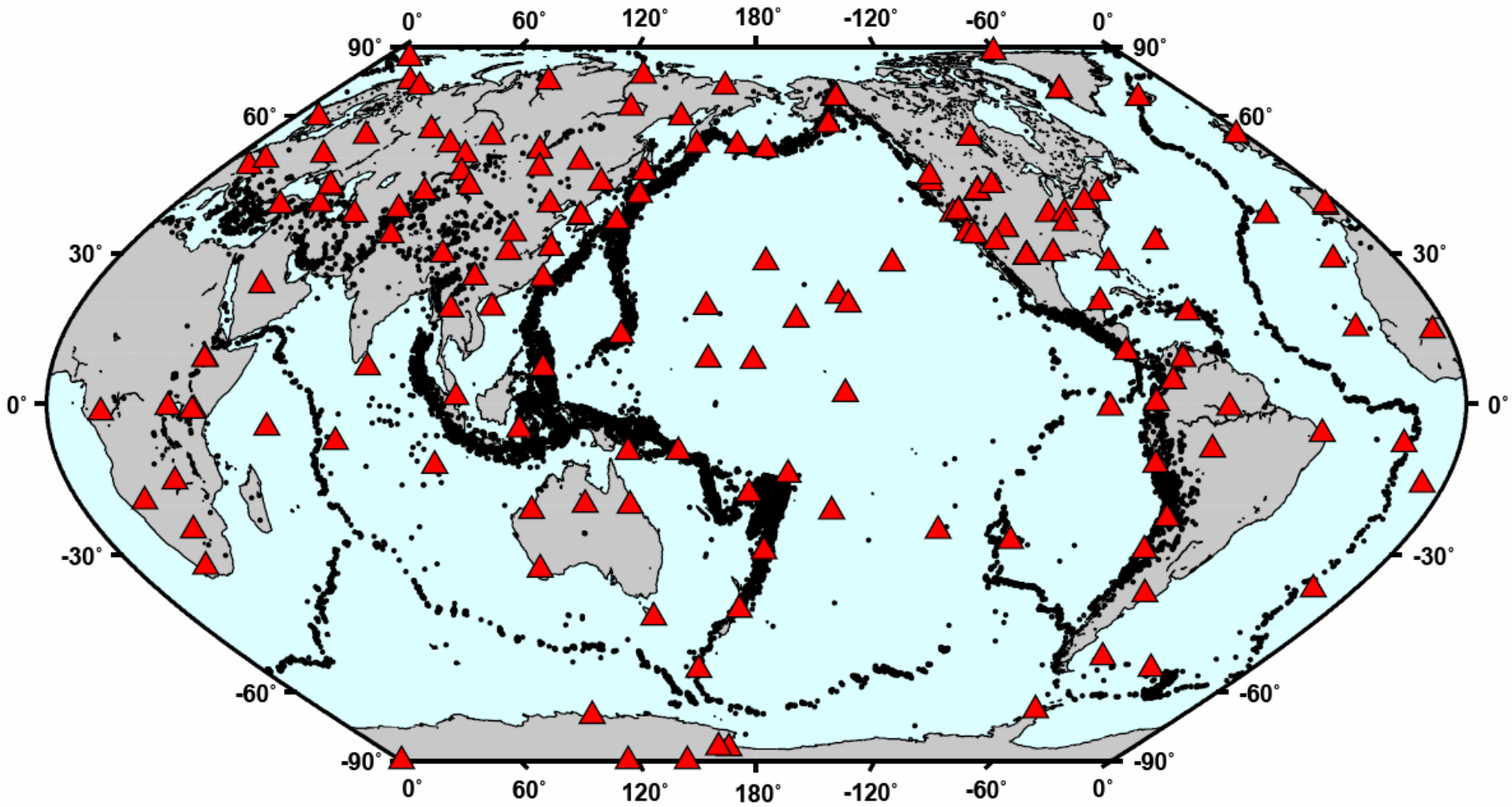




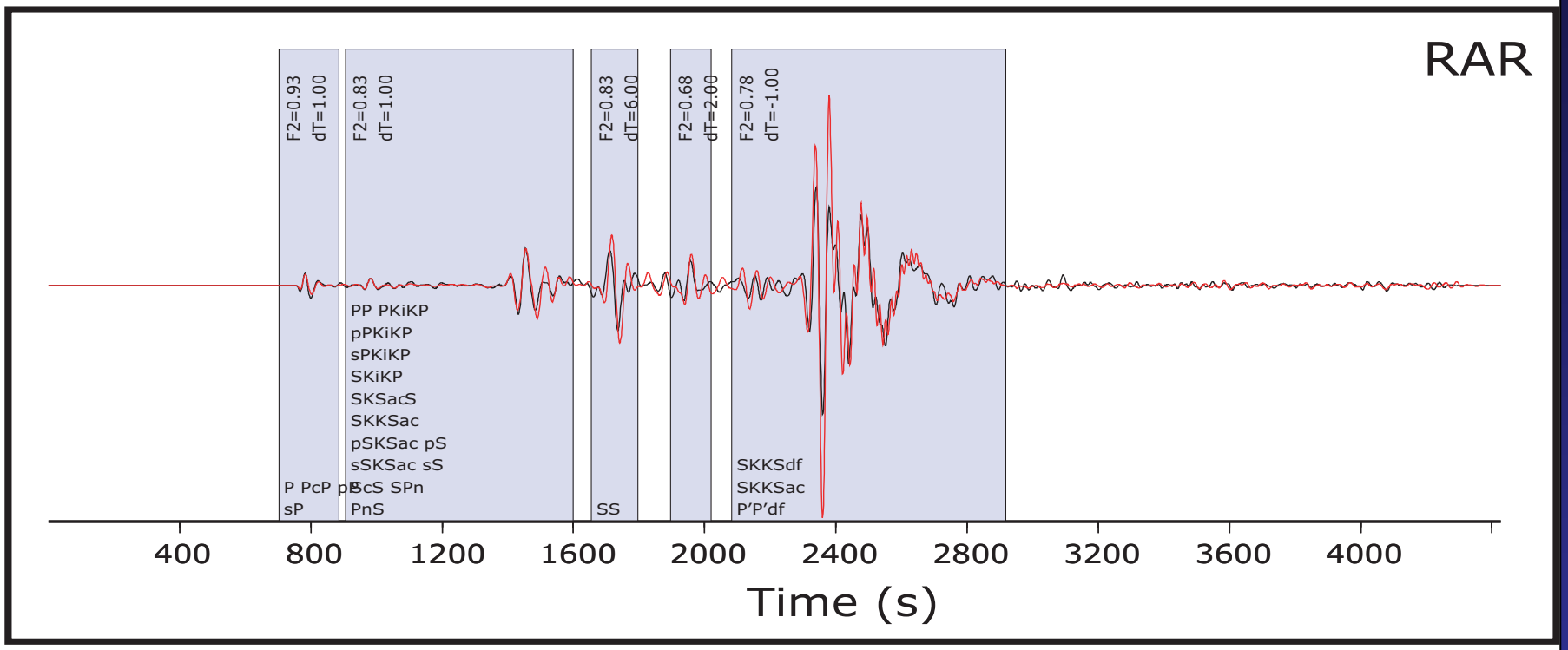
Carene Larmat

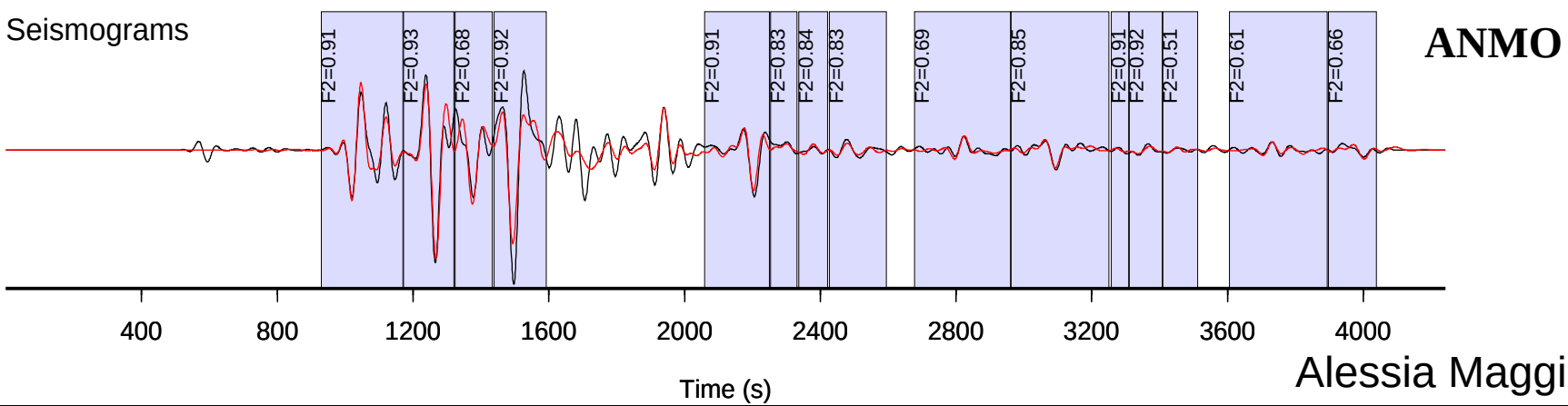
Measuring all available Phase & Amplitude Anomalies

Seismicity and seismometers



Measure all suitable phases





Alessia Maggi

Conclusions

Adjoint methods:

- Choose an observable, e.g., waveforms or cross-correlation traveltimes
- Choose a measure of misfit, e.g., least-squares
- Determine the appropriate adjoint source for this observable & measurement
- Use fully 3D reference models
- Any arrival suitable for measurement
- No dependence on the number of stations, components, or measurements
- 3D sensitivity kernels may be calculated based upon two forward simulations for each earthquake
- Number of simulations: 3 * (# earthquakes) * (# iterations)
- Full anisotropy for the same cost
- Attenuation remains a challenge

Regional simulations:

- One 3 minute forward simulation accurate to 1.5 seconds takes 45 minutes on a 75 node cluster
- 150 events and 3 iterations would require 1800 simulations, i.e., three weeks of dedicated CPU time on 75 nodes
- Near real-time simulations

Global simulations:

- One 1 hour forward simulation accurate to 20 seconds takes 4 hours on a 75 node cluster
- 500 events and 3 iterations would require 6,000 simulations, i.e., 100 days on a 750 node cluster
- Near real-time simulations
- On-demand global seismology
- Petascale application