Teleseismic scattered wave imaging: considerations on array geometry and constraints from field conditions

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Presentation for Imaging Science Workshop
Washington University, October 31, 2006
outline

• equations of imaging I: Kirchhoff and Generalized Radon Transform for continuous functions

• reality check from field conditions

• equations of imaging II: discretization and loss of dimension

• resolution: frequency/wavenumber, ray coverage, multiples

• image robustness: source distribution, aliasing, departure from 2-D geometry

• case studies

• challenges: application to short period teleseismic data and 3-D imaging
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Kirchhoff migration

\[
I(x) = PS(x) = \frac{1}{4\pi^2} \int_{-\infty}^{\infty} d\omega (i\omega) \int_{S'} dS' \exp(-i\omega(t - (\tau_p + \tau_s))R_F(x', \omega) \frac{A_s(x, x') \cos \theta(x')}{A_p(x, x_e) \beta(x')} \bigg|_{t=t_e}
\]

Levander et al., 2005
Generalized Radon Transform (GRT)

**forward problem:**

\[
\Delta u_n [x', p^0, t] = - \int dx f(x, \theta) A_n(x', x') \delta'' [t - T(x, x')] \]

\[
\Delta u_n [x', p^0, t = T(x_0, x')] \approx - \frac{A_n(x_0, x')}{|\nabla_0 T(x_0, x')|^2} \int dx f(x, \theta) \delta'' [n \cdot (x - x_0)]
\]

Miller et al., 1987; Beylkin and Burridge, 1990; Bostock et al., 2001
Generalized Radon Transform (GRT)

forward problem:

\[ \langle f(x_0, \theta) \rangle = \frac{1}{4\pi} \int d\psi \frac{\nabla_0 T(x_0, x')^2}{|A(x_0, x')|^2} \sum_n A_n(x_0, x') \Delta u_n [x', p_0^\perp, t = T(x_0, x')] \]

inverse problem:
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event coverage
Cost of sensors

Short period (1 Hz): $2000

Broadband (50-0.0083 Hz): $13000
3-D imaging (2-D array)  

300 km x 300 km  
(10 km spacing ⇒ 900 sensors)

2-D imaging (1-D array)  

300 km  
(10 km spacing ⇒ 30 sensors)

PASSCAL broadband instruments pool (250 sensors)
ideal array geometry
realistic array geometry
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Kirchhoff migration

3-D:

\[ I(x) = PS(x) = \frac{1}{4\pi^2} \int d\omega (i\omega) \int S' \exp(-i\omega(t - (\tau_P + \tau_S))) R_F(x',\omega) \frac{A_S(x,x') \cos \theta(x')}{A_P(x,x_e) \beta(x')} \bigg|_{t=t_e} \]

2-D:

\[ I(x) = PS(x) = \frac{1}{4\pi^2} \int d\omega (\sqrt{i\omega}) \sum_{j=1}^{N} \Delta x_j' \exp(-i\omega(t - (\tau_P + \tau_S - p_j y')) R_F(x_j',\omega) \frac{A_S(x_n,x_j') \cos \theta(x_j')}{A_P(x_n,x_e) \beta(x_j')} \bigg|_{t=t_e} \]

limited frequency band ⇒ reduces resolution

45° phase shift ⇒ reduces resolution

Insures constructive/destructive interference in 2-D

sum over discrete station location

can cause aliasing and image distortion

Time-shift adjustment for 2-D geometry

Correct moveout for line scatterers
Generalized Radon Transform (GRT)

3-D:

\[
\langle f(x_0, \theta) \rangle = \frac{1}{4\pi} \int d\psi \frac{\left| \nabla_0 T(x_0, x') \right|^2}{|A(x_0, x')|^2} \sum_n A_n(x_0, x') \Delta u_n \left[ x', p^0, t = T(x_0, x') \right]
\]

2-D:

\[
\langle f(x_0, \theta) \rangle = \frac{1}{4\pi} \sum_{j=1}^N \Delta \psi_j \frac{\left| \nabla_0 T(x_0, x_j; p^0_\parallel) \right|^2}{|A(x_0, x_j; p^0_\parallel)|^2} \sum_n A_n(x_0, x_j; p^0_\parallel) \Delta \tilde{u}_n \left[ x_j, p^0_\perp, t = T(x_0, x_j; p^0_\parallel) \right]
\]

sum over discrete source geometry

Adjust for 2-D geometry

deteriorating images due to interference artifacts
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Resolution: frequency - wavelength

- lateral resolution

first Fresnel zone (vertical incidence): \[ R = 2\sqrt{z\lambda + \frac{\lambda^2}{4}} \]

using of the whole diffraction hyperbola: \[ R \rightarrow \frac{\lambda}{2} \]
Resolution: frequency - wavelength

- longitudinal resolution

transmission (forward scattering): \( R \approx \frac{\lambda}{2} \)

reflection (back scattering): \( R \approx \frac{\lambda}{4} \)
Resolution: frequency - wavenumber

FORWARD PROBLEM

Horizontal distance - \( x_i \) [km]

\[
\begin{bmatrix}
\tau = T(x', x, p')
\end{bmatrix}
\]

Effective wavenumber

\[ |k| = \omega |\nabla T| \]

INVERSE PROBLEM

Horizontal distance - \( x_i \) [km]

\[
\begin{bmatrix}
\Delta \alpha \\
\Delta \beta \\
\Delta \rho
\end{bmatrix}
\]

\( p^0 \)
Resolution: ray coverage

500km

30km

∇T

P
Idealized coverage

ABI96 experiment
Resolution: ray coverage and aperture

$z = L / 2 \Rightarrow -45^\circ \leq \psi \leq 45^\circ$
Resolution: ray coverage and aperture

\[ \frac{L}{2} \Rightarrow -45^\circ \leq \psi \leq 45^\circ \]

\[ L \Rightarrow -33^\circ \leq \psi \leq 33^\circ \]
Resolution: multiples

direct arrival

multiples

\[
P_{dp} \quad (q=1) \\
P_{ds} \quad (q=2)
\]

\[
P_{pp_{dp}} \quad (q=3) \\
P_{pp_{ds}} \quad (q=4)
\]

\[
P_{ps_{dp}} \quad (q=5) \\
P_{ps_{ds}} \quad (q=6,7)
\]
\( V_s: P_{ds} \)
\( V_s: P_{ds}, P_{pds} \)
$V_s$: $P_{ds}$, $P_{pds}$, $P_{sds}$
$V_s$: $P_{ds}$, $P_{pds}$, $P_{sds}$

$V_p$: $P_{pdp}$
images

d\(\frac{V_s}{V_0}\) (\(P_{ds}, P_{pds}, P_{sds}\))

d\(\frac{V_p}{V_0}\) (\(P_{dp}\))
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Image robustness: source distribution

continental crust
low V

mantle
high V

oceanic crust
low V

mantle
high V

subduction zone imaging
data

left incidence

right incidence

Time [seconds]

Horizontal distance [km]

Psds

Ppds

Pds
The frenzied excitement of World Series 2006 has ended with a victory in the city of St Louis

The World ended in St Louis
Image robustness: spatial aliasing

near apex of the diffraction hyperbola:

\[ \nabla x' = \sqrt{\left( \frac{z\lambda + \frac{\lambda^2}{4}}{4} \right)} \]

branches of the diffraction hyperbola:

\[ \nabla x' \rightarrow \frac{\lambda}{2} \]
Image robustness: migration operator aliasing
migration operator aliasing

rule of thumb:
station spacing $\leq \frac{z}{2}$
Image robustness: departure from 2-D geometry

\[ \Delta x_2? \]

\[ \Delta x_1 \]

line scatterer

\[ p_2 \text{ constant} \]

plane section
Image robustness: departure from 2-D geometry

rule of thumb: linear extent of scatterer $\geq z/2$ (on both sides of imaging plane)
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Cascadia subduction zone
BEAAR experiment
central Alaska

δβ/β perturbations

S-Velocity Perturbation [%]
ABITIBI 1996

Study area

Event distribution

- Station projection line:
- Stations:
  - ABI-94
  - ABI-96

- Superior
- Grenville
- Paleozoic COVER
2D GRT profile

$\delta \alpha / \alpha$ perturbations

- Depth [km]
- Horizontal Distance [km]

Color scale:
- 0.08
- 0
- -0.08
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3-D imaging: what are we waiting for?

- equations are well known

- more instruments, but they need to be cheaper (short period or new technologies)

- for short period data, we need to develop new tools to separate near-field and/or surface scattering effects

- source distribution will remain a problem!
Concluding remarks

• Imaging works despite numerous assumptions made about the geometry of the model and the background velocity model

• Careful examination of the equations we use can help to develop strategies for working around limitations introduced by field conditions, designing optimal arrays for any given target, and assessing the robustness of the resulting image