Teleseismic scattered wave imaging: considerations on array geometry and constraints from field conditions

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Presentation for Imaging Science Workshop
Washington University, October 31, 2006

outline

- equations of imaging I: Kirchhoff and Generalized Radon Transform for continuous functions
- reality check from field conditions
- equations of imaging II: discretization and loss of dimension
- resolution: frequency/wavenumber, ray coverage, multiples
- image robustness: source distribution, aliasing, departure from 2-D geometry
- case studies
- challenges: application to short period teleseismic data and 3-D imaging

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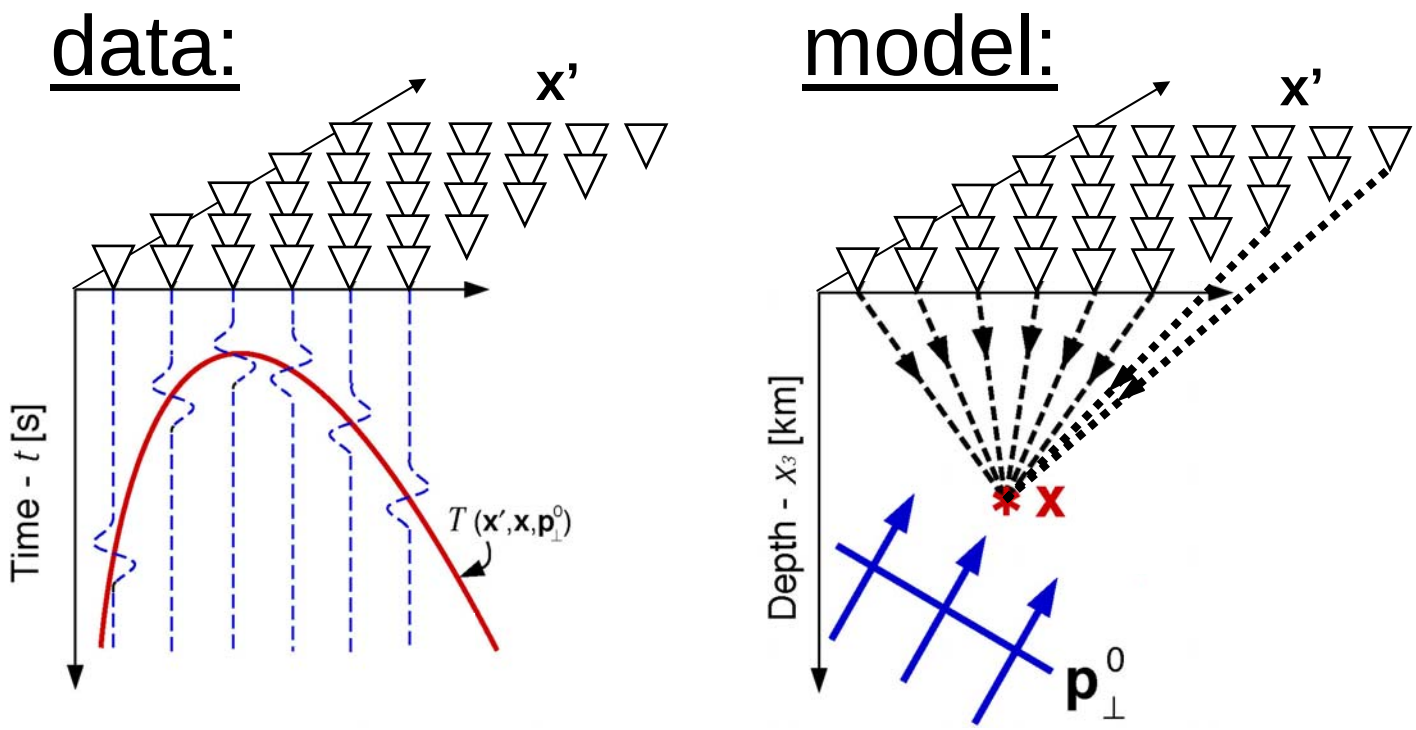
Kirchhoff migration

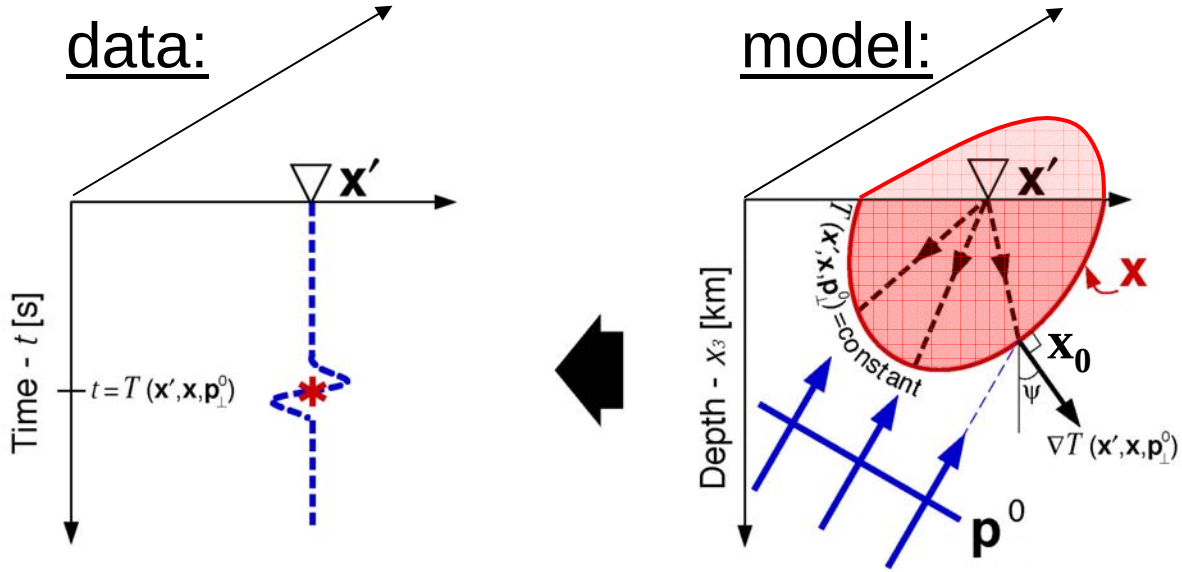


$$I(\mathbf{x}) = \overset{\square\square}{PS}(\mathbf{x}) = \frac{1}{4\pi^2}\int_{-\infty}^{\infty} d\omega(i\omega)\int_{S'} dS' \exp(-i\omega(t-(\tau_P+\tau_S))R_F(\mathbf{x}',\omega)\frac{A_S(\mathbf{x},\mathbf{x}')\cos\theta(\mathbf{x}')}{A_P(\mathbf{x},\mathbf{x_e})\beta(\mathbf{x}')}\Bigg|_{t=t_e}$$

Levander et al., 2005

Generalized Radon Transform (GRT)

forward problem:



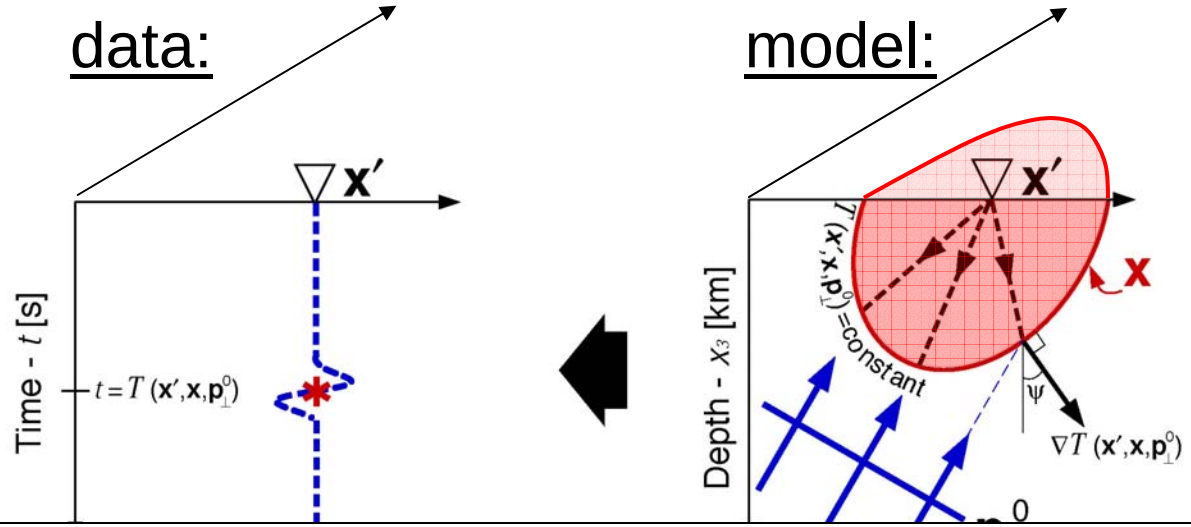
$$\Delta u_n[\mathbf{x}',\mathbf{p}^0,t] = -\int d\mathbf{x}\, f(\mathbf{x},\theta) A_n(\mathbf{x},\mathbf{x}')\delta'\left[t - T(\mathbf{x},\mathbf{x}')\right]$$

$$\Delta u_n[\mathbf{x}',\mathbf{p}^0,t = T(\mathbf{x}_0,\mathbf{x}')] \approx -\frac{A_n(\mathbf{x}_0,\mathbf{x}')}{\left|\nabla_0 \mathrm{T}(\mathbf{x}_0,\mathbf{x}')\right|^2} \int d\mathbf{x}\, f(\mathbf{x},\theta)\delta'\left[\mathbf{n}\cdot(\mathbf{x}-\mathbf{x}_0)\right]$$

Miller et al., 1987; Beylkin and Burridge, 1990; Bostock et al., 2001

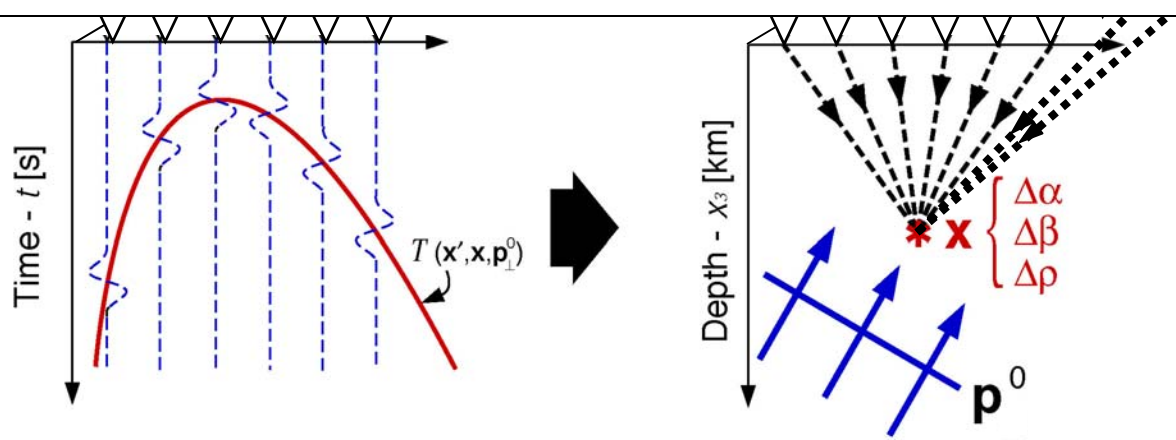
Generalized Radon Transform (GRT)

forward problem:



$$\langle f(\mathbf{x_0},\theta)\rangle = \frac{1}{4\pi}\int d\psi \frac{\left|\nabla_0 T(\mathbf{x_0},\mathbf{x'})\right|^2}{\left|A(\mathbf{x_0},\mathbf{x'})\right|^2}\sum_n A_n(\mathbf{x_0},\mathbf{x'})\Delta u_n\left[\mathbf{x'},\mathbf{p}_\perp^{\mathbf{0}},t=T(\mathbf{x_0},\mathbf{x'})\right]$$

inverse problem:



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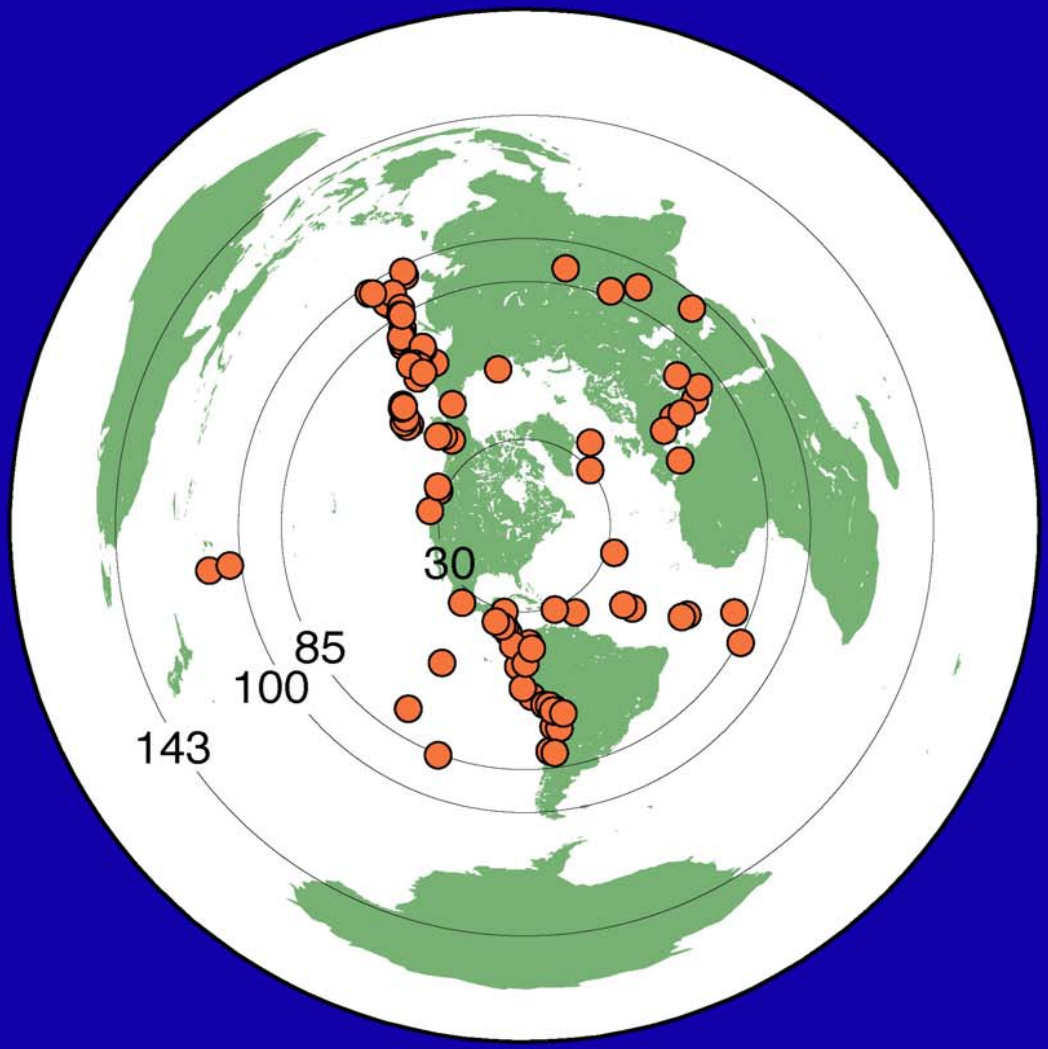
event coverage

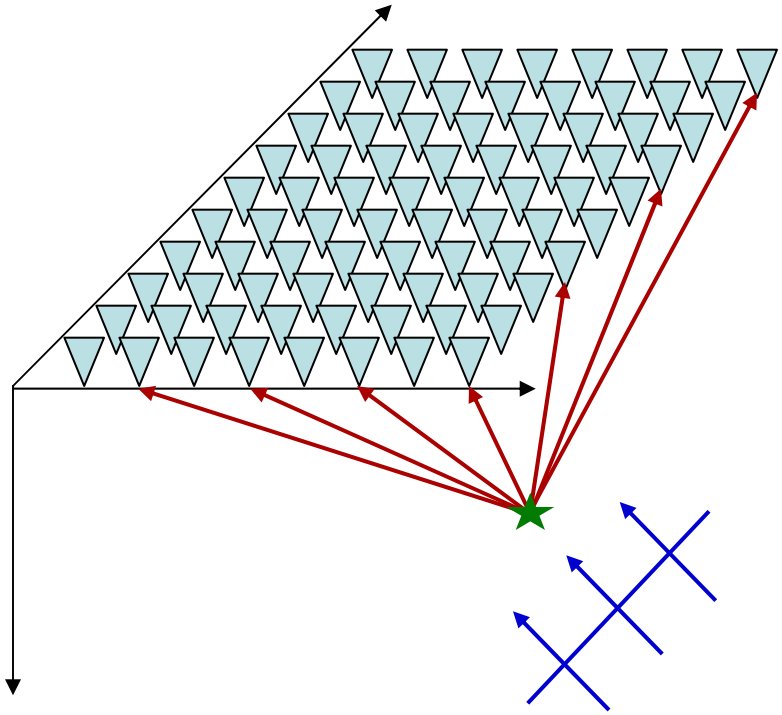


Cost of sensors

Short period (1 Hz): $2000

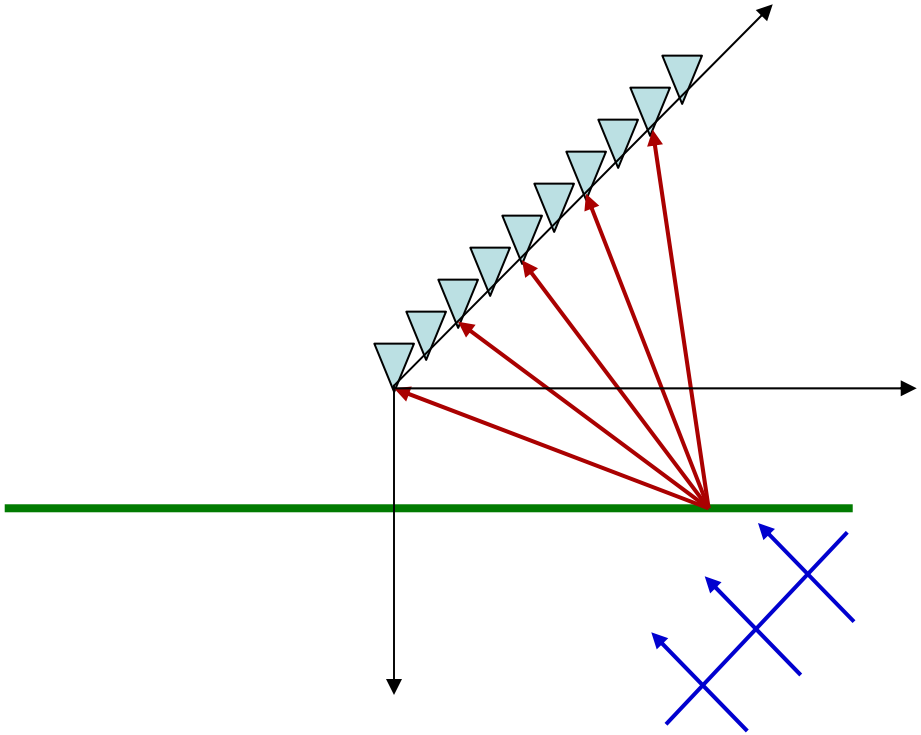
Broadband (50-0.0083 Hz): $13000

3-D imaging (2-D array)



300 km x 300km
(10km spacing ⇒900 sensors)

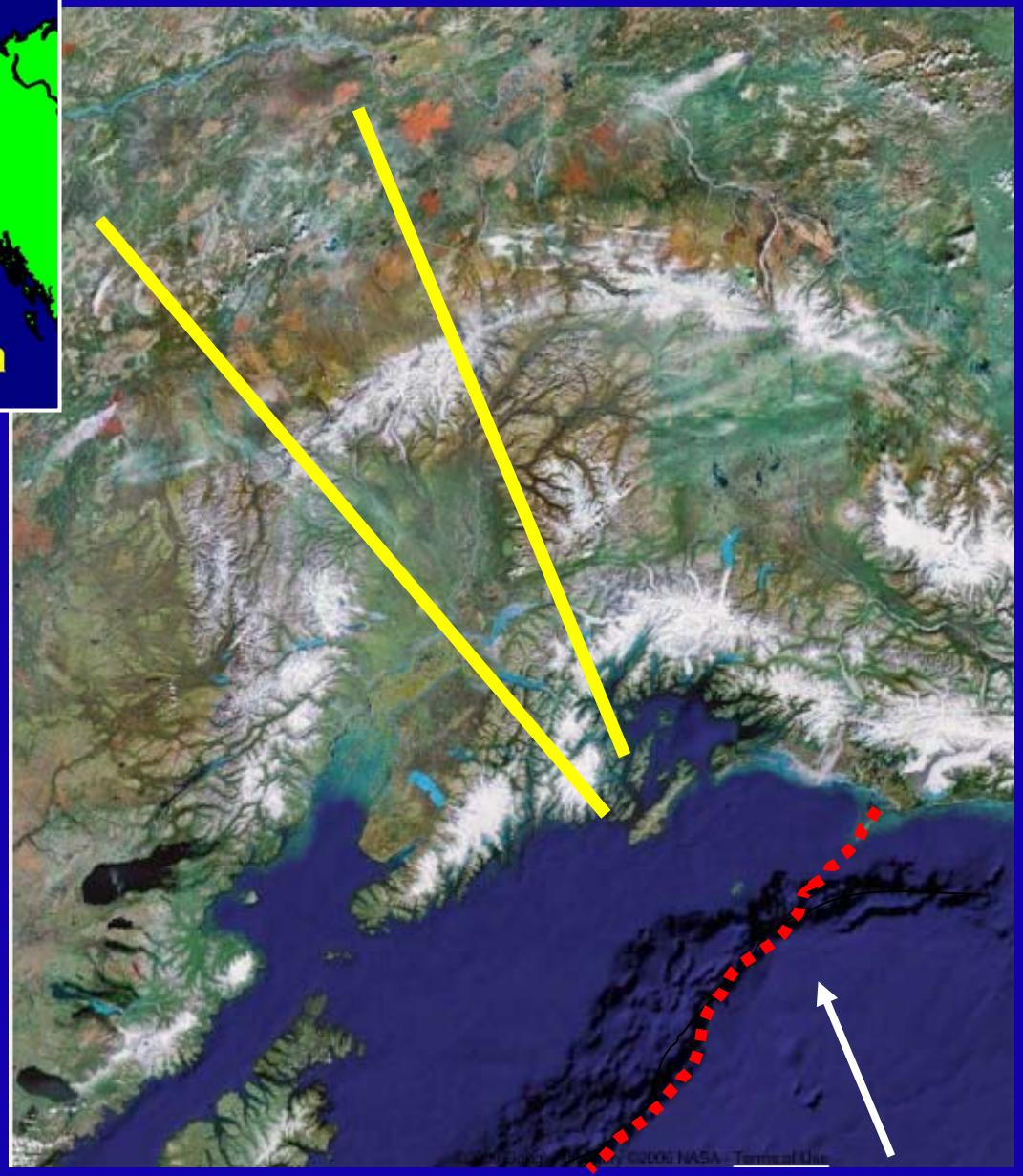
2-D imaging (1-D array)



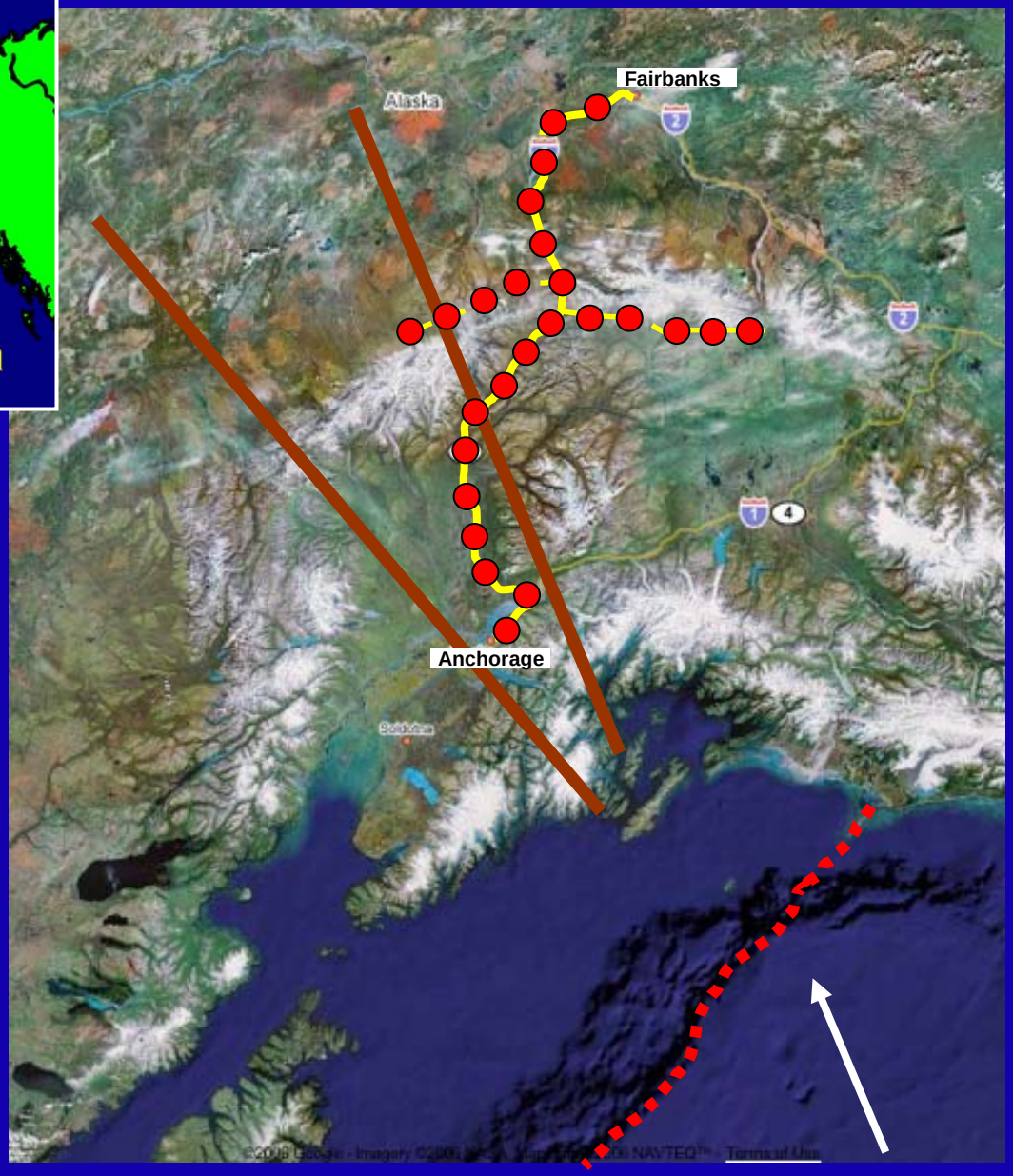
300 km
(10 km spacing ⇒ 30 sensors)

PASSCAL broadband instruments pool (250 sensors)

ideal array geometry



realistic array geometry



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Kirchhoff migration

3-D:

$$I(\mathbf{x}) = \overset{\square\square}{PS}(\mathbf{x}) = \frac{1}{4\pi^2}\int_{-\infty}^{\infty} d\omega(i\omega)\int_{S'} dS' \exp(-i\omega(t-(\tau_P+\tau_S))R_F(\mathbf{x}',\omega)\frac{A_S(\mathbf{x},\mathbf{x}')\cos\theta(\mathbf{x}')}{A_P(\mathbf{x},\mathbf{x_e})\beta(\mathbf{x}')}\Bigg|_{t=t_e}$$

2-D:

$$I(\mathbf{x}) = \overset{\square\square}{PS}(\mathbf{x}) = \frac{1}{4\pi^2}\int_{\omega_L}^{\omega_H} d\omega(\sqrt{i\omega})\sum_{j=1}^{N}\Delta\mathbf{x}_j' \exp(-i\omega(t-(\tau_P+\tau_S-p_y\mathbf{y}'))R_F(\mathbf{x}_j',\omega)\frac{A_S(\mathbf{x_n},\mathbf{x}_j')\cos\theta(\mathbf{x}_j')}{A_P(\mathbf{x_n},\mathbf{x_e})\beta(\mathbf{x}_j')}\Bigg|_{t=t_e}$$

limited frequency bandwidth

45° phase shift

Insures constructive/destructive interference

Discrete station spacing

Insures constructive/destructive interference

Time shift adjustment for 2-D geometry

Corrects 2-D moveout for line scatterers

Generalized Radon Transform (GRT)

3-D:

$$\left\langle f(\mathbf{x_0},\theta)\right\rangle = \frac{1}{4\pi}\int d\psi \frac{\left|\nabla_0 T(\mathbf{x_0},\mathbf{x'})\right|^2}{\left|A(\mathbf{x_0},\mathbf{x'})\right|^2}\sum_n A_n(\mathbf{x_0},\mathbf{x'})\Delta u_n\left[\mathbf{x'},\mathbf{p^0},t=T(\mathbf{x_0},\mathbf{x'})\right]$$

2-D:

$$\left\langle f(\mathbf{x_0},\theta)\right\rangle = \frac{1}{4\pi}\sum_{j=1}^{N}\Delta\psi_j \frac{\left|\nabla_0 T(\mathbf{x_0},\mathbf{x}_j';\mathbf{p}_{\|}^{\mathbf{0}})\right|^2}{\left|A(\mathbf{x_0},\mathbf{x}_j';\mathbf{p}_{\|}^{\mathbf{0}})\right|^2}\sum_n A_n(\mathbf{x_0},\mathbf{x}_j';\mathbf{p}_{\|}^{\mathbf{0}})\Delta\tilde{u}_n\left[\mathbf{x}_j',\mathbf{p}_{\perp}^{\mathbf{0}},t=T(\mathbf{x_0},\mathbf{x}_j';\mathbf{p}_{\|}^{\mathbf{0}})\right]$$

sum over discrete angles

Adjustment for 2D geometry

$\Downarrow$

$\Downarrow$

can cause reconstructive artifacts

Due to constructive interference in 2-D

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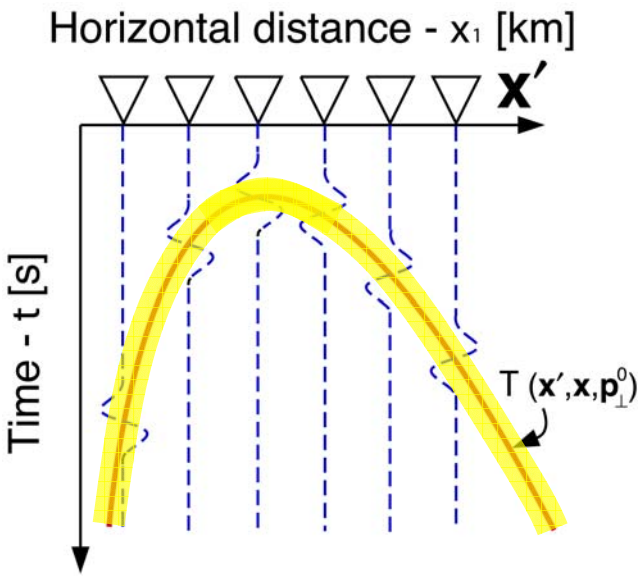
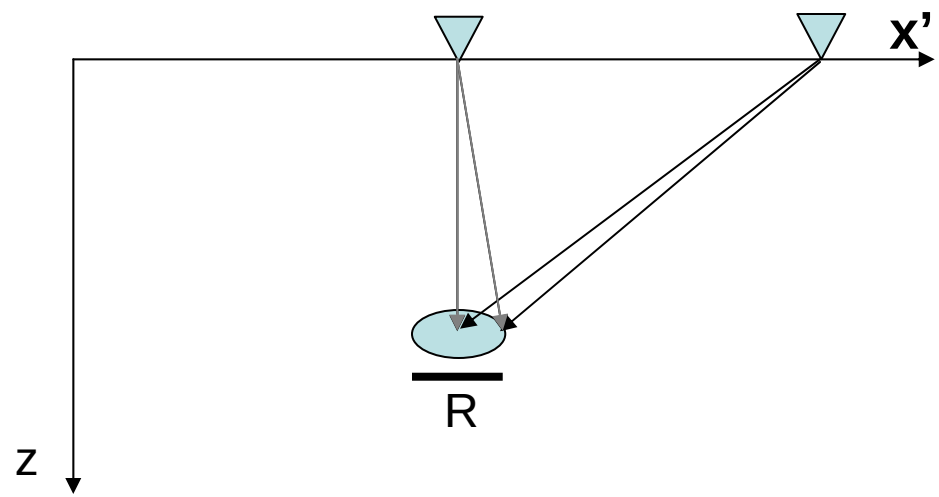
Resolution: frequency - wavelength

- lateral resolution

first Fresnel zone (vertical incidence) : $R = 2\sqrt{\left(z\lambda + \lambda^2/4\right)}$

using of the whole diffraction hyperbola: $R \rightarrow \frac{\lambda}{2}$



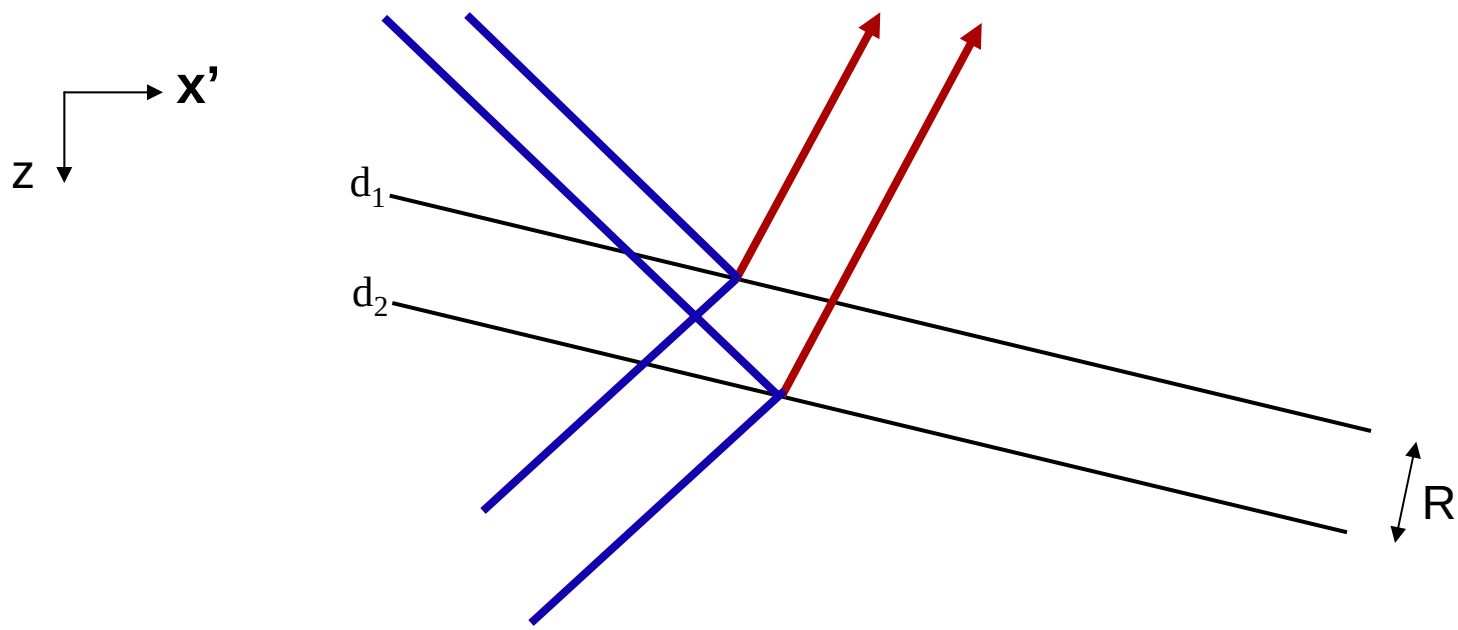


Resolution: frequency - wavelength

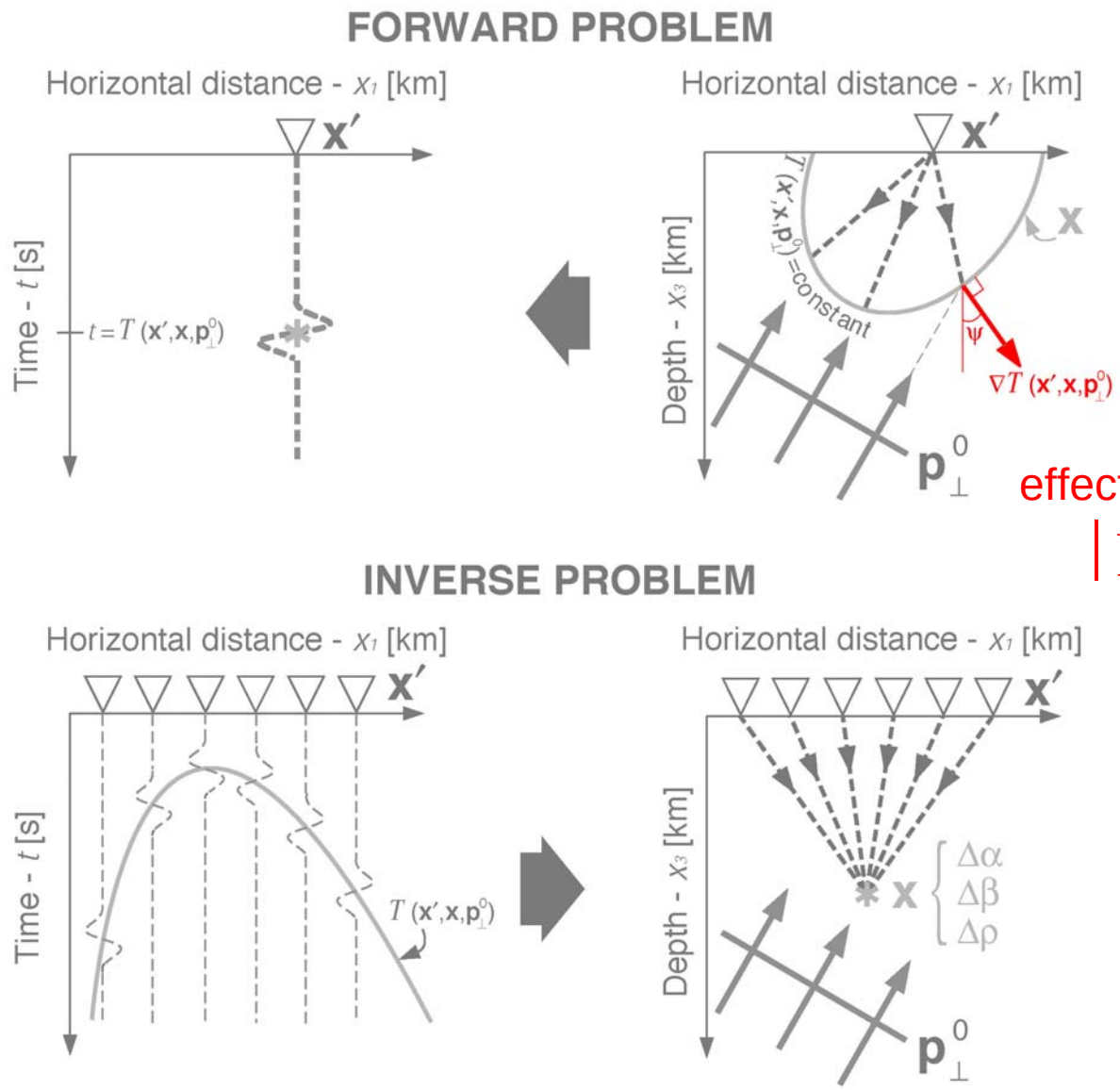
- longitudinal resolution

transmission (forward scattering) : $R \approx \frac{\lambda}{2}$

reflection (back scattering): $R \approx \frac{\lambda}{4}$



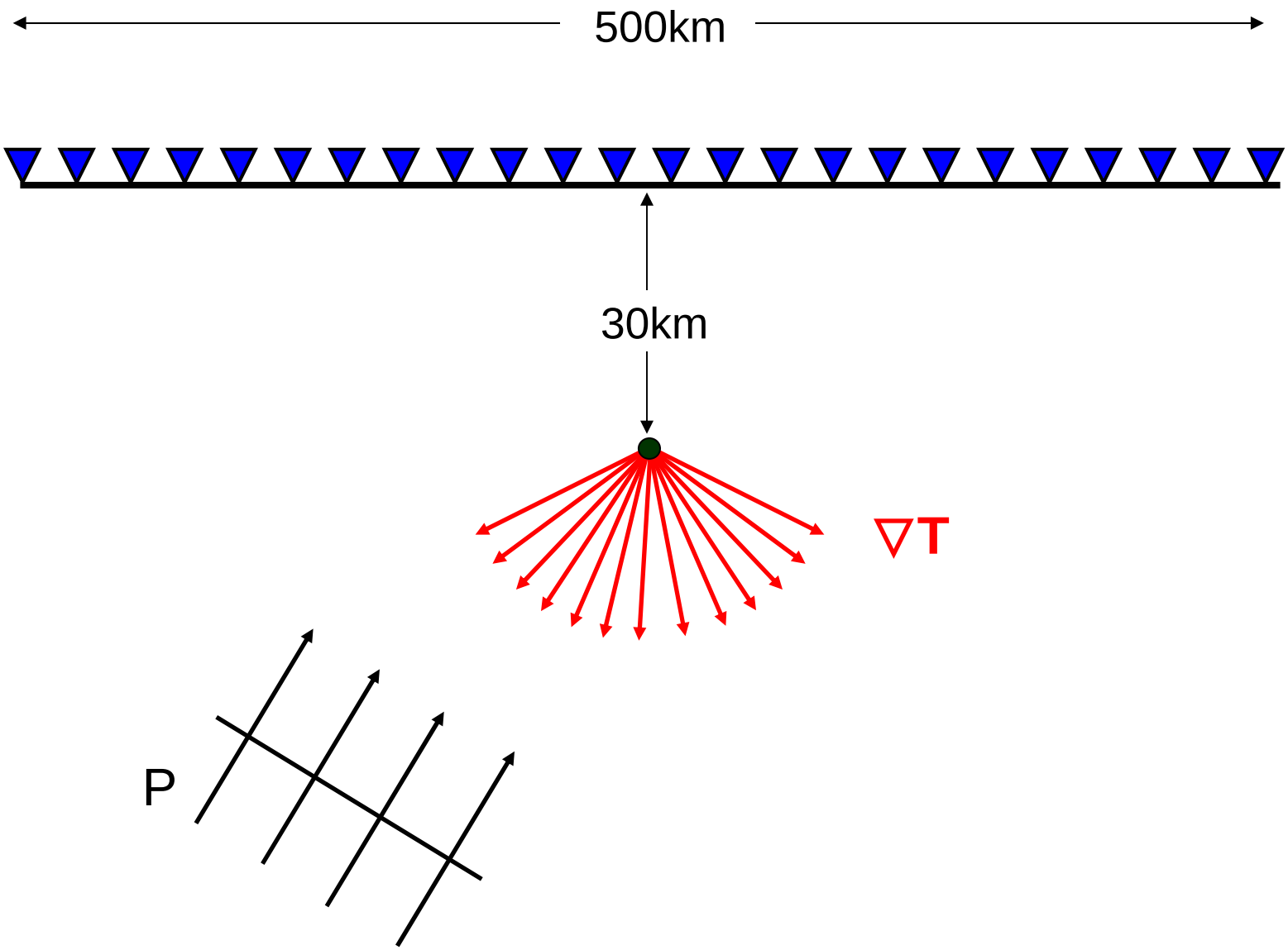
Resolution: frequency - wavenumber



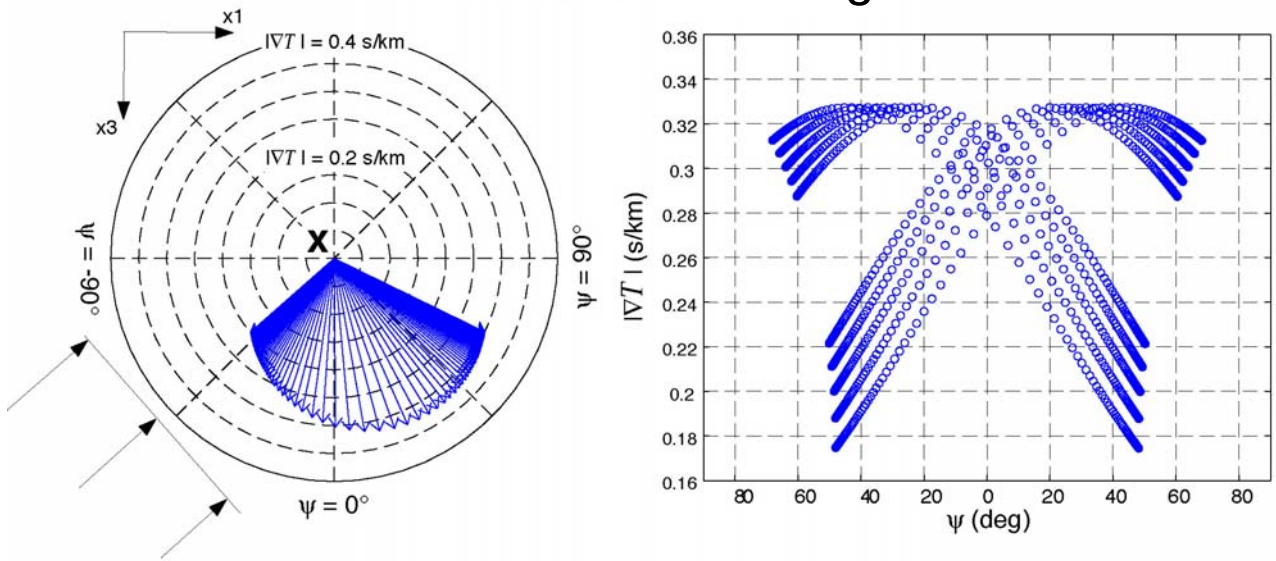
effective wavenumber

$$|k| = \omega |\nabla T|$$

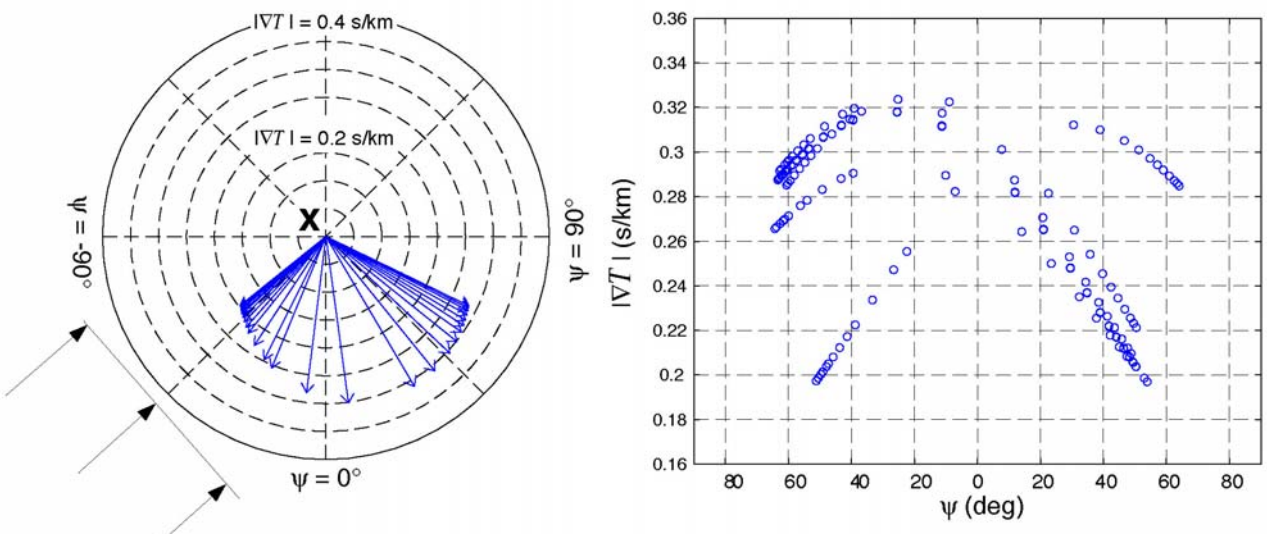
Resolution: ray coverage



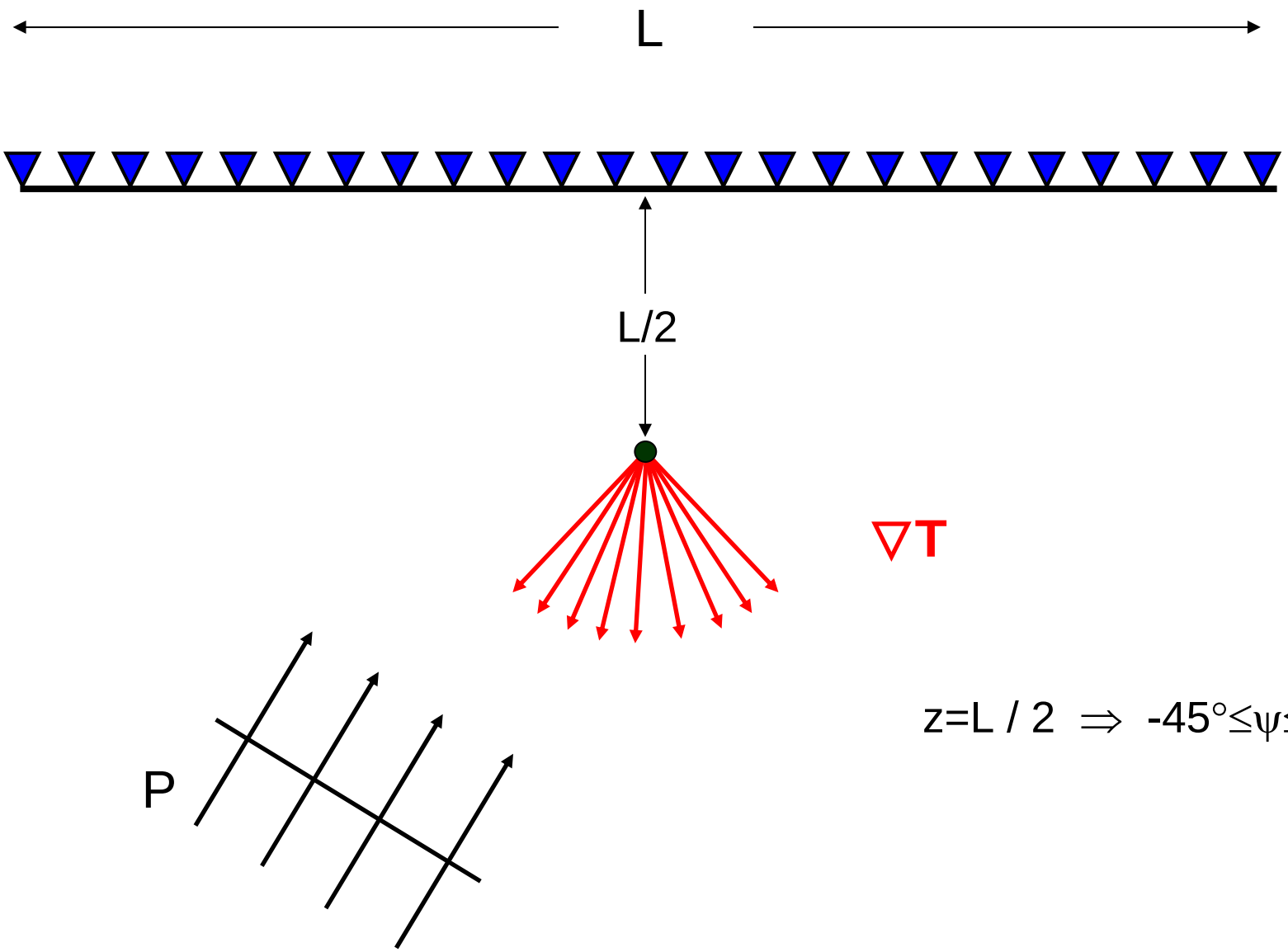
Idealized coverage



ABI96 experiment

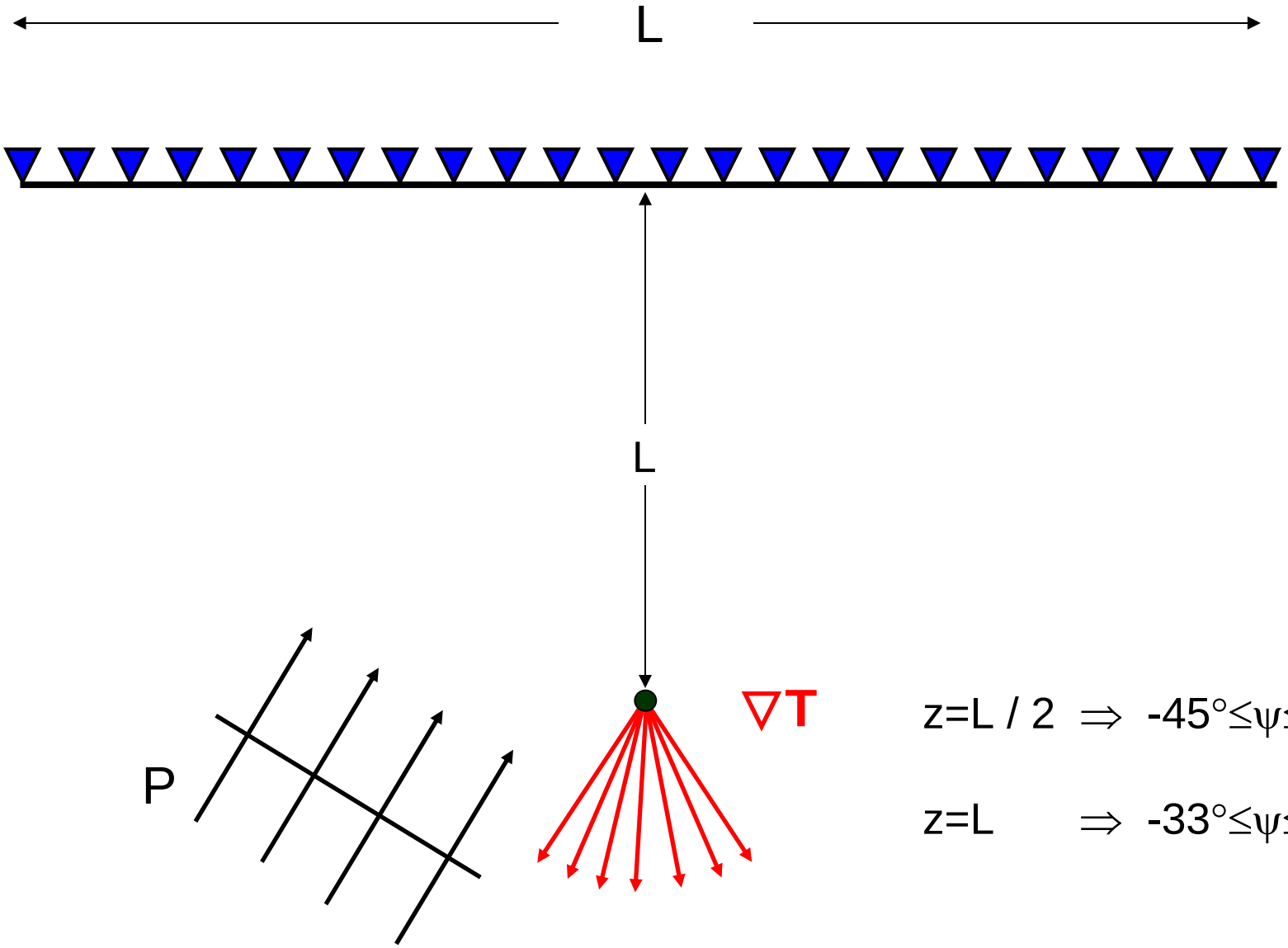


Resolution: ray coverage and aperture



$z = L / 2 \Rightarrow -45° \leq \psi \leq 45°$

Resolution: ray coverage and aperture

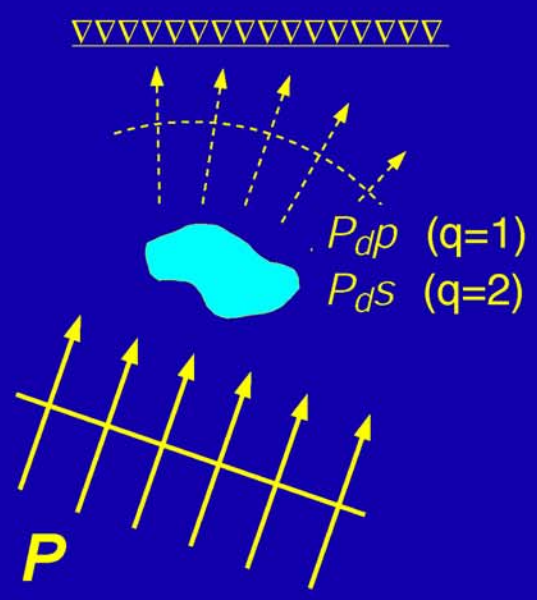


$z=L/2 \Rightarrow -45° \leq \psi \leq 45°$

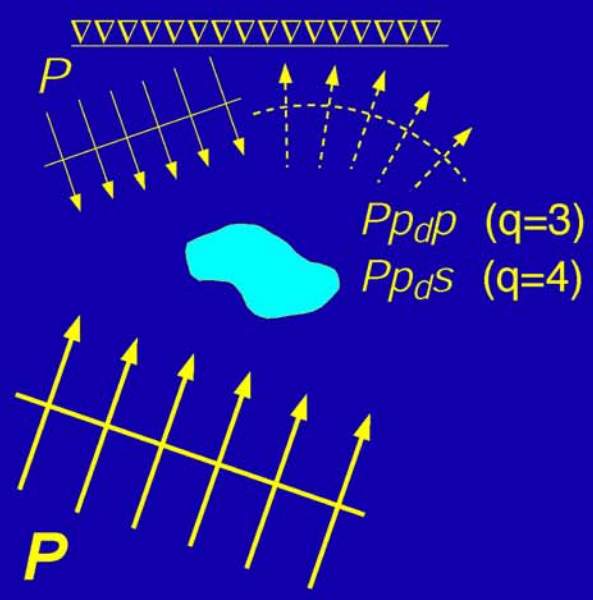
$z=L \Rightarrow -33° \leq \psi \leq 33°$

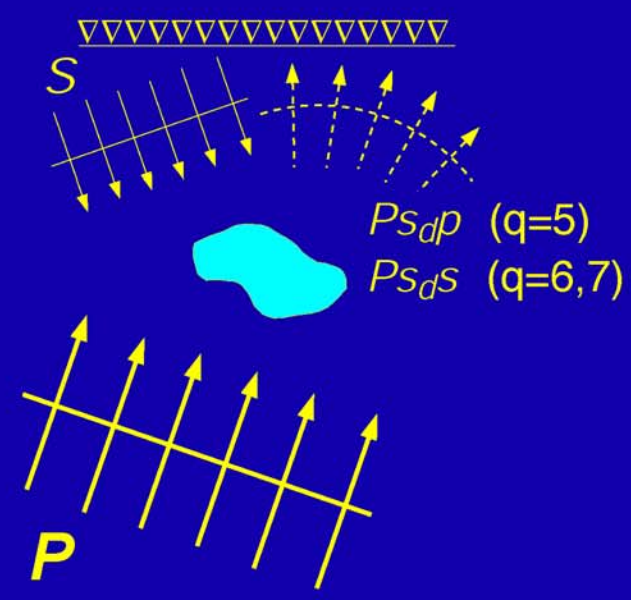
Resolution: multiples

direct arrival

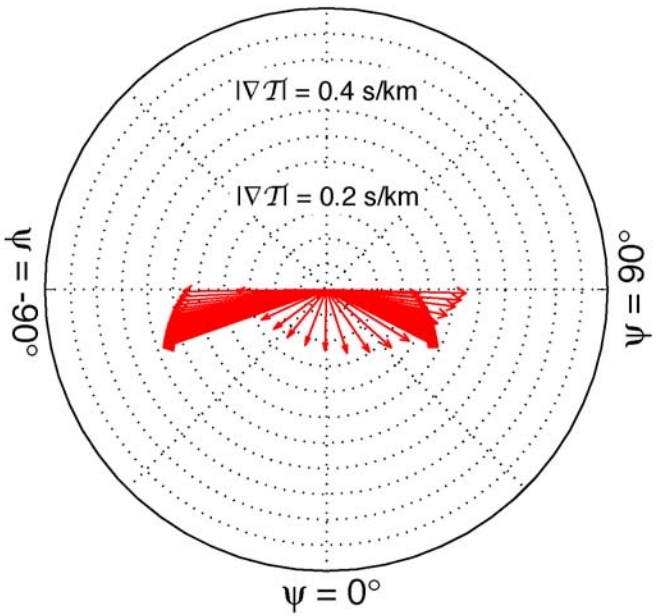


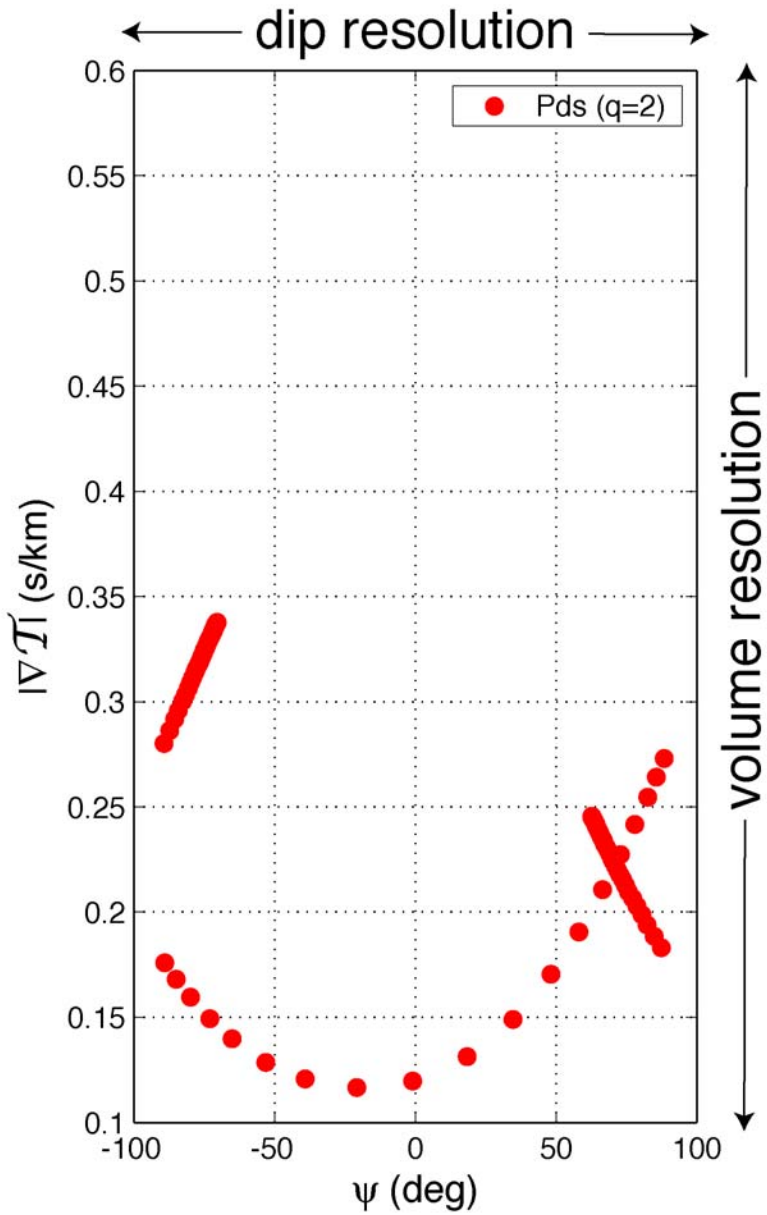
multiples



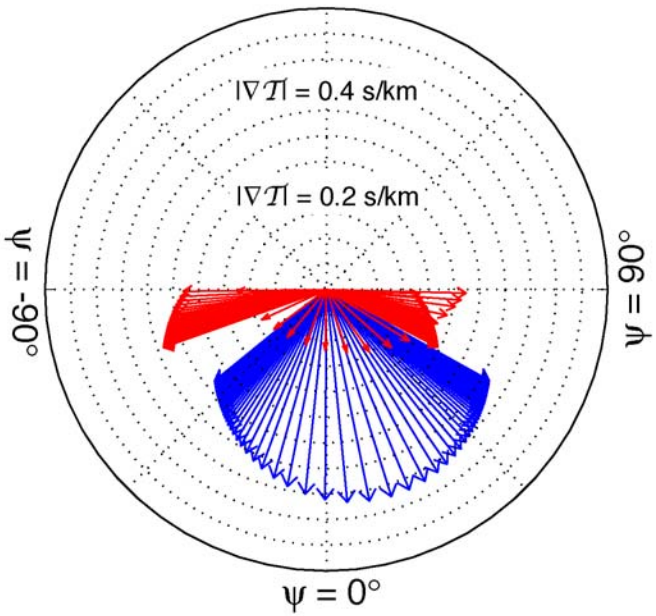


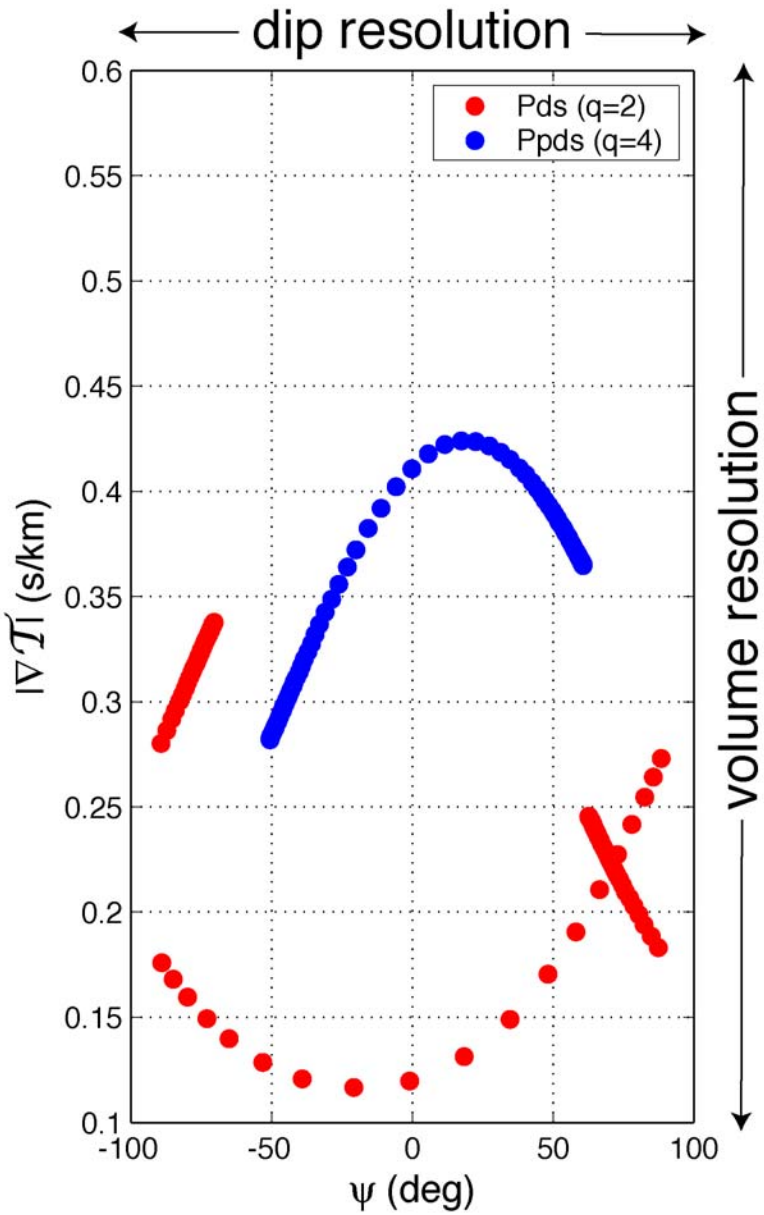
Vs: Pds


|∇T| = 0.4 s/km
|∇T| = 0.2 s/km
ψ = -90°
ψ = 90°
ψ = 0°

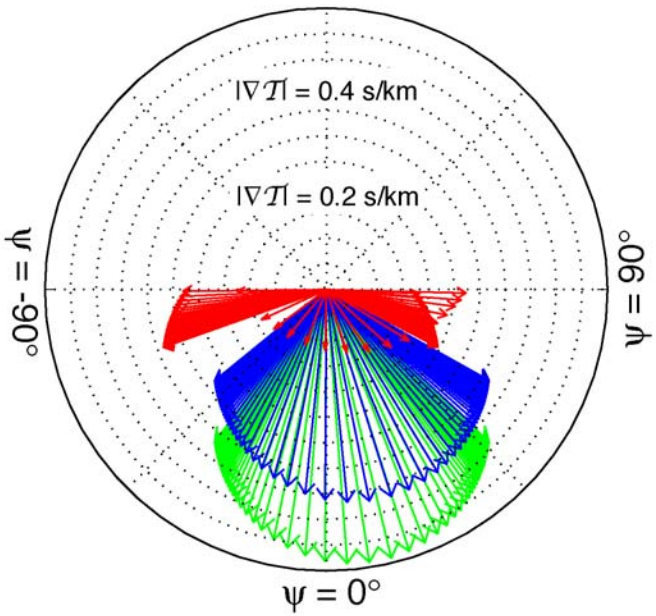

dip resolution
volume resolution
Pds (q=2)
|∇T| (s/km)
ψ (deg)
0.6
0.55
0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
-100
-50
0
50
100

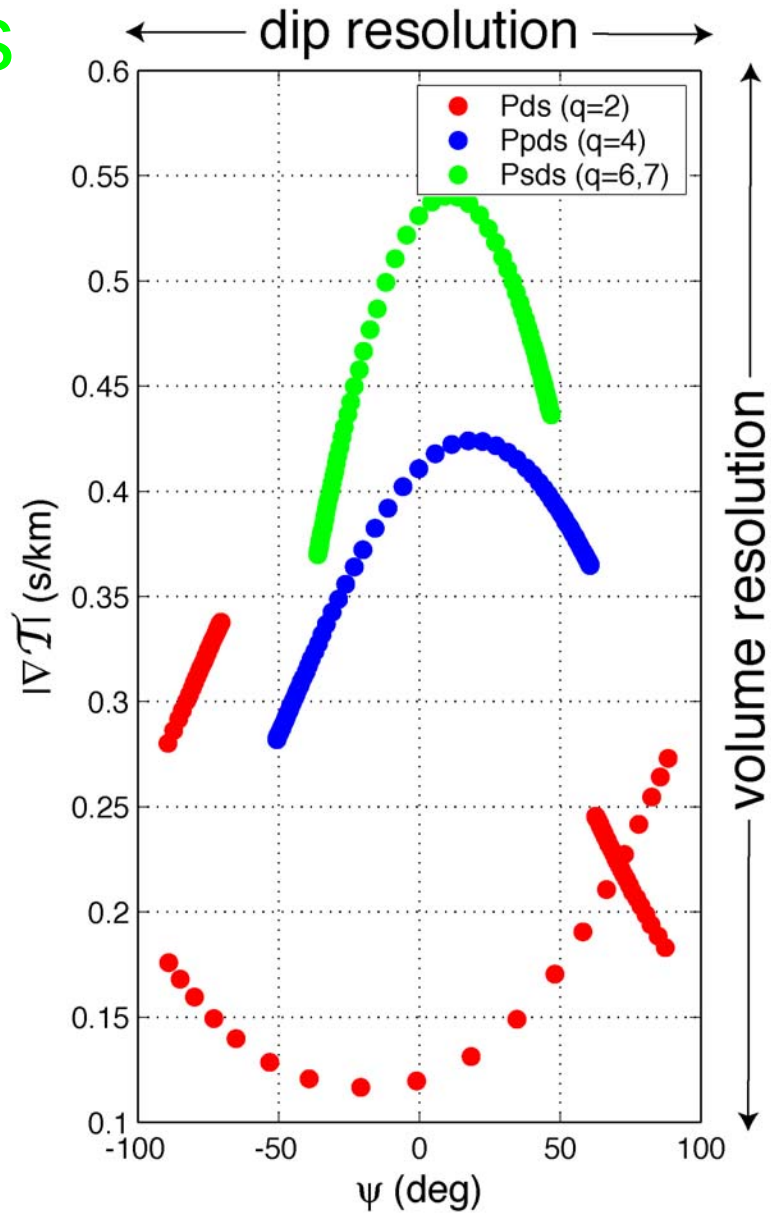
Vs: Pds, Ppds


|∇T| = 0.4 s/km
|∇T| = 0.2 s/km
ψ = -90°
ψ = 90°
ψ = 0°


dip resolution
volume resolution
Pds (q=2)
Ppds (q=4)
|∇T| (s/km)
ψ (deg)
0.6
0.55
0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
-100
-50
0
50
100

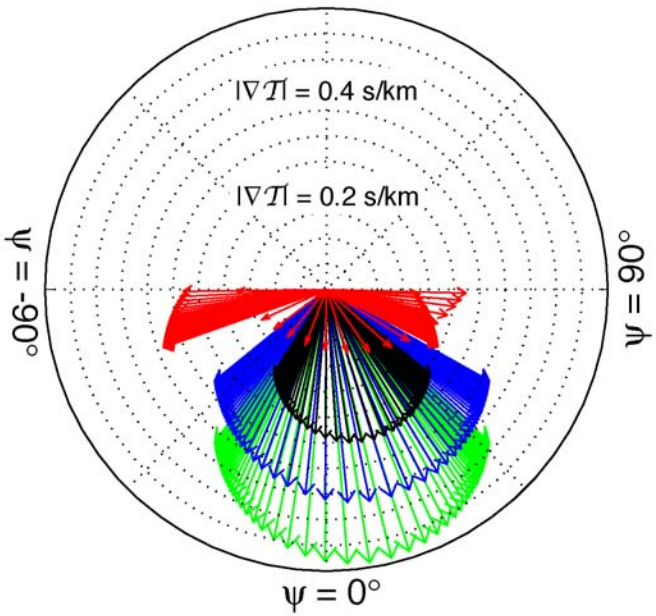
Vs: Pds, Ppds, Psds

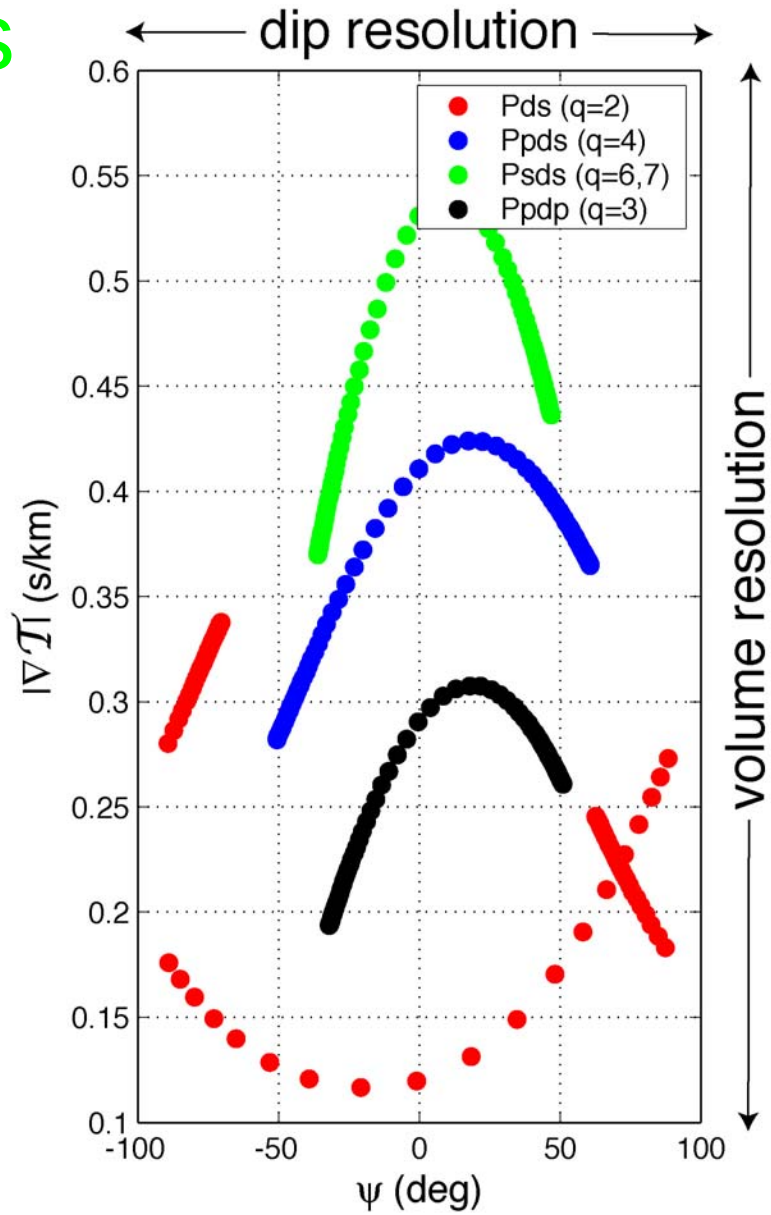




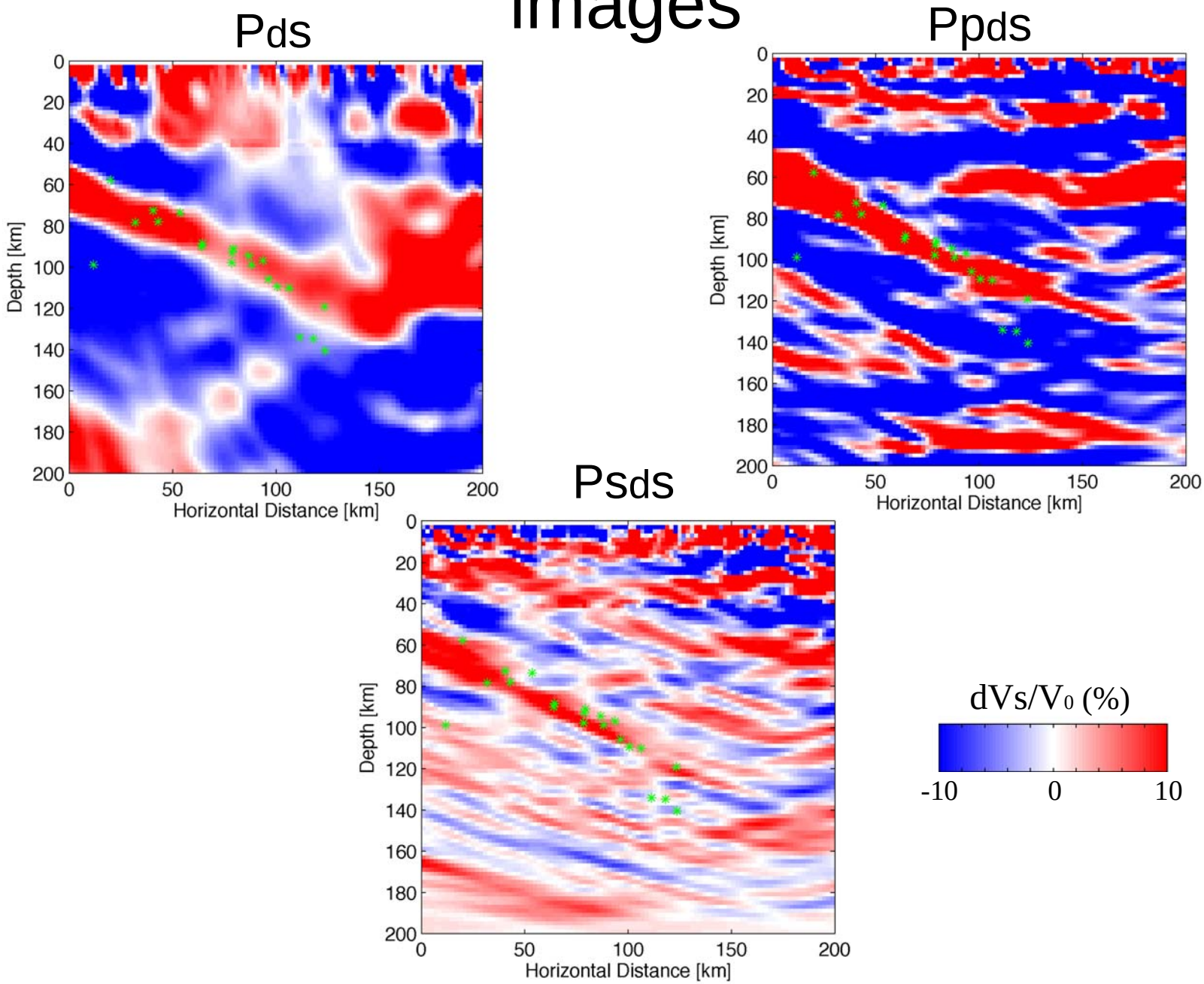
Vs: Pds, Ppds, Psds

Vp: Ppdp





images


Pds
Ppds
Psds
Depth [km]
Horizontal Distance [km]
0
20
40
60
80
100
120
140
160
180
200
0
50
100
150
200
dVs/V0 (%)
-10
0
10

images

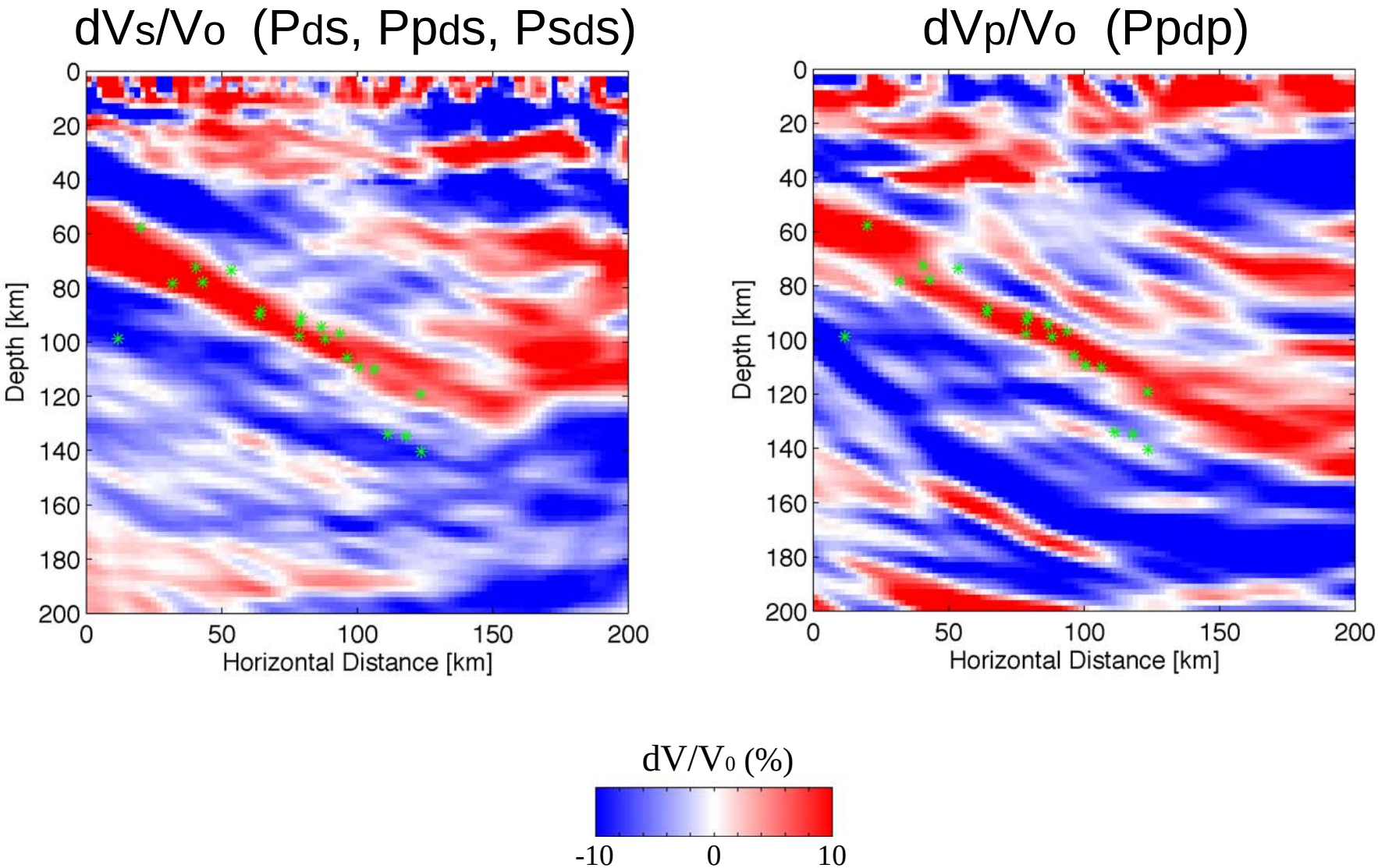

dVs/Vo (Pds, Ppds, Psds)
dVp/Vo (Ppdp)
Depth [km]
Horizontal Distance [km]
0
20
40
60
80
100
120
140
160
180
200
0
50
100
150
200
dV/V0 (%)
-10
0
10

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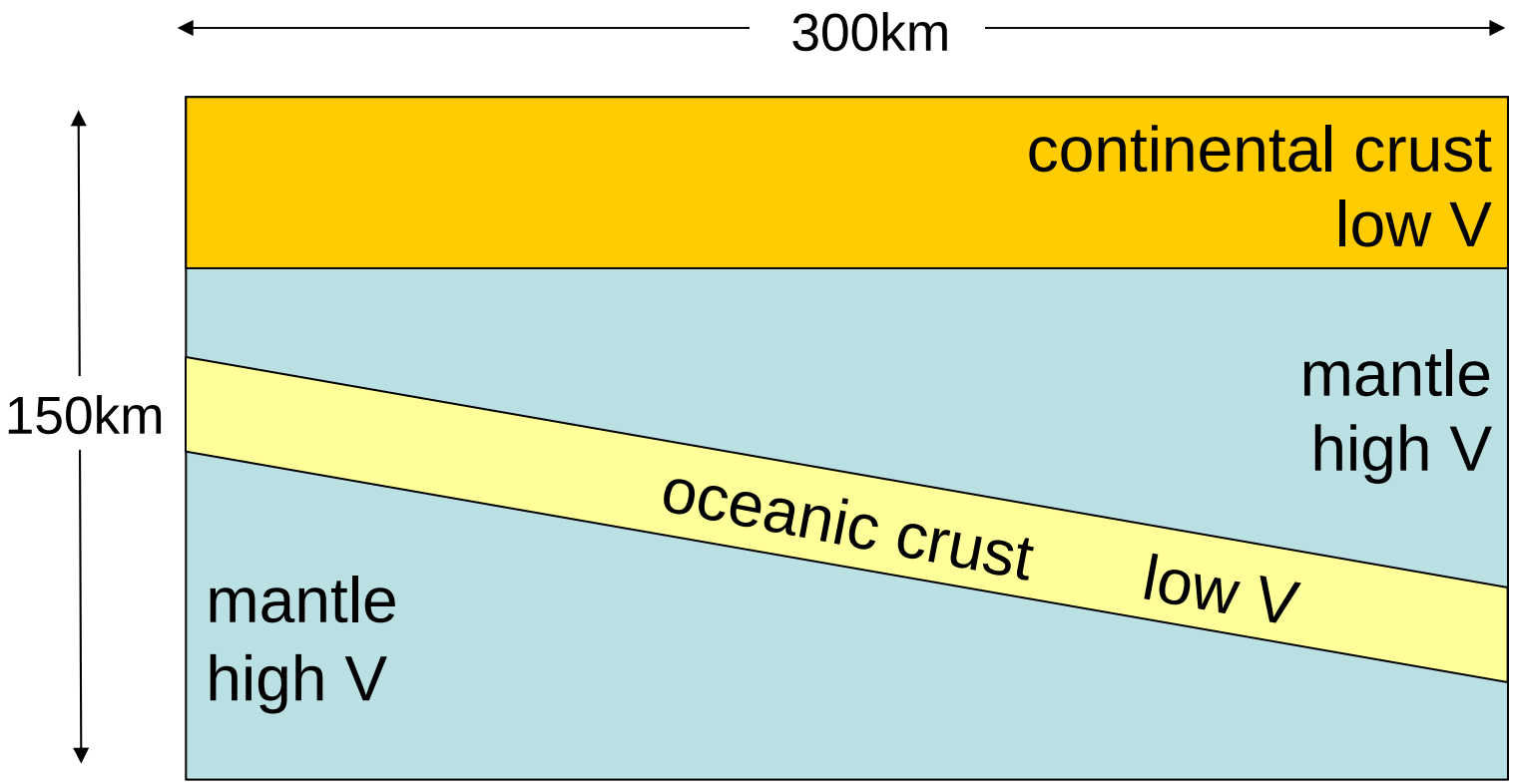
Image robustness: source distribution



subduction zone imaging

data

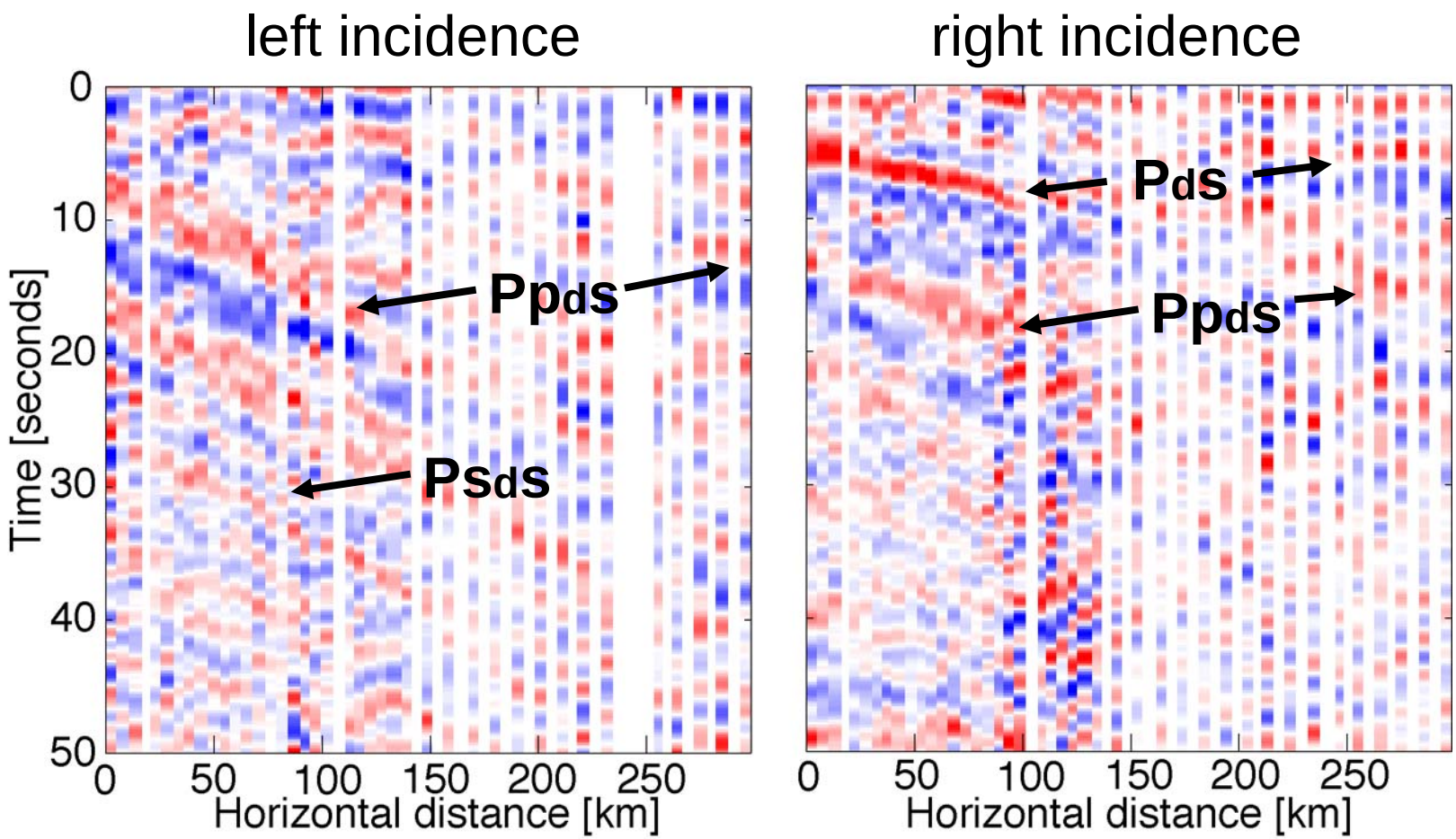

left incidence
right incidence
Time [seconds]
0
10
20
30
40
50
0
50
100
150
200
250
Horizontal distance [km]
Ppds
Psds
Pds
Ppds

Image robustness: spatial aliasing

<u>what is aliasing?</u>

"The frenzied excitement of World Series 2006 has ended with a victory in the city of St Louis"

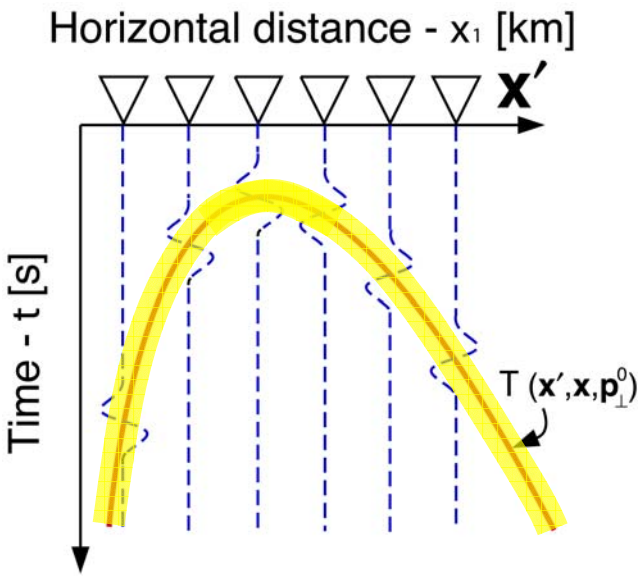
$\Downarrow$

The World ended in St Louis

Image robustness: spatial aliasing

near apex of the diffraction hyperbola : $\nabla \mathbf{x}' = \sqrt{\left(z\lambda + \lambda^2/4\right)}$

branches of the diffraction hyperbola: $\nabla \mathbf{x}' \rightarrow \dfrac{\lambda}{2}$



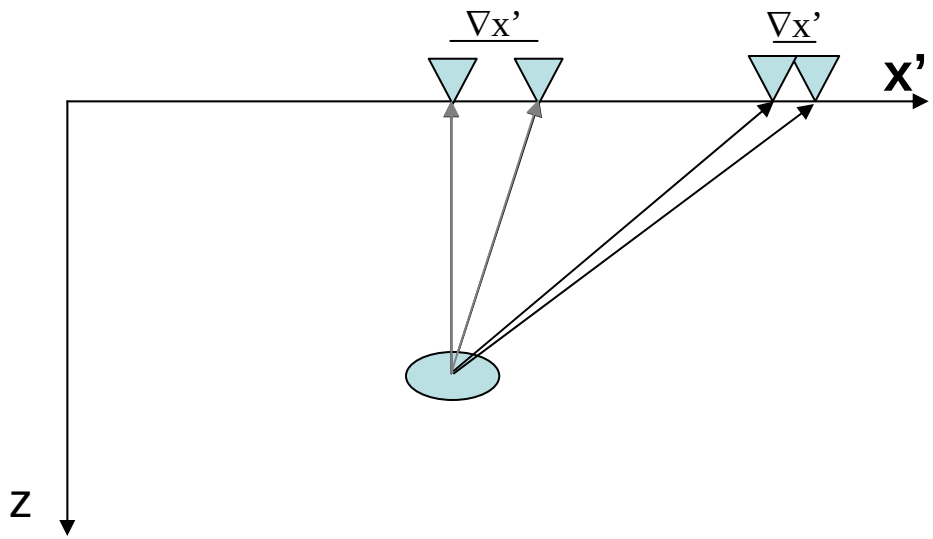
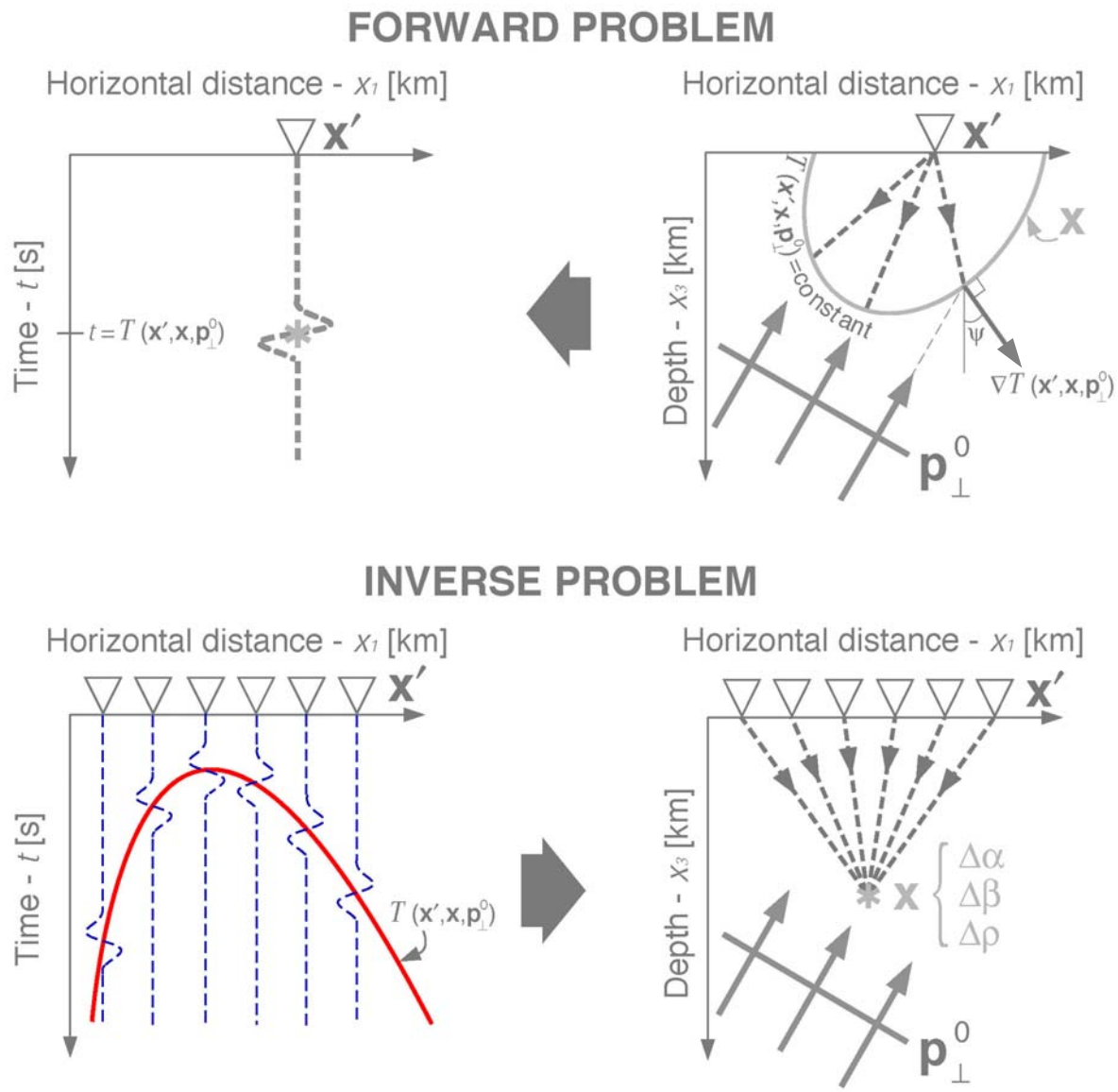
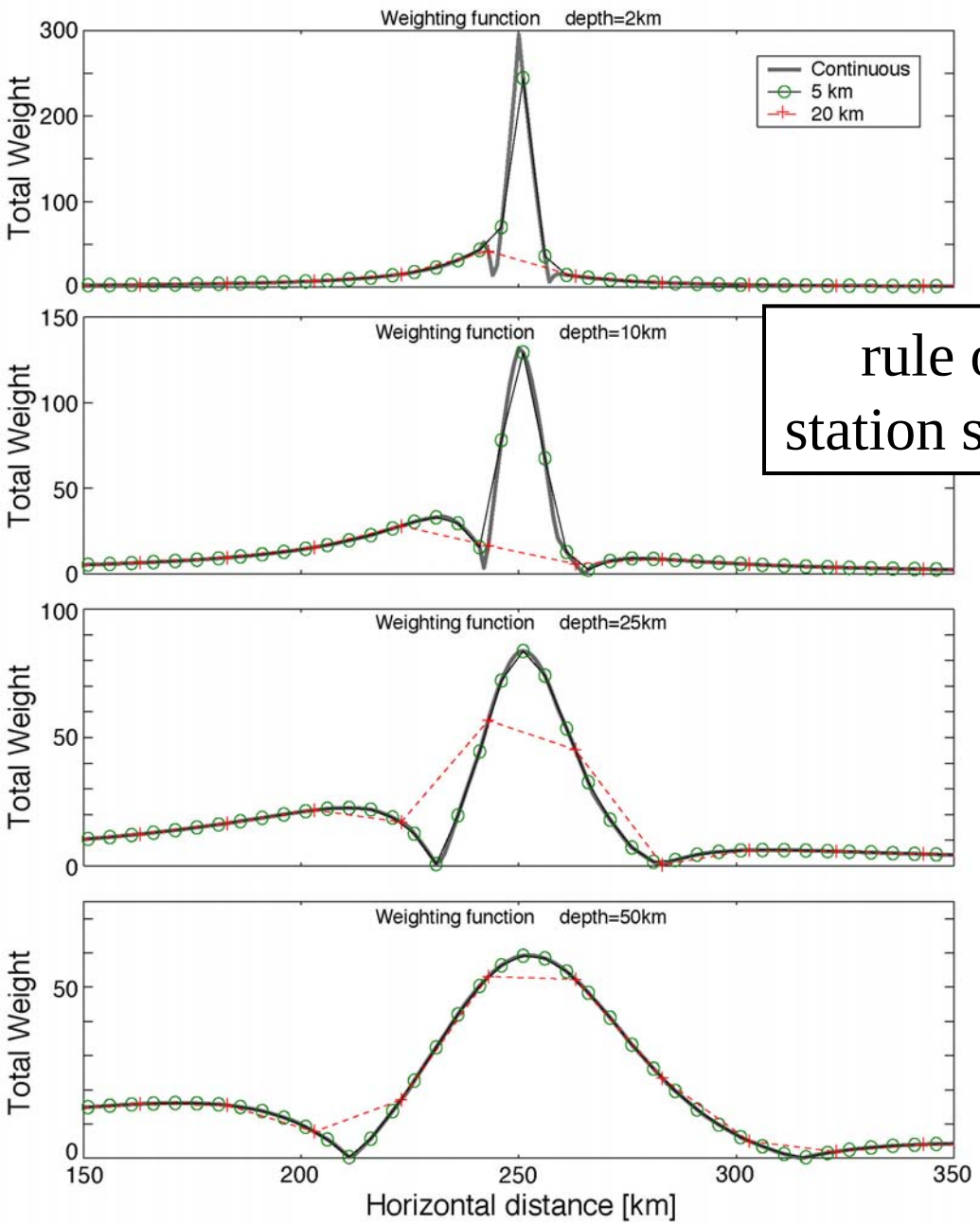


Image robustness: migration operator aliasing

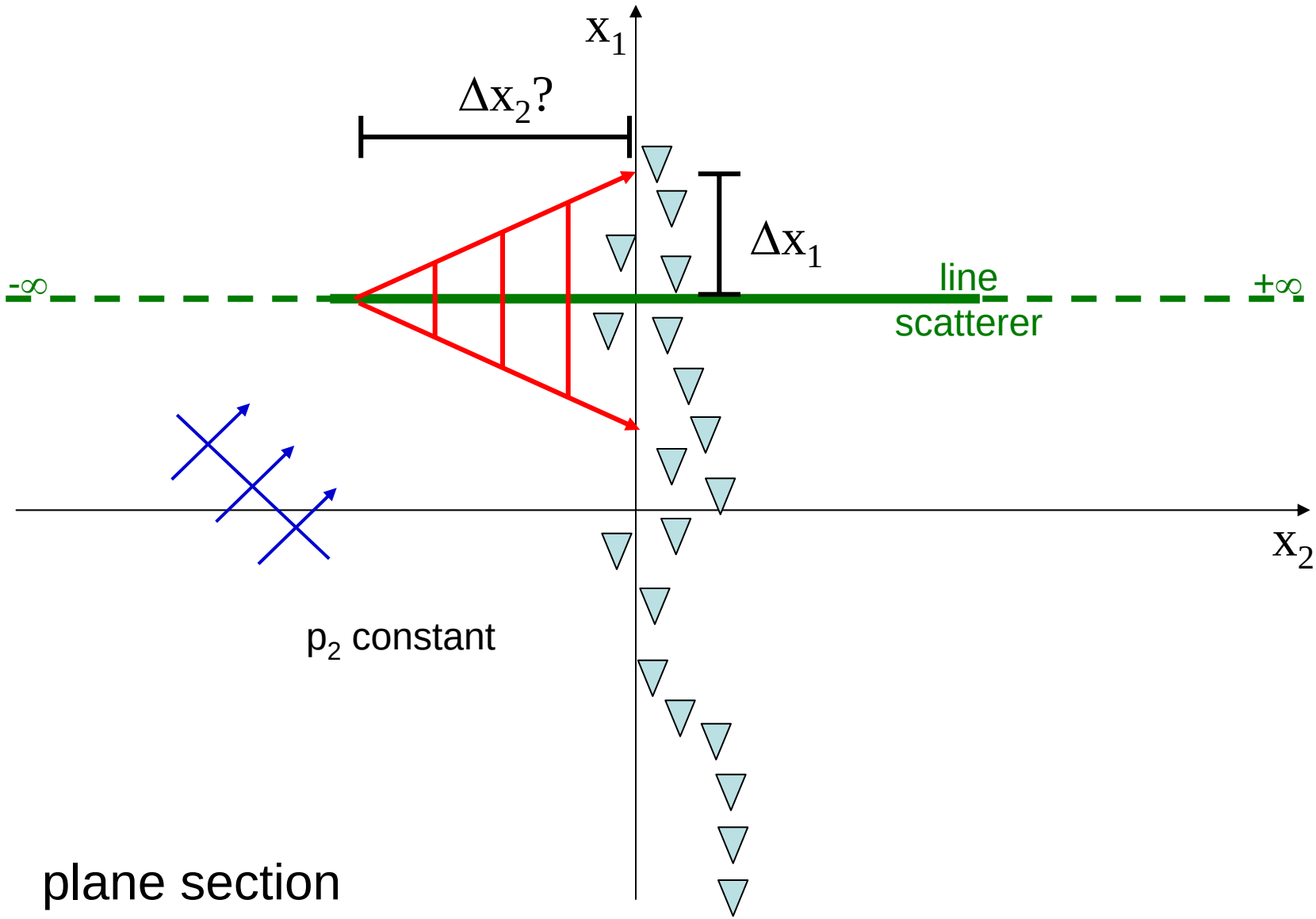


migration operator aliasing



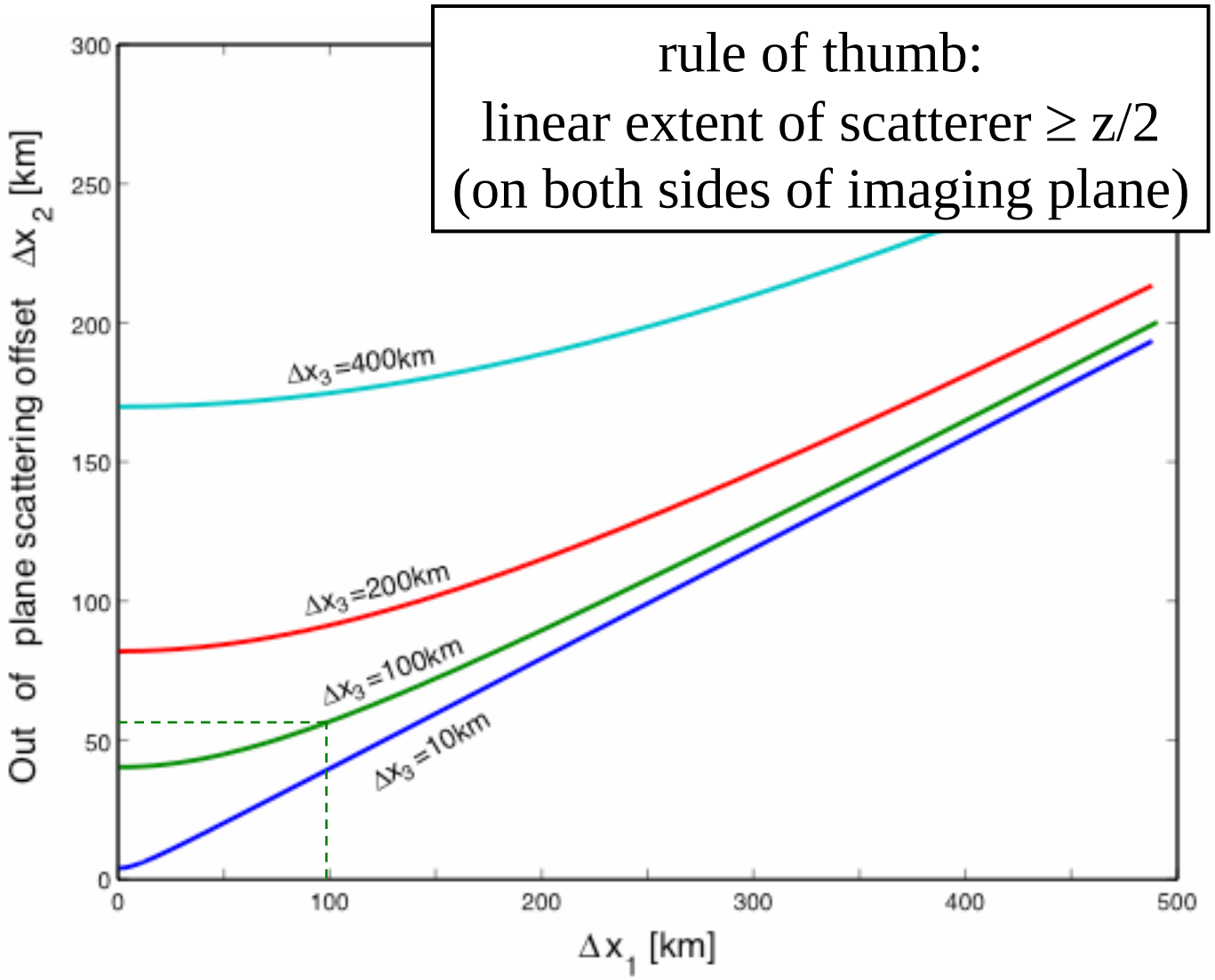
rule of thumb:
station spacing ≤ z/2

Image robustness: departure from 2-D geometry



plane section

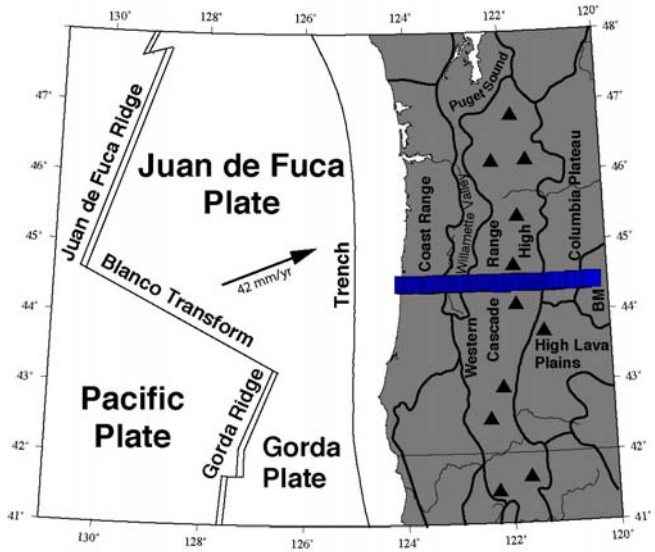
Image robustness: departure from 2-D geometry

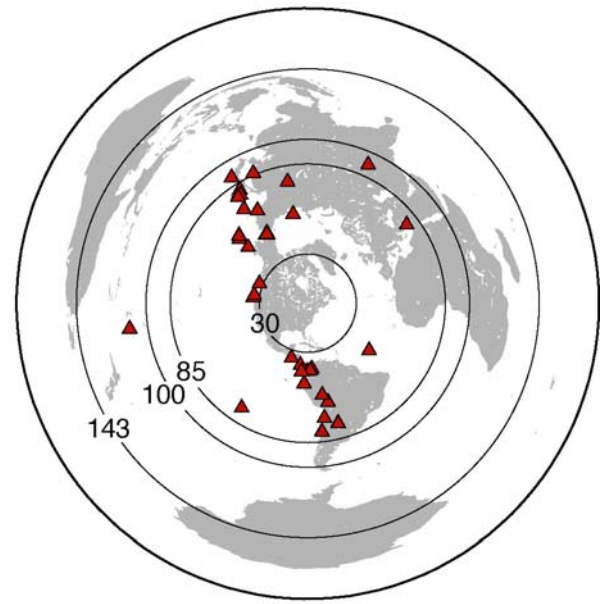


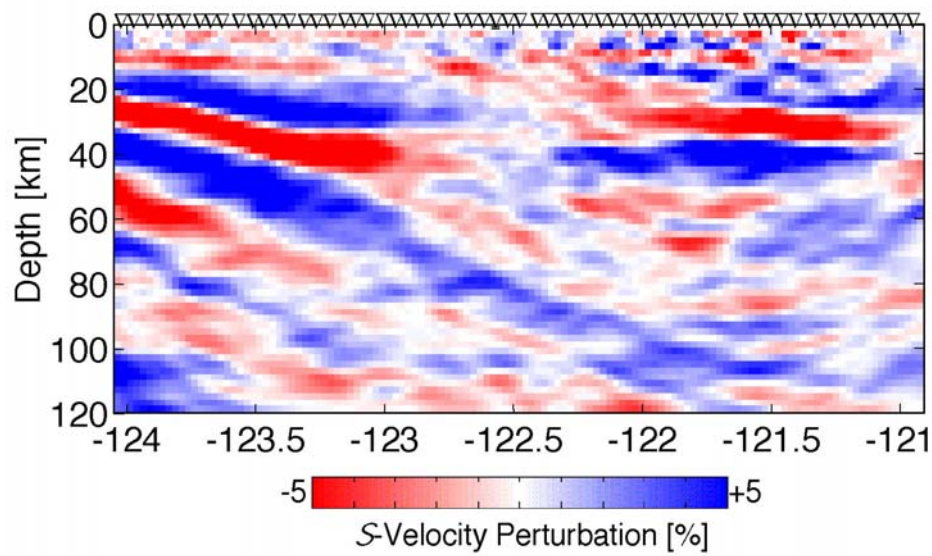
outline

- equations of imaging I: Kirchhoff and Generalized Radon Transform for continuous functions
- reality check from field conditions
- equations of imaging II: discretization and loss of dimension
- resolution: frequency/wavenumber, ray coverage, multiples
- image robustness: source distribution, aliasing, departure from 2-D geometry
- case studies
- challenges: application to short period teleseismic data and 3-D imaging

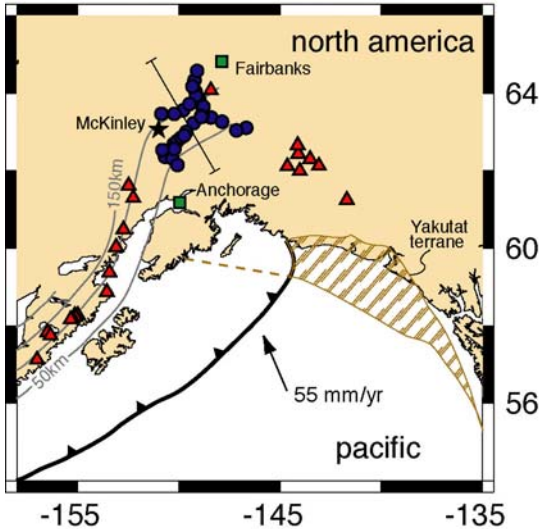
Cascadia subduction zone

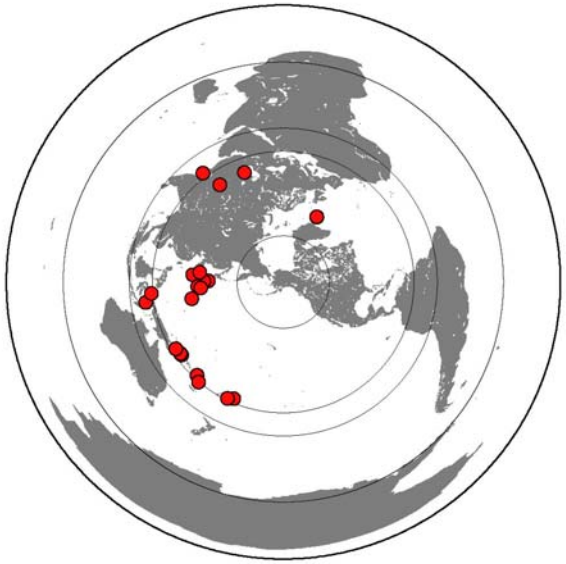




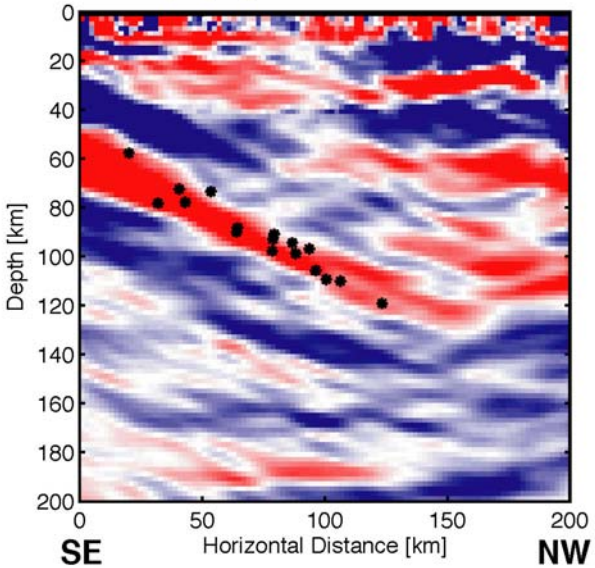


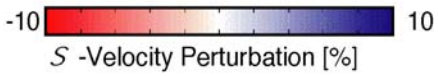
BEAAR experiment central Alaska





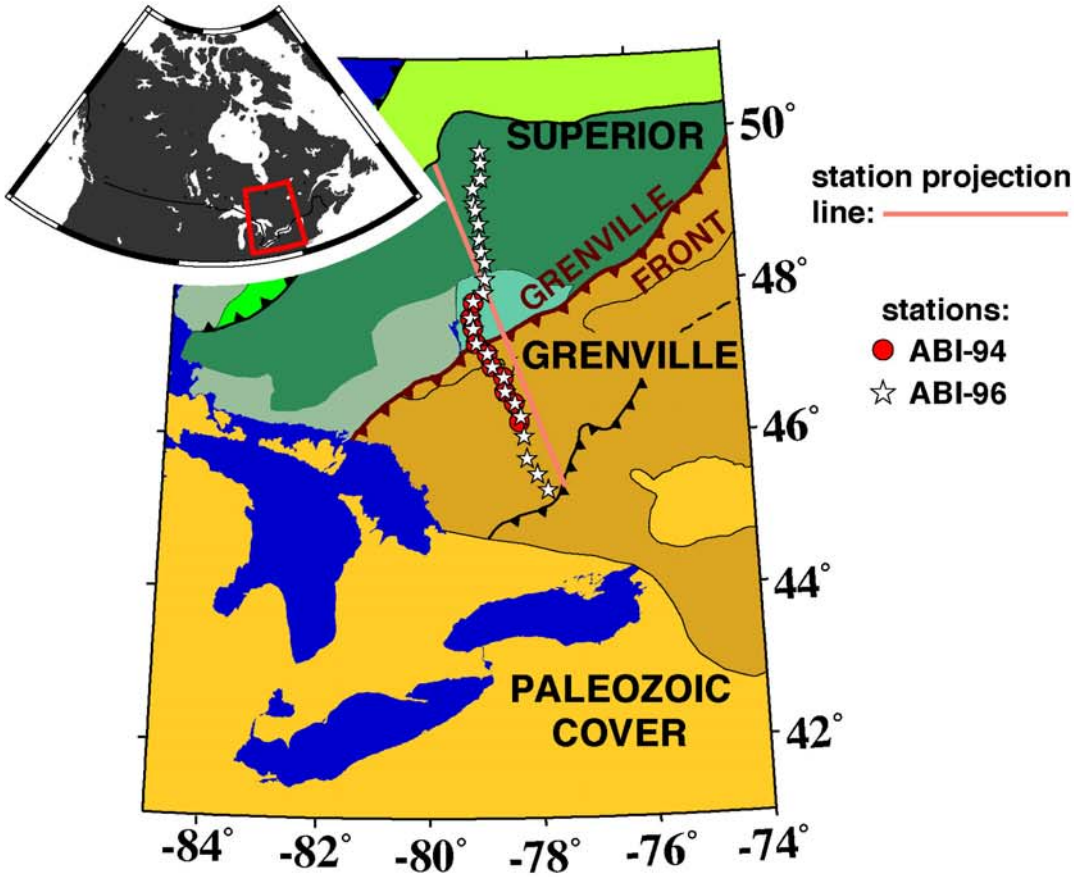
δβ/β perturbations



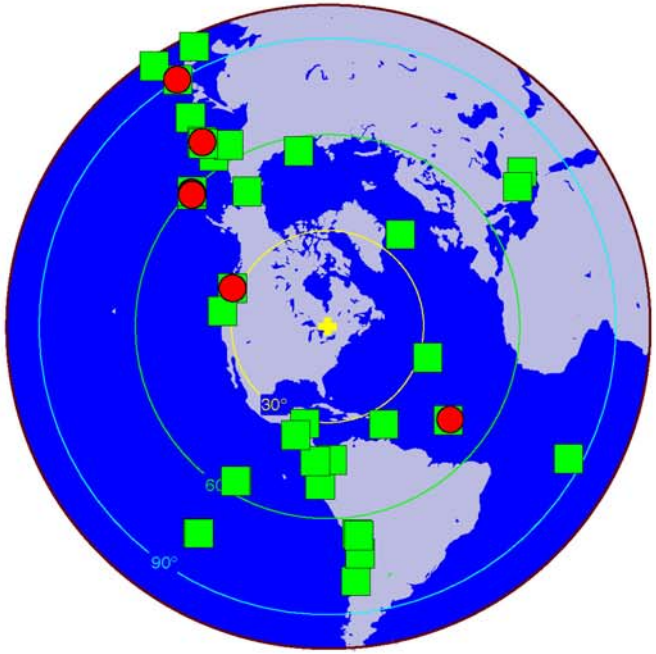


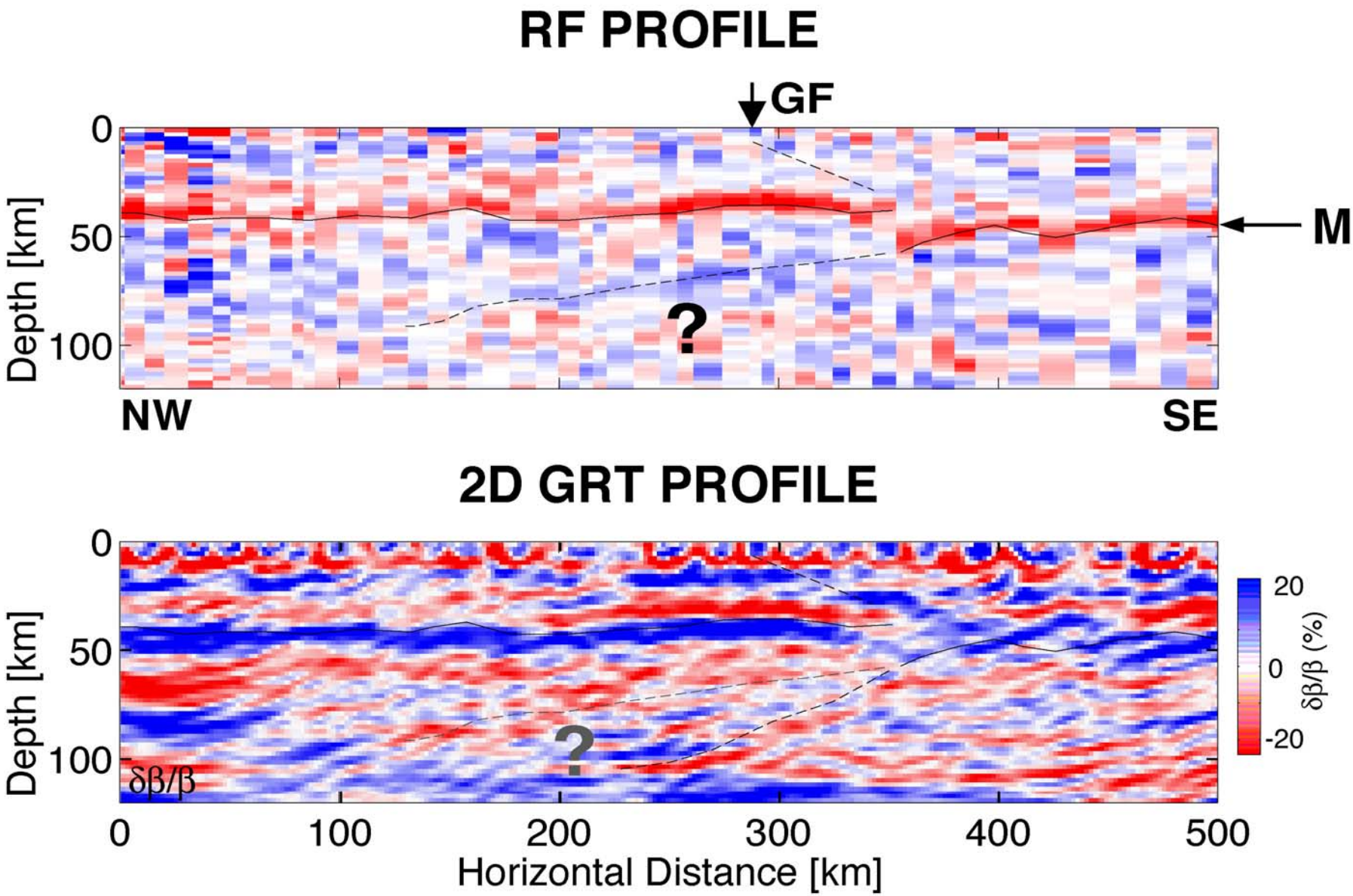
ABITIBI 1996

study area



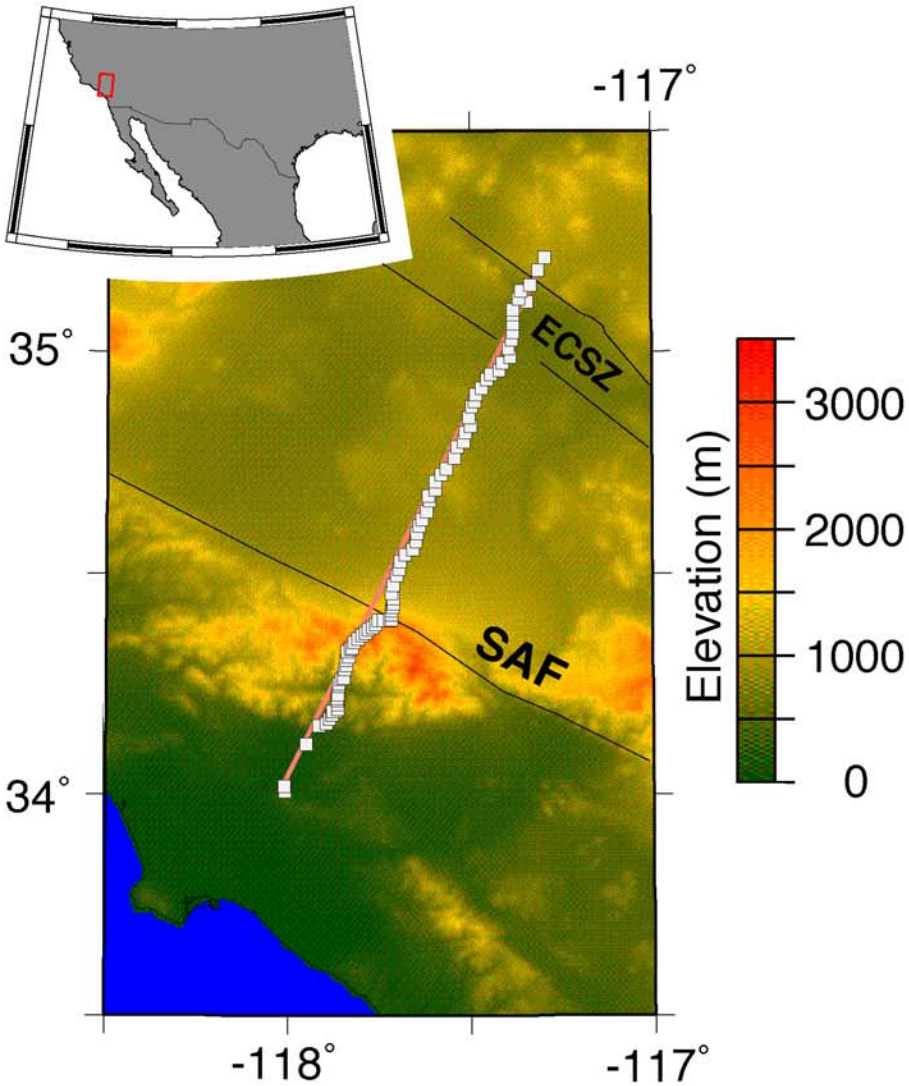
event distribution



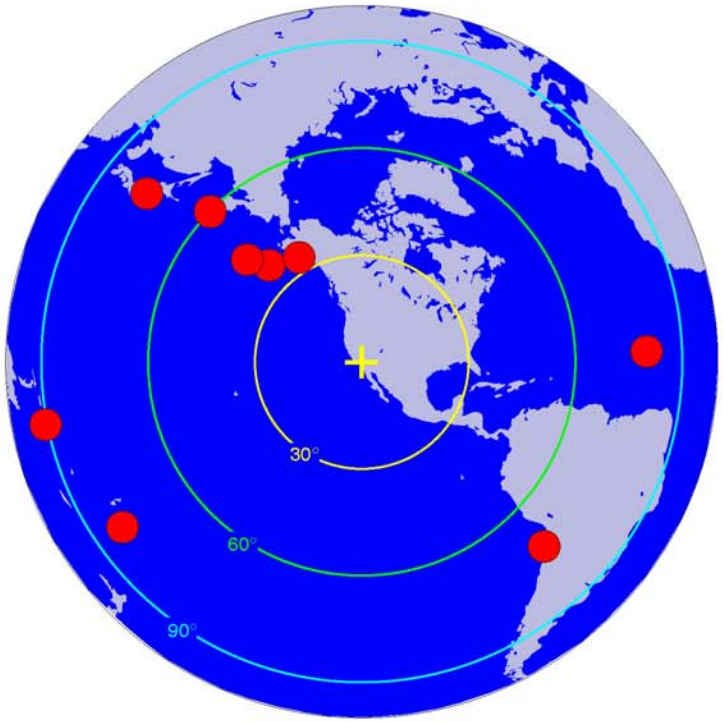

RF PROFILE
GF
M
0
50
100
Depth [km]
?
NW
SE
2D GRT PROFILE
0
50
100
Depth [km]
δβ/β
?
0
100
200
300
400
500
Horizontal Distance [km]
20
0
-20
δβ/β (%)

LARSE

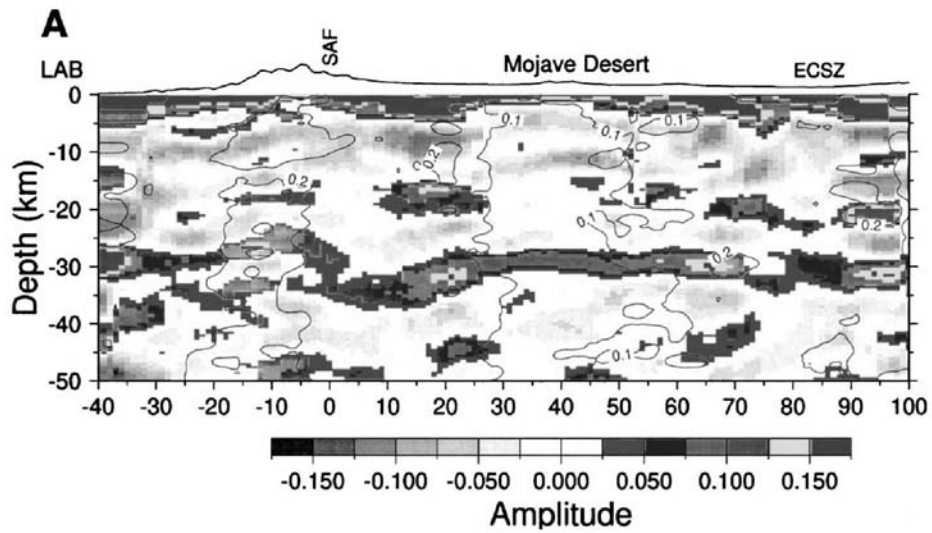
study area



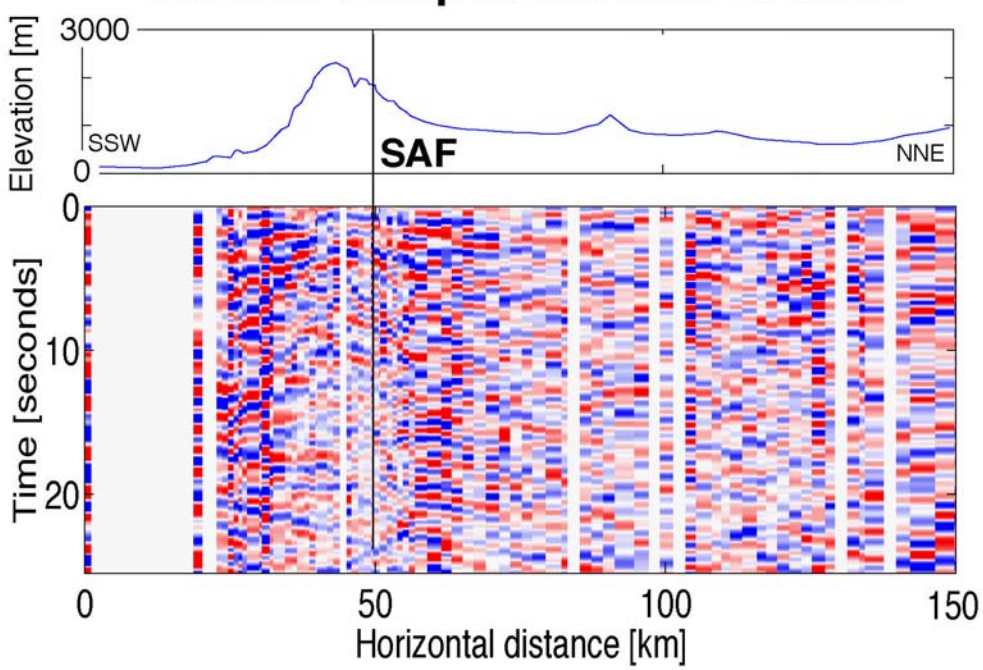
event distribution



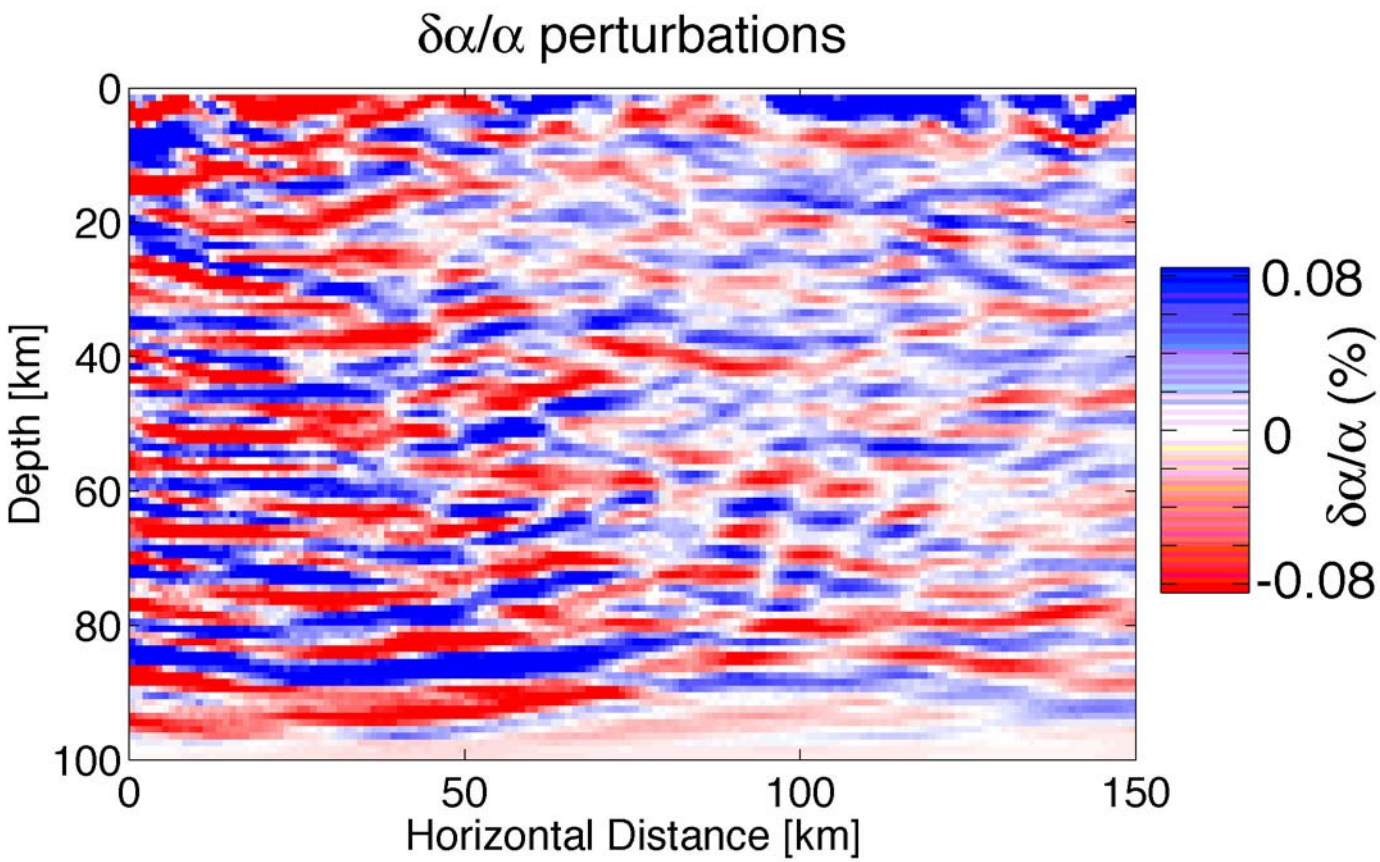
RF profile (Zhu, 2000)

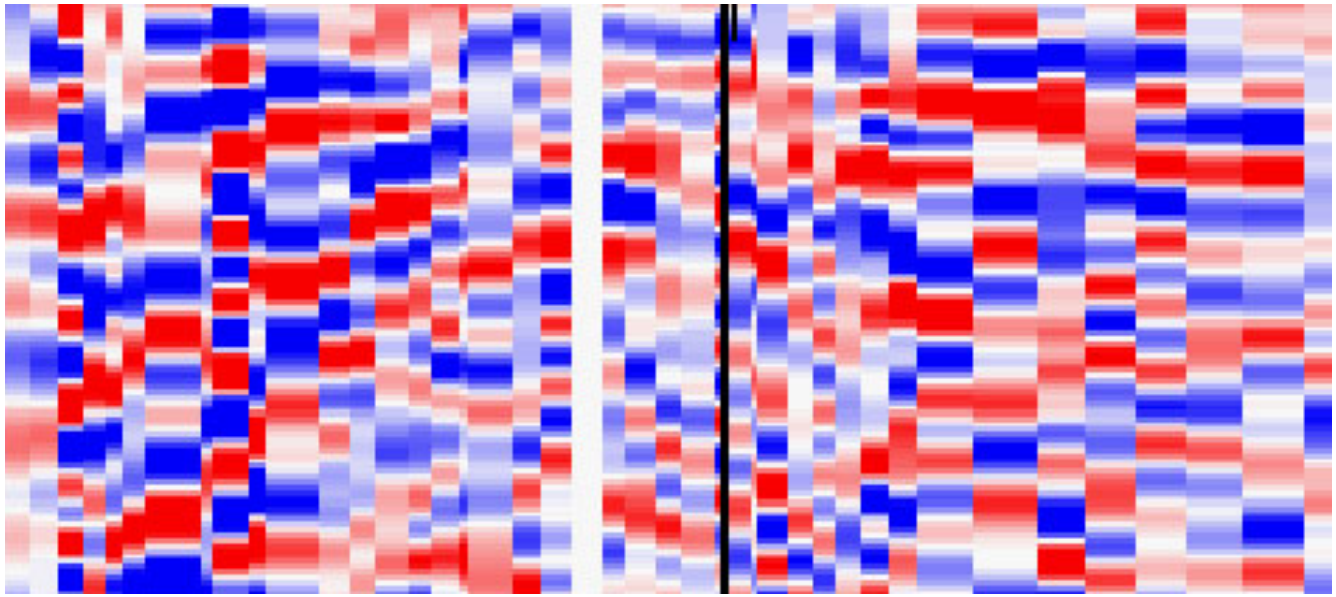


vertical component data section



2D GRT profile





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3-D imaging: what are we waiting for?

- equations are well known

- more instruments, but they need to be cheaper (short period or new technologies)

- for short period data, we need to develop new tools to separate near-field and/or surface scattering effects

- source distribution will remain a problem!

Concluding remarks

- Imaging works despite numerous assumptions made about the geometry of the model and the background velocity model
- Careful examination of the equations we use can help to develop strategies for working around limitations introduced by field conditions, designing optimal arrays for any given target, and assessing the robustness of the resulting image