

Seismic imaging of the crust and upper mantle using regularized joint receiver functions, frequency–wave number filtering, and multimode Kirchhoff migration

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[1] Receiver functions provide an indispensable tool for producing discontinuity images of the crust and upper mantle from teleseismic earthquake arrivals. However, image quality can be significantly compromised by instabilities inherent in the deconvolution process and by irregularities in data coverage driven by source and/or receiver geometries. We present receiver function estimation and prestack migration techniques which both reduce receiver function deconvolution instability and produce regularized, multimode receiver function images. The receiver function estimation technique exploits frequency–wave number filtering methods using frequency–pseudowave number domain filtering of gathers to enhance arrivals with consistent moveout characteristics. Synthetic tests confirm robust recovery of features in the presence of high noise levels. To process regularized receiver functions into accurate images of the subsurface seismic scattering potential, we employ a regularized Kirchhoff migration that migrates both direct and reverberated *P*-to-*S* converted modes to their correct subsurface locations. The subsequent images can be stacked to mute mismigrated modes. Synthetic studies and model resolution tests demonstrate that this methodology is well suited to irregularly spaced stations and uneven data coverage and does not require the recording of a single earthquake at multiple stations.

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1. Introduction

[2] Teleseismic body waves from earthquakes produce a series of reflections, refractions, and conversions as they traverse boundaries between regions of differing seismic velocity and/or impedance. Radial receiver function analysis emphasizes *P*-to-*S* converted arrivals at these interfaces via signal deconvolution between horizontal and vertical component seismograms [Langston, 1977; Ammon, 1991; Cassidy, 1992]. In receiver function analysis, the vertical component seismogram of the *P* wave is characterized as a convolution of instrument response, vertical site response, and earthquake source function. The radial component seismogram of the *P* wave is characterized as the convolution of the instrument response, radial site response, and earthquake source function. Deconvolution of the horizontal signal by the vertical signal thus removes the common factors of instrument response and source function from the radial seismogram. The resulting receiver function is

predominantly controlled by the velocity and impedance discontinuity structure of the upper mantle and crust along the ray path, and can be conceptualized as a scaled version of the radial component of displacement with *P* multiples removed [Ammon, 1991].

[3] An inherent problem in calculating receiver functions is proper regularization of the commonly ill-posed deconvolution problem. In this paper we develop receiver function estimation techniques which take advantage of characteristic moveout to enhance receiver function arrivals, while removing features attributable to noise and deconvolution instability.

[4] Receiver functions were first applied to solitary stations to obtain local one-dimensional structure estimates [e.g., Langston, 1977]. Receiver functions at a single station can be moveout corrected to a reference epicentral distance, to vertical travel time, or to a common depth axis, and subsequently stacked to emphasize consistent features. As the number of stations available for deep seismic imaging has increased, receiver function techniques have been extended to create (to date, frequently two-dimensional) images of fundamental structures such as the Moho or upper mantle transition zone discontinuities [e.g., Sheehan *et al.*, 1995; Yuan *et al.*, 1997; Li *et al.*, 2002]. A commonly used method of producing cross sections referenced to depth

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from receiver functions is the exploration common conversion point (CCP) image [Tessmer and Behle, 1988; Dueker and Sheehan, 1998]. The CCP image is created by first back projecting the receiver function along a theoretical ray path. The depth cross section is calculated by taking the mean sample value (or some other central tendency measure) within bins. Although the CCP method is easily implemented and transforms the data into offset and depth space, it does not correct for diffracted energy or mismatched multiples (such as commonly arise from free surface reverberations).

[5] Migration seeks to map seismic energy to its true subsurface origin (either in time or in depth) while collapsing diffraction artifacts, and is necessary to produce structural images that properly correct for wave propagation effects due to lateral heterogeneity [e.g., Abers, 1998; Eaton et al., 2002]. Recent efforts have included the application of seismic data migration techniques to teleseismic wave fields, including both prestack [Sheehan et al., 2000; Poppeliers and Pavlis, 2003; Wilson et al., 2003, 2005] and poststack [Ryberg and Weber, 2000] methodologies. Other promising methodologies implement prestack migration and scattering inversion of the three-component teleseismic wave field [e.g., Bostock and Rondenay, 1999; Bostock et al., 2001; Fan et al., 2003; Shragge and Artman, 2003].

[6] Here we present a method of receiver function migration that uses information contained in both direct and reverberated P -to- S converted energy while suppressing reverberation artifacts. The migrated image is constructed using receiver functions calculated independently at each station, making it unnecessary to simultaneously record a given earthquake teleseismic wave field at multiple stations. Migration with receiver functions rather than the three-component wave field has the practical advantage of reducing a vector imaging problem to a scalar imaging problem [e.g., Levander, 2003]. Receiver function migration produces images of the subsurface P -to- S scattering potential [Poppeliers and Pavlis, 2003] as estimated in receiver function amplitudes. Receiver function amplitudes are, in turn, determined by the amplitude of shear wave arrivals on the radial component scaled by the P wave arrival on the vertical component [Ammon, 1991; Cassidy, 1992] and thus contain information about the relative S wave and P wave transmission and reflection coefficients which are directly tied to subsurface velocity and impedance contrasts. In particular, forward scattering of the incoming wave field (direct converted P -to- S energy) is most sensitive to contrasts in velocity, while back scattered phases (reverberated receiver function phases) are most sensitive to P and S impedance contrasts [Wu and Aki, 1985]. We present a receiver function migration methodology based on regularized Kirchhoff methodology [e.g., Duquet et al., 2000; Nemeth et al., 1999] that has been shown to produce subsurface images with higher resolution and fewer migration artifacts and acquisition geometry artifacts than standard migration algorithms. We develop and apply this technique to synthetic receiver functions to demonstrate the benefits for common recording geometries where imaging can be degraded by irregular station spacing and uneven data coverage. These receiver function estimation and migration methods are subsequently applied to data

from the RISTRA experiment in the southwestern United States by Wilson et al. [2005].

2. Receiver Function Estimation

[7] In receiver function modeling incorporating plane waves and flat-lying interfaces, four dominant converted/refracted propagation modes are commonly observed (Figure 1a):

[8] 1. The direct P mode, observed as a prominent pulse near zero time. The deconvolution process should place this pulse exactly at zero time, however shallow structure such as sedimentary layers can cause a slight delay (typically less than 1 s) and a decrease in direct P pulse amplitude [Cassidy, 1992].

[9] 2. Direct P -to- S (Ps) modes generated at each discontinuity. In a flat-layered velocity structure, the arrival time for the Ps mode relative to the direct P arrival is

$$T_{Ps} = T_s - T_p \quad (1)$$

where

$$T_s = H(V_s^{-2} - p^2)^{1/2}, \quad T_p = H(V_p^{-2} - p^2)^{1/2} \quad (2)$$

H is the depth to the converting interface, V_p and V_s are the reciprocal average P and S wave slownesses to the converting interface, and p is the horizontal slowness (ray parameter) of the teleseismic event. For conversions from continental Moho, the Ps mode typically arrives at an arrival time around 4–6 s.

[10] 3. The $PpPs$ mode resulting from incoming P refracting as P at a discontinuity, reflecting downward as P off the free surface, then converting/reflecting upward as S at each discontinuity. The travel time for this arrival is

$$T_{PpPs} = T_s + T_p \quad (3)$$

and typically occurs around 15–20 s for the Moho.

[11] 4. The (simultaneously arriving for a flat structure) $PpSs$ and $PsPs$ arrivals resulting from incoming P , refracting/converting as P or S at a discontinuity, reflecting downward off the Earth's free surface as P or S , and finally reflecting upward as S at each discontinuity. The travel time for this mode is

$$T_{(PpSs+PsPs)} = 2T_s \quad (4)$$

and is typically observed several seconds after the $PpPs$ Moho arrival. Although $PpSs + PsPs$ energy propagates along two simultaneously arriving ray paths for flat-lying geometries, in laterally heterogeneous situations the two modes will not be simultaneous, resulting in destructive interference and/or produce a multiple arrival.

[12] Receiver function deconvolution may be performed either in the time [e.g., Sheehan et al., 1995; Zandt and Ammon, 1995; Poppeliers and Pavlis, 2003] or frequency domains [e.g., Ammon, 1991; Cassidy, 1992; Dueker and Sheehan, 1998; Park and Levin, 2000]. In either strategy, an inherent problem is regularizing of the commonly ill-posed deconvolution [e.g., Aster et al., 2004], where substantial artifacts may be introduced by high noise levels and/or

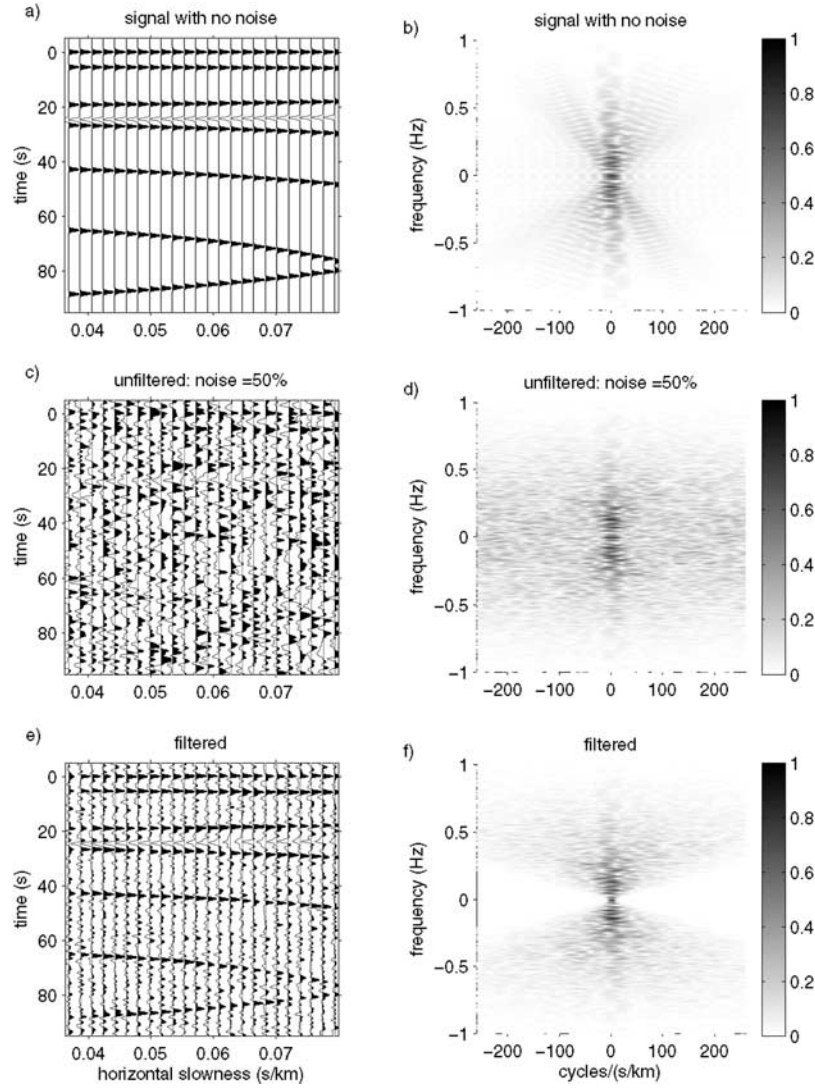


Figure 1. (a) Predicted receiver functions for interface depths of 45, 250, 410, and 660 km plotted as a function of horizontal slowness. Receiver functions are calculated using a reference Earth velocity model (TNA [Grand and Helmberger, 1984]), and amplitudes have been normalized for display purposes. (b) Receiver function amplitude transformed to frequency–pseudowave number ($f - \kappa$) space (5). (c) Receiver functions in the time domain with noise added (Gaussian noise with a standard deviation of 0.5 times the maximum receiver function amplitude); (d) Receiver functions from Figure 1c in the $f - \kappa$ domain with noise added. (e) Time domain, filtered. (f) The $f - \kappa$ domain, filtered.

spectral holes in the vertical component spectrum. A commonly applied regularization technique used to stabilize the deconvolution is water level deconvolution [Clayton and Wiggins, 1976; Ammon, 1991], which sets a lower bound on the magnitude of the denominator terms (the vertical seismogram spectral elements) in a frequency domain spectral division. To further stabilize this problem we implement a variation of frequency–wave number filtering that takes advantage of moveouts predicted by equations (1)–(4). We do this by first transforming receiver functions calculated at a single station from the time–slowness ($t - p$) domain to the frequency–pseudowave number ($f - \kappa$) domain using a two-dimensional Fourier transform

$$\mathcal{U}(f, \kappa) = \int \int u(t, p) e^{i2\pi\kappa p - i2\pi f t} dp dt \quad (5)$$

where the pseudowave number κ has units of cycles/slowness, or cycles/(s/km). This transform, and the filtering discussed below, is analogous to common frequency–wave number ($f - k$) techniques. The advantage of $f - \kappa$ transformation is that mapping to the $f - \kappa$ domain is determined by the moveout in the $t - p$ domain, which facilitates discrimination of moveout characteristics. For example, a receiver function arrival with a linear moveout dt/dp and frequency f is mapped to pseudowave number $\kappa = 2\pi f dt/dp$. Thus receiver function arrivals with positive moveouts as a function of p , such as predicted for P_s (equation (1)), will be mapped to positive κ , and receiver function arrivals with negative moveouts as a function of p , such as predicted for $PpPs$ and $PpSs + PsPs$ (equations (3) and (4)), will be mapped to negative κ . Figure 2 shows the predicted mapping to the $f - \kappa$ domain predicted for various arrivals predicted from equations (1)–(4).

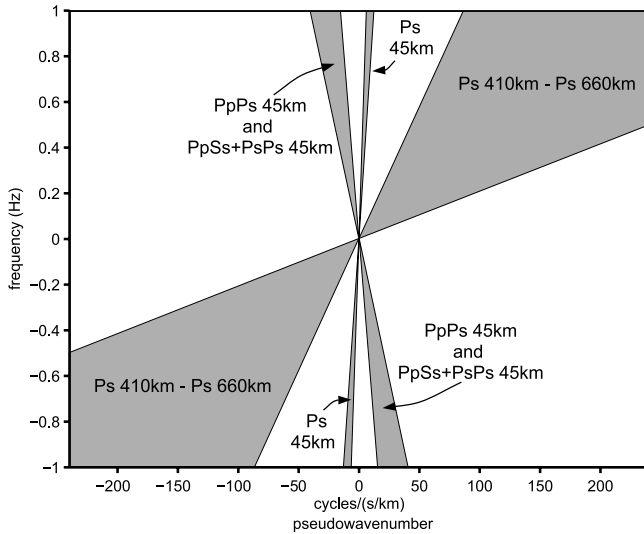


Figure 2. Predicted mapping to $f - \kappa$ space of receiver function arrivals.

[13] Receiver function amplitudes that have large variability as a function of p will map to high κ values in the $f - \kappa$ domain at a given frequency. This could occur due to either high-moveout arrivals (large dt/dp), or to receiver function features that do not appear on adjacent receiver functions (and may in fact be caused by high background noise amplified by the instability of the receiver function deconvolution). By calculating the maximum predicted dt/dp from (1)–(4) we can calculate the maximum physically reasonable range of κ and design an $f - \kappa$ filter to attenuate receiver function amplitudes with high variability as a function of horizontal slowness while preserving arrivals with consistent moveout characteristics.

[14] The $f - \kappa$ domain filtering of receiver function amplitudes is accomplished by multiplication by a Gaussian filter response

$$F(f, \kappa) = e^{-(2\pi\kappa)^2 / (4\beta^2 \alpha^2)} \quad (6)$$

where $\alpha = 2\pi f \max(dt/dp)$. In this study we use a scaling factor of $\beta = 3.75$, which results in approximate filter amplitudes of 0.1 when $\kappa = 1.88 \times 2\pi f \max(dt/dp)$, 0.5 when $\kappa = 2\pi f \max(dt/dp)$, and 0.9 when $\kappa = 0.375 \times 2\pi f \max(dt/dp)$. We use a broad Gaussian filter rather than a narrower realization targeted at specific arrivals to allow curvature in predicted moveout times due to dipping or laterally heterogeneous velocity geometries. The filter applies the strongest attenuation to receiver function amplitudes that do not vary smoothly as a function of p , such as would be produced by uncorrelated noise.

[15] Figure 1 illustrates the filtering process. Figure 1a shows receiver function amplitudes in $t - p$ domain as predicted by equations (1)–(4) using the TNA reference Earth velocity model [Grand and Helmberger, 1984], and Figure 1b shows the same receiver function amplitudes transformed to the $f - \kappa$ domain. For demonstration purposes, receiver function arrival amplitudes at all depths have been normalized. In Figures 1c and 1d we add white uncorrelated Gaussian noise with a standard deviation of

0.5 times the maximum receiver function amplitude. The added white noise makes it difficult to discern the lateral coherency of many of the receiver function arrivals (Figure 1c), and disrupts amplitudes throughout the $f - \kappa$ plane (Figure 1d).

[16] We design a filter using (6) by calculating the maximum predicted dt/dp from (1)–(4) using the TNA reference Earth velocity model. The maximum time moveout, $(dt/dp) = 520$ s/(s/km) for the predominant phases occurs for the Ps arrival from the 660 km upper mantle discontinuity. Resulting filtered receiver function arrivals are shown in Figures 1e and 1f in the $t - p$ and $f - \kappa$ domains, respectively. Improvement is clear in the $t - p$ domain, with a significant increase in the lateral coherency of receiver function arrivals. The correlation coefficient between Figure 1a and 1c is 0.39, and the correlation coefficient between Figure 1a and 1e is 0.76.

3. Receiver Function Imaging

3.1. Migration

[17] We next present a method of receiver function migration for both direct and reverberated P -to- S converted arrivals. We cast the problem as a Kirchhoff-style prestack migration [Yilmaz, 1987; Dellinger *et al.*, 2000] by characterizing the model space as a grid of point scatterers. For each station we ray trace recorded receiver function arrivals (direct converted, as well as reverberated) to all possible scattering points within the model space. Receiver function amplitudes are scaled and summed onto an output migration grid according to the Kirchhoff integral, which relates the wave field measured at the surface to the wave field at depth. The far-field term of the (single-component) Kirchhoff integral can be expressed as [Yilmaz, 1987]

$$u_{out}(x, z) = \frac{1}{2\pi} \int S_r \frac{\cos \theta}{vr} \frac{\partial}{\partial t} u_{in}(x_{in}, z = 0, t) dx \quad (7)$$

where v is the RMS velocity at the imaging point (x, z) , r is the distance between the input $(x_{in}, z = 0)$ and the output (x, z) imaging points, and the time t is the sum of the source-to-imaging point and imaging point-to-receiver travel times. The obliquity factor $\cos \theta$ arises from recording geometry (Figure 3). For vertical component P wave migration, θ is the incidence angle of the ray being imaged, since P waves will have maximum amplitude on vertical components for near vertical arrivals. Similarly, for radial receiver function migration, θ is also the incidence angle of the ray being

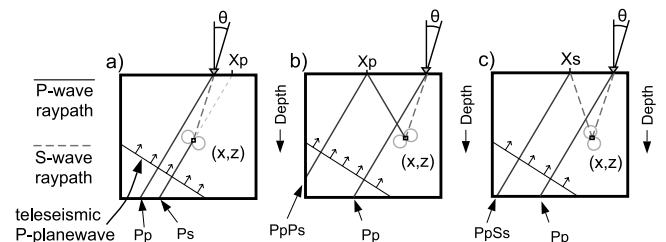


Figure 3. Ray paths for the (a) Ps , (b) $PpPs$, and (c) $PpSs$ arrivals used for calculating migration travel times and scaling. Point diffractor scattering radiation patterns [e.g., Wu and Aki, 1985] are indicated at the point scatterers.

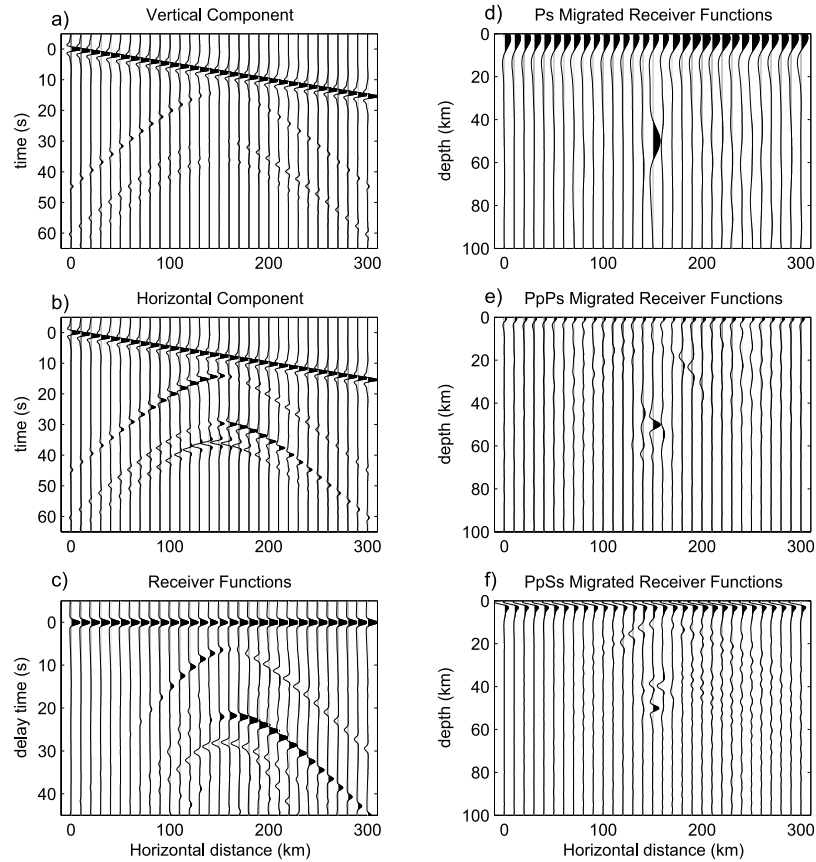


Figure 4. Application of receiver function migration to simulated diffractions from a single point scatterer. (a), (b), and (c) Vertical component, horizontal radial component, and receiver functions, respectively, for an evenly spaced (10 km) recording network. The point diffractor is located at 50 km depth and at 150 km on the horizontal axis. (d), (e), and (f) *Ps*, *PpPs*, and *PpSs* migrated receiver functions, respectively.

imaged, since a shear wave arrival will have maximum amplitude on horizontal components for near vertical arrivals. The scaling term S_{tr} corrects for *S* wave point scattering patterns [Wu and Aki, 1985; Levander, 2003] (Figure 3) for forward *P*-to-*S* scattering or back scattered *P*-to-*S* or *S*-to-*S* phases, depending on which phase is being migrated. For the *Ps* and *PpPs* phases (Figures 3a and 3b), S_{tr} varies as a function of the sine of the angle between the direction of scattering and the *P* wave propagation direction. For *PpSs* phase (Figure 3c), S_{tr} varies as a function of the cosine of the angle between the direction of scattering and the *S* wave propagation direction. The factor $(vr)^{-1}$ reflects spherical spreading and the time derivative provides the phase shift required by Huygens' principle [Yilmaz, 1987]. In two dimensions the half-order derivative (multiplication by $(i\omega)^{1/2}$ in the frequency domain) is used.

[18] Figure 4 illustrates receiver function migration applied to a lithosphere-scale model featuring a point scatterer at 50 km depth. Figures 4a and 4b show synthetic vertical and horizontal-radial seismograms, respectively, containing characteristic teleseismic spectral energy between 15 s and 2 Hz. Figure 4c shows corresponding receiver functions calculated from Figures 4a and 4b. Note that the phase associated with the incoming *P* wave field is now aligned at time zero by the receiver function deconvolu-

tion. Diffractions resulting from the point scatterer can be seen for the *Ps*, *PpPs*, and *PpSs* phases between 5 and 45 s. Figures 4d, 4e, and 4f shows migrations using *Ps*, *PpPs*, and *PpSs* ray path geometries, respectively, using (7).

[19] Seismic data migration can be analyzed as an inverse problem by considering a forward operator $\mathbf{G}$, that maps the velocity discontinuity structure of the subsurface, $\mathbf{m}$, onto data, $\mathbf{d}$. For linear situations, this gives the relationship $\mathbf{d} = \mathbf{G}\mathbf{m}$. In the context of this paper, an appropriate $\mathbf{G}$ may be constructed using the travel time and scaling relationships specified by (7). For Kirchhoff migration, the inverse operator that maps data back to the model estimate is $\mathbf{G}^T$ [Nemeth et al., 1999]. Regularized migration [e.g., Duquet et al., 2000; Nemeth et al., 1999] applies a generalized and regularized inverse operator, $\mathbf{G}^{-g}$ to produce a solution $\mathbf{m} = \mathbf{G}^{-g}\mathbf{d}$, and has been shown to produce subsurface images with higher resolution and fewer migration artifacts and acquisition geometry artifacts than standard migration algorithms [Duquet et al., 2000; Nemeth et al., 1999]. However, it is not routinely used in the exploration industry (where large surveys often have over 10^6 data traces) due to the computing power needed to compute and implement $\mathbf{G}^{-g}$. However, passive teleseismic experiments to date such as RISTRA [Wilson et al., 2002, 2003, 2005]) have on the order of 10^3 to 10^4 traces, making such inversion method-

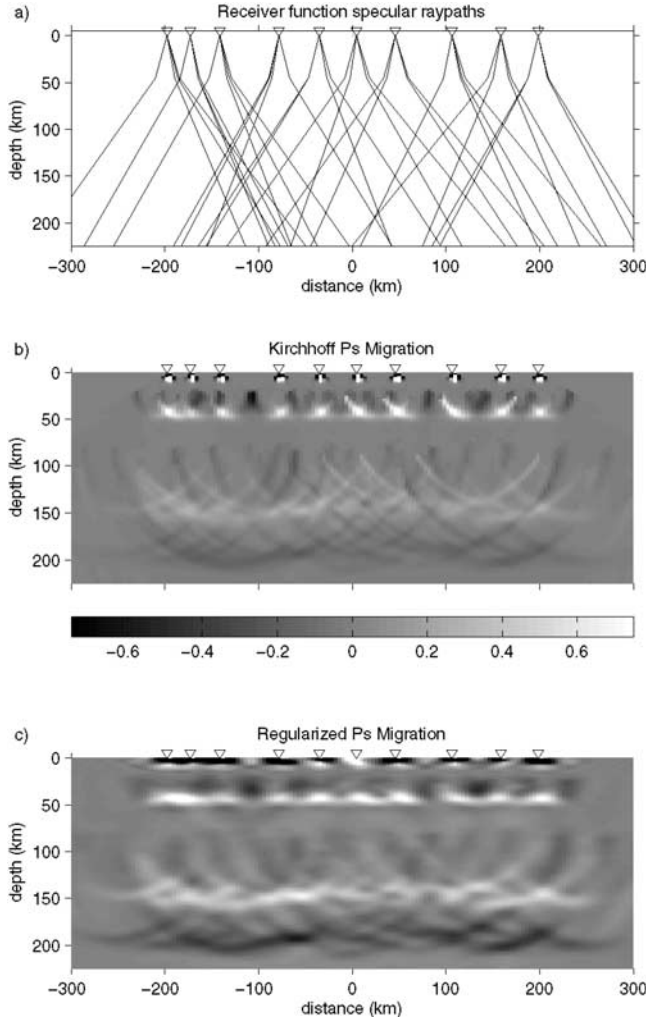


Figure 5. (a) Ray paths for the synthetic receiver function data set used for migration demonstration. (b) Kirchhoff-migrated cross sections for the P_s mode and (c) the corresponding regularized migrated cross sections. Gray scale indicates scattering potential.

ologies far more tractable. Setting up migration as an inverse problem allows for the use of diverse regularization techniques. One method is to minimize the Tikhonov objective function [Aster *et al.*, 2004; Nemeth *et al.*, 1999]

$$p(\mathbf{m}) = \|\mathbf{G}\mathbf{m} - \mathbf{d}\|^2 + \epsilon \|\mathbf{C}\mathbf{m}\|^2 \quad (8)$$

The first term in (8) is the 2-norm data misfit and the second term is a regularization term scaled by a regularization parameter, ϵ . $\mathbf{C}$ is a conditioning matrix used to penalize undesirable model properties. We implement a second-order Tikhonov regularization so that left multiplication of the model by $\mathbf{C}$ approximates the second-order spatial derivative. To determine the appropriate ϵ , we compute the solution to (8) for a range of values of ϵ , and then construct a tradeoff curve between model misfit ($\|\mathbf{G}\mathbf{m} - \mathbf{d}\|^2$) and roughness ($\|\mathbf{C}\mathbf{m}\|^2$). We employ a conjugate gradient iterative scheme [e.g., Aster *et al.*, 2004; Rowe *et al.*, 2002] to solve the system for the large, sparse $\mathbf{G}$ matrix arising in this application. Travel times relative to P for each

arrival of interest (Figure 3), are calculated for each seismic station.

[20] The travel time for P_s is

$$t_{P_s} = t_{\text{mod-sta}} - T_p + (t_{\text{src-}X_p} - t_{\text{src-sta}}) \quad (9)$$

where $t_{\text{mod-sta}}$ is the travel time from the model point to the station, T_p is given by (2), $t_{\text{src-}X_p}$ is the travel time from the epicenter to the point X_p (Figure 3), and $t_{\text{src-sta}}$ is the travel time from the epicenter to the recording station. If we assume a flat-layered velocity geometry, we can calculate the quantity $(t_{\text{src-}X_p} - t_{\text{src-sta}})$ directly from the horizontal slowness without actually calculating the travel time from the epicenter. For real data, we need to account for the curvature and velocity structure of the Earth. In this case, we can calculate the quantity $(t_{\text{src-}X_p} - t_{\text{src-sta}})$ based on travel times predicted for a reference Earth model as a function of epicentral distance. Because rays traveling from the epicenter to X_p , and from the epicenter to the recording station, have taken nearly the same path, the quantity $(t_{\text{src-}X_p} - t_{\text{src-sta}})$ is primarily a function of the distance between X_p and the station, and of the horizontal slowness at X_p and the station. Synthetic testing demonstrates that the fine details of the reference Earth velocity model do not have a large effect. Similarly, for the $PpPs$ migration, the travel time is

$$t_{PpPs} = t_{\text{mod-sta}} + T_p + (t_{\text{src-}X_p} - t_{\text{src-sta}}) \quad (10)$$

and for the $PpSs + PsPs$ migration it is

$$t_{PpSs+PsPs} = t_{\text{mod-sta}} + T_s + (t_{\text{src-}X_s} - t_{\text{src-sta}}) \quad (11)$$

where $t_{\text{src-}X_s}$ is the travel time from the epicenter to point X_s (Figure 3), and T_s is given by (2).

[21] Figures 5b, 5c, and 6 show migrated cross sections for noise-free synthetic receiver function data containing 10 stations with a spacing of 40 km and a standard deviation of 20 km. Each station is associated with four receiver functions with random back azimuths that are in line with the imaging plane (i.e., $\pm 180^\circ$), and random epicentral distances between 35° and 95° (corresponding to a range in values of horizontal slowness, p , between 0.039 and 0.078 s/km). Synthetic receiver functions were calculated for a 45 km thick crust over a half-space. Ray paths for the synthetic receiver function data set are shown in Figure 5a. For this synthetic migration example, we have chosen this simple velocity model in order to emphasize the benefits of applying regularized migration to a data set with irregular station spacing and sparse data coverage. Figures 5b, 6a, and 6c show Kirchhoff migrated cross sections ($\mathbf{m} = \mathbf{G}^T \mathbf{d}$) of the P_s , $PpPs$, and $PpSs + PsPs$ modes, and the corresponding regularized migration cross sections are shown in Figures 5c, 6b, and 6d. For each mode the regularized migration shows improved lateral coherency of imaged receiver function amplitudes. In areas where there is a large gap between adjacent stations, the regularized migration implements a consistent interpolation to produce a more spatially continuous image while fitting the data. The regularized migration also shows a reduction in migration artifacts (“smiles”) caused by the sparse data

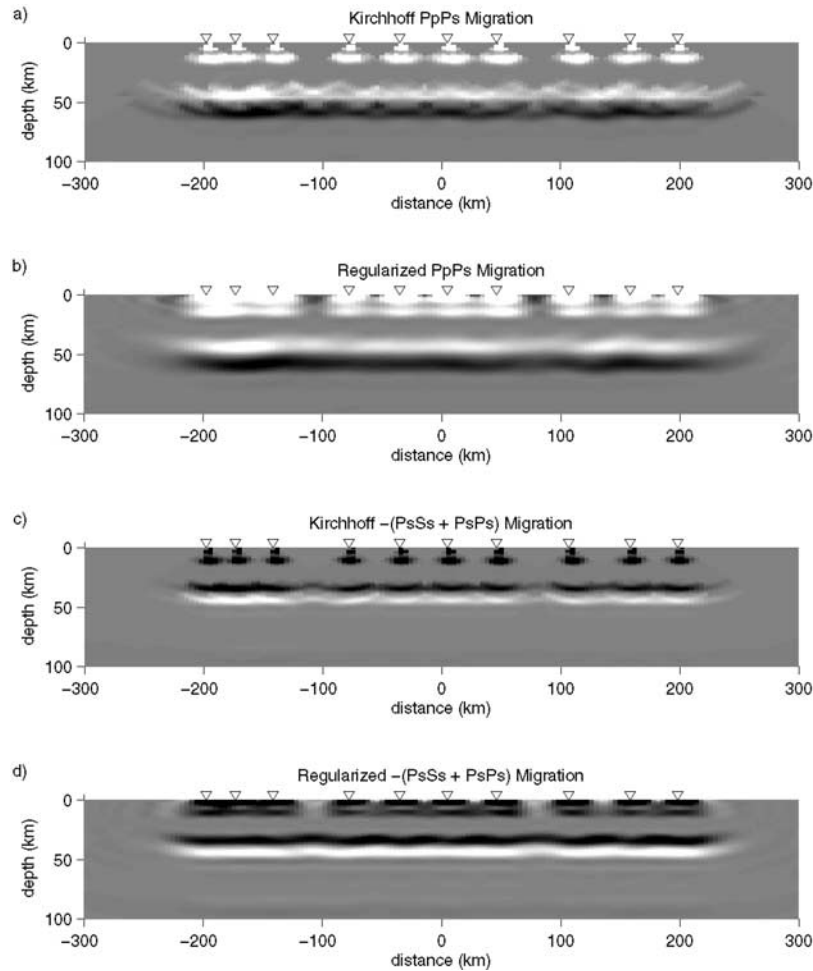


Figure 6. Kirchhoff-migrated cross sections for the (a) $PpPs$ and (c) $PpSs + PsPs$ modes and (b) and (d) the corresponding regularized migrated cross sections. Gray scale is the same as that given in Figure 5.

coverage, which are most evident in the Ps migration (Figure 5b). It is important to note that the regularized migration does not simply smooth the data, but rather determines a “best” tradeoff between fitting the data and smoothing the model. The objective function curves for the Ps , $PpPs$, and $PpSs + PsPs$ mode regularized migrations are shown in Figure 7. The optimal value of ϵ for each curve was selected by finding the corner, or “knee”, of the trade-off curve using the MATLAB function `l-corner` [Hansen, 1994].

[22] The depth to which each of the receiver function modes are migrated depends on the velocity model. However, because each mode has differing sensitivity to velocity model errors we can use any such systematic inconsistencies to assess the velocity model accuracy. Figure 8 shows the Moho imaging depth error for each of the receiver function modes as a function of errors in the velocity model, calculated from (1)–(4) for the simple synthetic model with a 45 km thick crust. We find that the inferred Moho depth in the migrated images is least

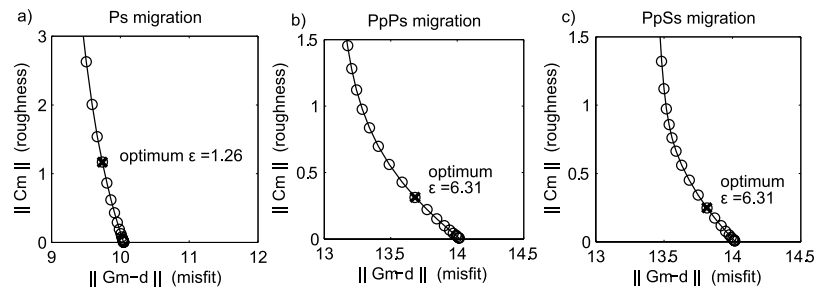


Figure 7. Objective function curve for the (a) Ps , (b) $PpPs$, and (c) $PpSs + PsPs$ synthetic migrations. Each data point on the curve represents a solution to (8) for a different value of ϵ . The optimal value of ϵ is selected by finding the corner, or “knee”, of the curve [Hansen, 1994].

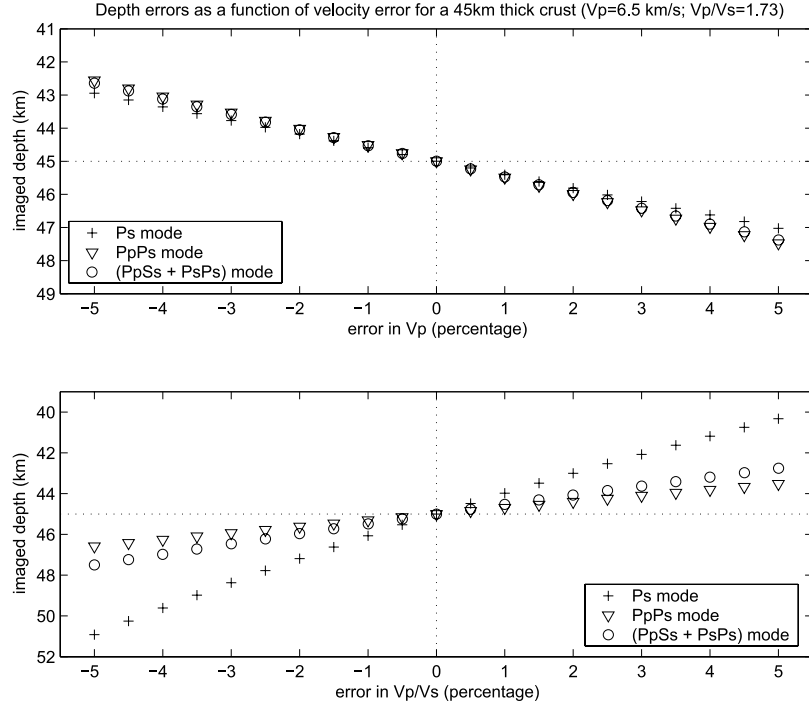


Figure 8. Imaging Moho depth error as a function of velocity errors.

sensitive to the starting P wave velocity model. In this case, a 5% over/underestimation of V_P shifts each modes images of the Moho approximately uniformly 2–2.5 km deeper/shallower. Errors in the V_P/V_S ratio can have more significant effects, and do not affect all modes in a uniform fashion. For this simple model, a 5% overestimate of the V_P/V_S ratio results in underestimating the Moho depth from the Ps , $PpPs$, and $PpSs + PsPs$ modes by 4.5, 1.5, and 2.3 km, respectively. A 5% underestimate of V_P/V_S results in overestimating the Moho depth from the Ps , $PpPs$, and $PpSs + PsPs$ modes by 6.0, 1.5, and 2.5 km, respectively.

3.2. Stacking Multiple Migrated Images

[23] Sharp velocity contrasts at the Moho and deep sedimentary basins introduce large amplitude multiples into the images that produce imaging artifacts and can overprint deeper structure when migrated. To reduce imaging artifacts the three migrated output grids are stacked so that large amplitudes in the final image are produced only when the Ps , $PpPs$, and the negative (due to reflection polarity reversal) of the $PpSs + PsPs$ migration images have identical polarity. To emphasize common polarity and amplitude images, we thus employ a multi-image nonlinear stacking algorithm [e.g., Kanasewich *et al.*, 1973] of the form

$$s = ab^n \quad (12)$$

where

$$a = \frac{3|\text{median}(m_{Ps} + m_{PpPs} - m_{PpSs+PsPs})|}{(|m_{Ps}| + |m_{PpPs}| + |m_{PpSs+PsPs}|)} \quad (13)$$

$$b = \frac{|m_{Ps} + m_{PpPs} - m_{PpSs+PsPs}|}{(|m_{Ps}| + |m_{PpPs}| + |m_{PpSs+PsPs}|)}, \quad (14)$$

and m_{Ps} , m_{PpPs} , and $m_{PpSs+PsPs}$ are the results of the Ps , $PpPs$, and $PpSs + PsPs$ migrations, respectively. The a factor is a measure of the relative amplitude of the three images, and will be maximized when the three images have the same polarity and amplitude. The b^n factor is exclusively a nonlinear measure of polarity consistence. In regions where all three images have consistent polarity, b^n will have a (maximum) value of 1. The parameter n controls the degree of nonlinear scaling applied to the summed images. As $n \rightarrow \infty$, the summed images are multiplied by 1 where they all have the same polarity, and multiplied by 0 where they do not. The choice of n will depend on the characteristics of a particular data set. In this case, we chose a value of $n = 8$, so that when b has the value of 0.75, b^n will equal 0.1.

[24] Figure 9 provides a simple illustration of the multi-mode stacking process using a single receiver function recorded at a single station, where the velocity model consists of a 45 km thick crust and a horizontal slowness of 0.05 s/km. The first three panels show the depth-mapped receiver functions for Ps , $PpPs$, and $PpSs + PsPs$, respectively, and the fourth panel shows the nonlinear stack. Each of the three single-mode depth-converted traces shows a positive amplitude at the Moho depth of 45 km which produces a positive stack trace. Imaging artifacts present in the single-mode images are severely downweighted by the stacking algorithm. Figure 10 shows the multimode, stacked images computed from the Kirchhoff-migrated and regularized-migrated single-mode images of Figures 5 and 6. Energy from receiver function multiples (e.g., from $PpPs$ and $PpSs + PsPs$) which migrated to incorrect depths (e.g., 125–200 km) in the Ps migration (Figures 5b and 5c) as well as energy which was incorrectly migrated by mode misattribution in the $PpPs$ and $PpSs + PsPs$ migrations (Figure 6) has been essentially removed by the stacking

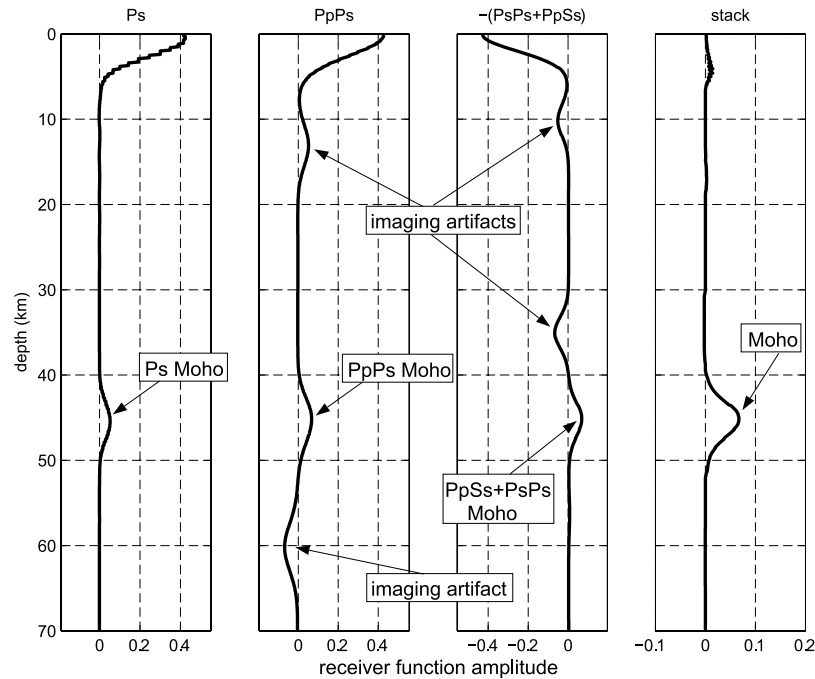


Figure 9. Example stack of the Ps , $PpPs$, and $PpSs + PsPs$ phases for a single synthetic receiver function calculated for a 45 km thick crust and horizontal slowness of 0.05 s/km.

process. The benefits of the regularized technique, versus standard Kirchhoff migration, are most evident in the stacked images (Figure 10), where the regularized migration shows much better lateral coherency of receiver function amplitudes.

[25] This stacking algorithm is a first-order attempt to distinguish correctly imaged data from misimaged data. The algorithm may not work well in all cases (i.e., complex structural geometries), so the stacked image should be interpreted in concert with the individual mode migrations.

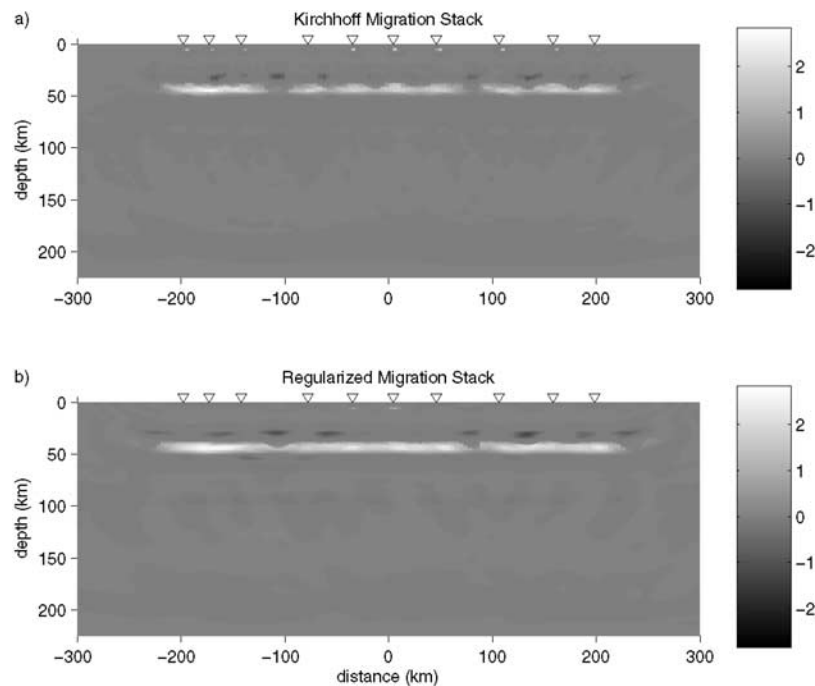


Figure 10. (a) Stack of the Kirchhoff-migrated cross sections in Figures 5b, 6a, and 6c. (b) Stack of the regularized migrated cross sections in Figures 5c, 6b, and 6d. Gray scale indicates relative scattering potential. Note that the near-zero amplitudes and relative uniformity of the image at depths of 75–200 km indicate that the imaging artifacts at these depths (Figures 5b and 5c) have been properly cancelled.

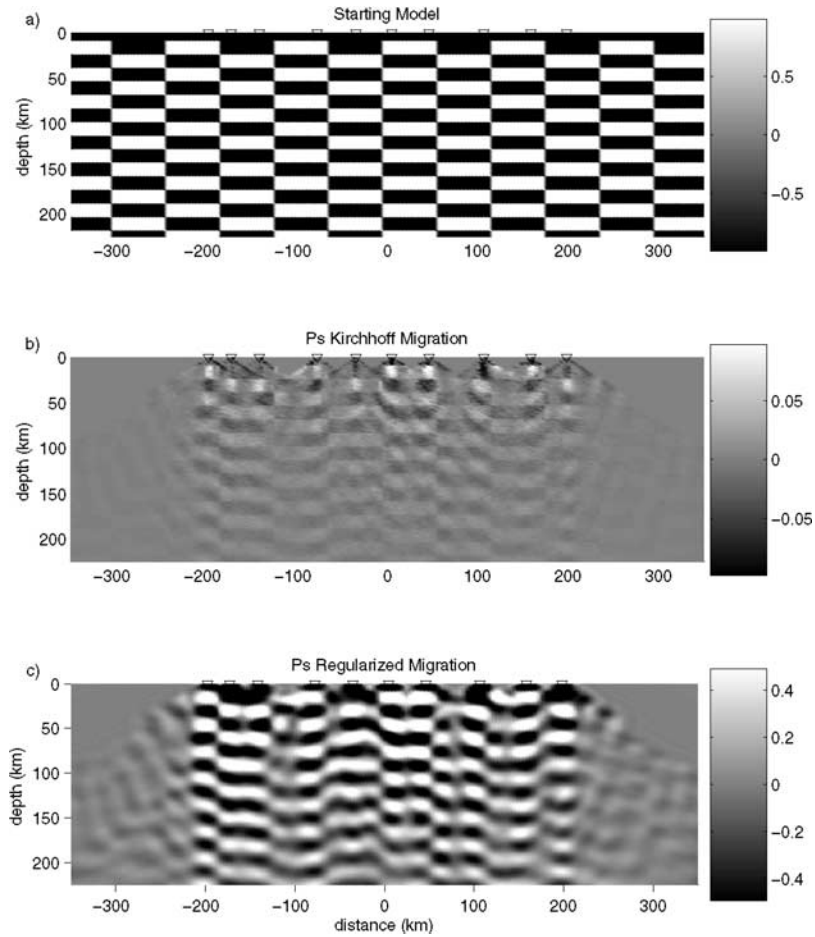


Figure 11. P_s migration resolution test consisting of (a) the scattering potential Earth model used for forward modeling, (b) the corresponding Kirchhoff-migrated cross section, and (c) the regularized migrated cross section. Gray scale indicates scattering potential.

One such case is the presence of higher-order multiples, sometimes referred to as “signal-generated noise” [e.g., *Park and Levin, 2000*], from near strong near-surface low-velocity zones such as sedimentary basins. If an accurate velocity model is used, the first-order multiples will stack destructively, but the higher-order multiples will not, resulting in imaging artifacts. However, higher-order multiples will have considerably lower amplitude than the primary phases and first-order multiples, so that associated imaging artifacts will typically be low in amplitude.

3.3. Migration Resolution

[26] For imaging at crustal levels (e.g., up to approximately 60 km depth) the direct converted phase (P_s) for the basins and Moho, as well as basin multiples, typically have maximum energy at approximately 0.5–2 Hz, while the crustal multiples have most of their energy at 0.15–0.5 Hz. At Moho depths, this translates to about 2 to 6 km vertical resolution ($1/2 P$ wavelength [Bostock, 1999]) for the P_s phase, and 4 km vertical resolution for the crustal multiples ($1/4$ wavelength). Fresnel zone widths at Moho depths are approximately 8 km at 1 Hz, and 18 km at 4 s periods. However, if we assume that the migration has properly collapsed diffracted energy, the horizontal resolution after migration is improved to approximately 1 wavelength [Stolt

and Benson, 1986], or to about 4 km horizontal resolution at 1 Hz and 15 km horizontal resolution at 4 s periods.

[27] The resolution of stacked migrated images will depend on station spacing, data coverage, and inverse methodology (e.g., the choice of the regularization parameter ϵ in [8]). We assess the resolution of the migration process in a standard “checkerboard test” manner by considering a synthetic scattering potential Earth model, $\mathbf{m}_{\text{true}}$, and a corresponding data set, $\mathbf{d} = \mathbf{G}\mathbf{m}_{\text{true}}$. Applying a linear generalized inverse operator $\mathbf{G}^{-g}$ results in an estimated model

$$\mathbf{m} = \mathbf{G}^{-g}\mathbf{d} \quad (15)$$

which can be compared to $\mathbf{m}_{\text{true}}$ to assess the recovery of model and, conversely, the details and the presence of smoothing or other artifacts resulting from regularization. Further image degradation may also come about due to noise, and these influences can be assessed by the addition of noise to the synthetic data. This analysis is analogous to resolution tests commonly performed in tomographic inversions, and assesses bias introduced by regularization and/or rank deficiency into the linear inverse problem for a general weakly scattering Earth structure (which is a solely function of station spacing and data coverage [Aster *et al.*,

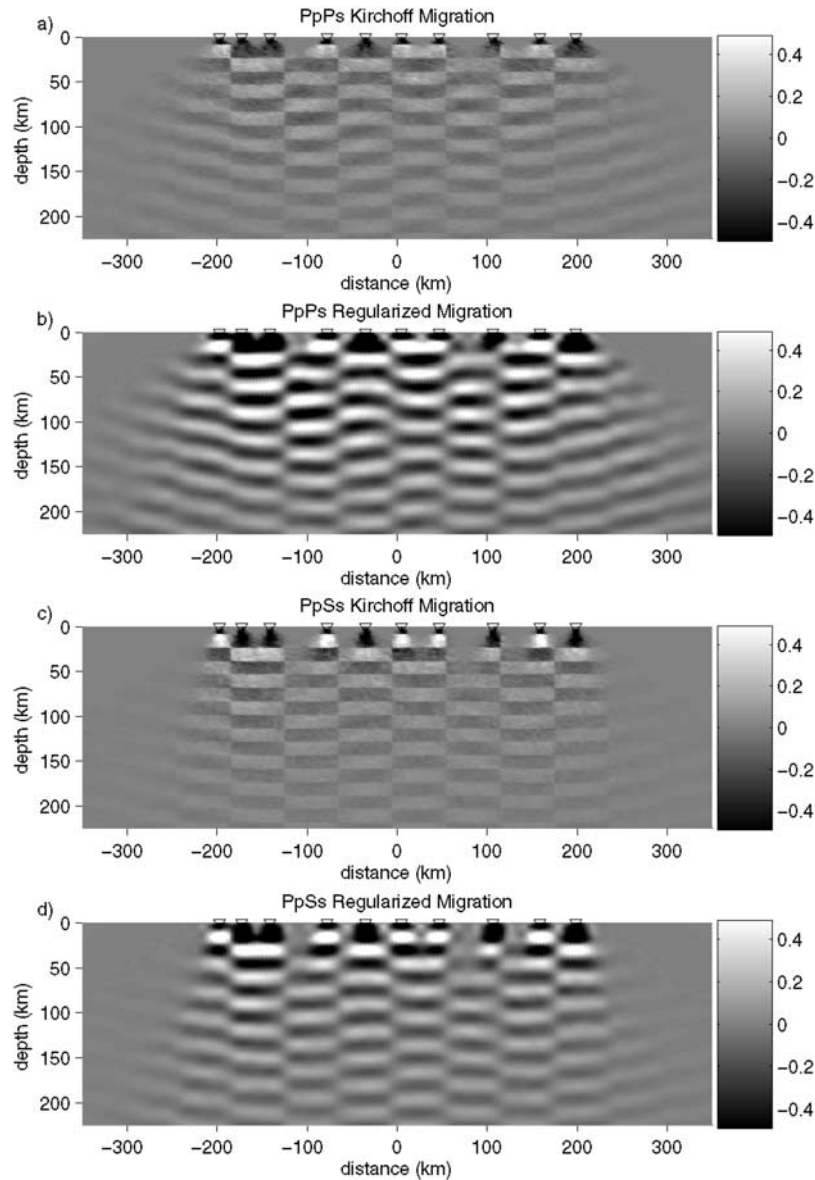


Figure 12. Migration resolution tests for the $PpPs$ and $PpSs+PsPs$ phases consisting of (a), (c) the Kirchhoff-migrated cross sections, and (b), (d) the corresponding regularized migrated cross sections. Gray scale indicates scattering potential.

2004]). However, note that this resolution analysis methodology is not an assessment of the ability of the migration to properly image the subsurface using the complex, multiply scattered wave field that would be produced by an actual “checkerboard” earth.

[28] Figures 11 and 12 show forward modeled resolution for the station spacing and data coverage used in the synthetic data migration of Figures 5 and 6. The scattering potential Earth model used for forward modeling is shown in Figure 11a and corresponding Kirchhoff-migrated and regularized migrated cross sections are shown in Figures 11b and 11c, respectively, for the Ps migration and in Figures 12a and 12c and Figures 12b and 12d, respectively, for the migration of reverberated phases. The Kirchhoff-migrated images (Figures 11b, 12a, and 12c) exhibit considerable amplitude variation, with the highest amplitudes occurring at shallow levels in regions with close station

spacing. Regularized migration (Figures 11c, 12b, and 12d) produces an image with less variation in amplitude, more closely resembling the starting model (note the differing color scales between the Kirchhoff and regularized migrations), and does not show strong amplitude variations attributable to the irregular station spacing. The effect of regularization is also evident, giving the regularized migrations a smoother appearance than the relatively grainy Kirchhoff migrations. Both the Kirchhoff and regularized migrated images show that migration resolution degrades rapidly off the ends of the network but the recovery near the edges is improved by the regularized algorithm.

4. Conclusions

[29] Instability in receiver functions produced either by high background noise or by holes in the vertical compo-

nent spectra, can create large receiver function artifacts and highly inconsistent receiver functions corresponding to nearby ray paths. By filtering receiver function gathers in the frequency–pseudowave number domain we can successfully remove spurious apparent high-moveout receiver function features and other artifacts while preserving true receiver function phases. Tests of this methodology using synthetic receiver functions further demonstrate that we can recover receiver function phases in the presence of high noise levels (e.g., 50%). Regularized migration, combined with multimode stacking, substantially reduces receiver function artifacts arising from reverberations and mode misattribution effects, as well as due to uneven data coverage arising from nonuniform station and source distributions.

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