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Performance Characteristics of a Rotational Seismometer for Near-Field and Engineering Applications

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Abstract Based on the concept that torsion balances act as good filters for rotational oscillations at frequencies above their natural frequency of oscillations, we have fabricated a simple prototype to serve as a rotational seismometer for near-field and engineering studies. This instrument displays a nearly flat response at frequencies above 10 mHz, and the preliminary data taken with this instrument show a sensitivity of $5 \times 10^{-6}(\nu/0.01\ \text{Hz})$ rad sec^{-1} Hz$^{-1/2}$. Marginal improvements of the fabrication and housing will be needed to fully cover the sensitivities recommended by a U.S. Geological Survey (USGS) panel for studies at these frequencies.

Introduction

The motivations for the development and characterization of instruments capable of recording the rotations associated with seismic activity and normal mode oscillations were detailed extensively at the 2006 Fall meeting of the American Geophysical Union in a sequence of sessions (Igel *et al.*, 2006) and in a special workshop at U.S. Geological Survey (USGS)-Menlo Park (Lee *et al.*, 2007). Where as the translational motions induced by seismic activity along the three axes have been observed extensively, yielding essentially all the information we currently know about earthquakes and the structure of the Earth, the rotations about these axes have proven to be very difficult to observe directly, mainly because of the lack of suitable instruments. During the past few years the pioneering efforts of Igel *et al.* (2005, 2007) and Cochard *et al.* (2006) observing the Sagnac effect caused by rotations on ring lasers have started yielding important new results. Similarly, Wiens *et al.* (2005) have been able to carry out episodic measurements of tilts in the strong fields in the proximity of volcanoes. In this article we outline an alternate scheme for the measurement of the rotations, about the local zenith axis, and show that it is possible to extend this design for the measurement of tilts as well. This new scheme is based on the study of responses of torsion balances of low-natural frequencies and shows great promise with high sensitivity and moderate cost (Cowsik, 2007). Such an instrument could provide complements or viable alternatives to ring lasers for studies in rotational seismology. It is possible to achieve very high sensitivities, and in this article we present the design and performance of a prototype developed for near-field seismic rotations and engineering applications. We review the design concept and report preliminary results obtained with a very simple realization of the basic design, quickly put together as a proof of concept. Substantial improvements are possible with better fabrication and standard engineering practices.[1] Such an instrument would be suitable for studies of near-field seismic rotations, which have larger amplitudes (Igel *et al.*, 2005, 2007) and have spectra peaked at higher frequencies (Kanamori, 1994; Evans *et al.*, 2007). This article is devoted primarily to the description of the design and performance of this prototype instrument.

Torsion Balance as the Instrument of Choice

Keeping in mind that torsion balances may appear to be exotic instruments for seismology, we begin with a brief historical overview of their design and use. Perhaps the first applications of torsion balances were by Pierre Augustin de Coulomb and Henry Cavendish, who used the extraordinary sensitivity of a torsion balance to measure electrostatic and gravitational forces. It is a remarkable instrument where the gravitational force of the whole Earth is exactly compensated by the tension in the suspension fiber, and the small forces in the horizontal plane are configured to induce torques causing an angular deflection of the balance. This angular deflection is measured with high resolution by an optical lever, which exerts negligible torques on the balance. In the beginning of the twentieth century, Eötvos made a highly innovative use of the balance to measure gradients in Earth's gravity and to study Einstein's equivalence principle. During the recent decades scientists led by Dicke (Roll *et al.*, 1964), Braginsky (Braginsky and Panov, 1972), Cowsik (Cowsik, 1981; Cowsik *et al.*, 1988, 1989), Ritter (Ritter *et al.*, 1990), Adelberger (Cowsik *et al.*, 1988, 1990; Adelberger *et al.*, 2003), Boyn-

[1] The instrumentation effort described here emerged from an ongoing program to study short-range gravity and Einstein's equivalence principle at Washington University in St. Louis, Missouri.

ton (Boynton *et al.*, 1987), and Newman (Newman *et al.*, 1990) have adapted the torsion balance for their investigations of the equivalence principle and to search for new extremely feeble forces that couple to baryon or lepton number, to nuclear isospin or spin, etc. Indeed, torsion balances are instruments of choice for the study of feeble forces of mesoscopic range. A new innovation of the torsion balance was introduced by Cowsik (2007). "The torsion balance with a sufficiently low natural frequency of torsional oscillations acts as a filter, providing a target that remains stationary even though the surface of the Earth might be undergoing torsional oscillations at frequencies significantly above the natural frequency of the balance. A sensitive optical lever, fixed to the Earth, observing such a balance can therefore faithfully measure the rotational oscillations of the Earth (Jones and Richard, 1959; Cowsik *et al.*, 2007)."

Nature of Rotational Motions and the Design Target for Their Observations

Torsional oscillations of the Earth, sometimes called toroidal oscillations, were predicted in the nineteenth century and have been calculated with increasing precision during the recent decades. The lowest of these modes involve the oscillations of the entire mantle and have periods of about 3000 sec and the strong motions in the near-field zones of earthquakes have much shorter periods ~100 sec or less. Careful measurement of these oscillations is of considerable importance in understanding the Earth's structure and dynamics and the mechanism of earthquakes.

A theoretical description of the rotational motions with respect to the local zenith occurring within the Earth may be found in several places. The rotations around the $\hat{z}$ axis in which we are interested are given (Stein and Wysession, 2003) by the curl of the displacement field; $u^T(r,\theta,\phi) = \sum_n \sum_l \sum_m {}_nA_l^m W_l(r) T_l^m(\theta,\phi) e^{i({}_n\omega_l^m)t}$, where ${}_nA_l^m$ are the appropriate amplitudes in the spherical harmonic expansion and $W(r)$ gives the radial dependence of the amplitudes:

$$\begin{aligned} R(\omega, r, \theta, \phi) &= (\nabla \times u^T) \cdot \hat{r} \\ &= \sum_n \sum_l \sum_m {}_nA_l^m W_l(r) e^{i({}_n\omega_l^m)t} \frac{1}{r \sin\theta} \\ &\quad \times \left[-\frac{1}{\sin\theta} \frac{\partial^2 Y_l^m}{\partial \phi^2} + \cos\theta \frac{\partial Y_l^m}{\partial \theta} + \sin\theta \frac{\partial^2 Y_l^m}{\partial \theta^2} \right] \\ &\equiv \sum_n \sum_l \sum_m R(\omega)(\cos\omega t + i \sin\omega t). \qquad (1) \end{aligned}$$

In the final part of this equation we have written for convenience ${}_n\omega_l^m = \omega$. In later discussions we will focus attention on the real part of $R(\omega, r, \theta, \phi)$, or simply $R(\omega)$, at the location of the instrument. The typical frequencies, $\nu = \omega/2\pi$, range from $\sim 3.7 \times 10^{-4}$ Hz for the gravest normal modes (see Fig. 1) to several tens of millihertz for the near-field strong motions (Igel *et al.*, 2005; Lee *et al.*, 2007). The near-field amplitudes are expected to be considerably larger than those of the normal modes, and based on the observations of

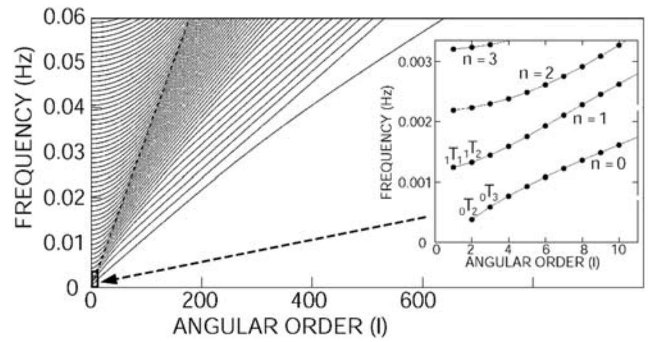

Figure 1. Frequency of torsional normal modes as a function of angular order computed from PREM model of Dziewonski and Anderson and/or estimated from the study of translational motions. The inset depicts the region of low-frequency and low-angular order (Dziewonski and Anderson, 1981).

Igel *et al.* (2005, 2007), they appear to be strong at frequencies around ~0.01–0.1 Hz. The importance of the near-field studies for the understanding of the mechanics of earthquakes is extensively reviewed by Kanamori (1994). The targeted sensitivity requirements for the sensors were discussed in a panel chaired by J. R. Evans during the First International Workshop on Rotational Seismology (Evans *et al.*, 2007); the summary recommendations are reproduced here in Figure 2.

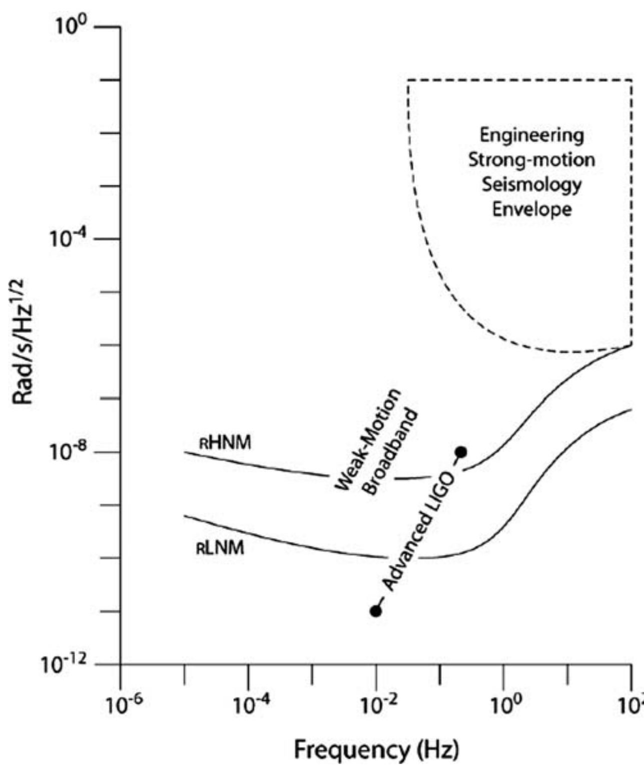

Figure 2. Principal design goal: in units of rms, $\mathrm{rad/sec}/\sqrt{\mathrm{Hz}}$, characterize both available and desirable rotational-sensor frequency-sensitive ranges for particular applications. (LIGO stands for Laser Interferometer Gravitational-Wave Observatory, rHMN stands for rotational high noise model, and rLNM stands for rotational low noise model.)

Brief Description of the Importance of Studies of Rotational Motion

Seismological studies have been mostly responsible for our understanding of the nature and dynamics of the interior of the Earth, with additional information coming from other channels such as gravity-based geoid measurements, palaeomagnetism, very long baseline interferometry, and Global Positioning System. At present the seismological studies are essentially limited to monitoring global and local wave fields at various frequencies, measuring only the three components of the translational velocity of ground displacement, and in some cases directly measuring the strains. The Fourier transform of the horizontal translations yields the frequency spectrum of these oscillations, and their correspondence with theoretical calculations carried out with reference Earth models like that of Dziewonski and Anderson (1981) allow us to identify the frequencies and Q-values associated with the rotational oscillations of the Earth's mantle. The rotations associated with the normal modes of the Earth have not been directly observed with present-day instruments. In a unique effort Igel *et al.* (2005, 2007) have been able to detect the rotational motions induced by the M 8.1 Tokachi-Oki earthquake at frequencies on the order of 30–60 mHz using ring laser gyroscopes.

Theoretical studies, for example, by Cochard *et al.* (2006), indicate that the observations of seismic rotational motions will provide important new information pertaining to the Earth sciences, complementary to those obtained from the observations of the translational motions of the Earth's surface using conventional seismometers. They note that combining the observation of rotational motion with the concomitant translational motion yields wave-field properties such as phase velocities and the direction of propagation. Even though it is possible to derive some of this information from the studies with arrays of seismographs (for which they provide the requisite analysis procedures), the advantages of direct observations of the rotational motions will be of great value for overcoming the limitations in the use of arrays for deciphering rotational motions.

Castellini and Zembaty (1996) have pointed out that generally not only the amplitude of the rotational motions but also the damages caused by them are underestimated. Accordingly, Cochard *et al.* (2006) have emphasized that recording even small rotational motions with low potential for causing damage would be useful, especially because of possible cross couplings between rotational motions and tilt motions (M. Todorovska and M. Trifunac, personal comm., 2006). The amplitudes of these rotational motions were estimated to be as high as 100 μrad by Bouchon and Aki (1982). Deuss and Woodhouse (2001) have calculated theoretically the free-oscillation spectra and have discussed the effect of the coupling of large groups of normal modes. They have particularly emphasized the cross coupling between spheroidal modes and toroidal modes, in inhomogeneous and anisotropic media, and have shown that a detailed study of the spectra would be useful in developing a 3D structural model of the Earth. In this context we note that the pioneering observations of Suda, *et al.* (1998) and their interpretation by Rhie and Romanowicz (2004) have indicated the presence of a continuous hum in the spheroidal mode oscillations of the Earth in the frequency range of 2–7 mHz, which is probably generated by the interactions amongst the ocean floor, the oceans, and the atmosphere. Thus, we may expect a hum in the toroidal modes as well, either generated by cross couplings with other modes, or, more interestingly, generated directly. At present we have several theoretical models with explicit predictions for rotational seismic motions but do not have observationally confirmed answers to several basic questions specific to rotational motions: What are the angular amplitudes of the torsional normal modes of the Earth in its quiescent state? What is the spectrum of excitations triggered by an earthquake? What are the differences that signify the different underlying mechanisms for earthquakes? How much energy of an earthquake is channeled into these modes? How is this energy dissipated, by cross coupling to S modes or by transfers to high frequencies or low frequencies before thermalization?

Recently Aki and Richards (2002) bemoaned that seismology still awaited suitable instruments for making measurements of the rotational motions. Except for the recent pioneering efforts of Igel *et al.* (2005, 2007) with ring lasers, the current situation may be summarized by noting that, by and large, seismic observations have been confined to the measurement of the three translational motions along the $\hat{x}$, $\hat{y}$, and $\hat{z}$ axes, and the measurements of the rotations about these three axes are either nonexistent or limited to occasional measurements of strong fields carried out in close proximity to earthquakes or volcanoes (Kanamori, 1994; Wiens *et al.*, 2005).

Proposed Seismological Measurements

Naturally when a new sensor is developed that makes measurements that previously could be carried out only with great difficulty, it is hard to anticipate the results. In many cases, new phenomena are discovered or new theories are developed. Simply allowing the widespread deployment of sensors that were originally confined to a few locations can lead to unanticipated breakthroughs, as demonstrated by recent deployments of large broadband seismograph arrays. In the case of rotational seismic motions, there has been some theoretical discussion of the usefulness of the rotational components of ground motion (Bouchon and Aki, 1982; Takao, 1998), and some concepts have been proven using rotational measurements made by arrays of translational seismographs (Huang, 2003; Suryanto *et al.*, 2006) or by ring lasers (Pancha *et al.*, 2000; Igel *et al.*, 2005, 2007). However, Aki and Richards (2002) note that "seismology still awaits a suitable instrument for making such measurements." In this section we describe some of the uses we anticipate for the proposed sensors.

Rotational Strong Ground Motion Measurements. Although calculations by Bouchon and Aki (1982) suggested that rotational motions near a fault are minor compared to translational motions, rather large rotational motions have been recorded within a few kilometers of several earthquakes (Takao, 1998; Huang, 2003). Rotational components of strong ground motion are expected to pose considerable problems for earthquake engineering (Oliveira and Bolt, 1989) but are seldom recorded because complex instrumentation like ring lasers or arrays are required. Near-source recordings of rotational motion may also place important constraints on the earthquake source processes (Pancha *et al.*, 2000; Huang, 2003). We suggest that, if inexpensive rotational strong ground motion sensors are developed, they would greatly increase the total number of available rotational strong ground motion records that would observationally constrain this hazard.

Measurement of Tilt Associated with Geodetic Deformation. Geodetic deformation such as that accompanying the inflation of a volcanic magma chamber may cause changes in ground tilt (Dvorak and Dzurisin, 1997). Tiltmeters are often deployed in volcanic regions for this purpose, and in rare cases large amplitude geodetic tilt can be determined from the horizontal components of translational broadband seismographs (Wiens *et al.*, 2005). The widespread deployment of rotational seismographs would allow accurate, routine recording of geodetic tilt.

Teleseismic Applications. Igel *et al.* (2007) report several possible applications of rotational sensors for analysis of teleseismic waveforms. Colocated rotational and translational seismographs allow single station determination of surface wave phase velocity, whereas multiple stations are necessary using traditional translational seismographs alone. Additional interesting teleseismic observations include rotational motions in the *P*-wave coda, between the *P*-wave and *S*-wave arrival times where no rotational motions are predicted for a spherically symmetric isotropic velocity model. These rotational motions must result from 3D structural complexity or anisotropy, and further investigation should lead to constraints on these properties. For these applications, especially for the low-frequency weak signals, it may be advantageous to use the design given by Cowsik (2007).

Correction of Translational Horizontal Seismograph Components for Tilt. The horizontal components of ordinary translational broadband sensors actually record a combination of translation and tilt motions, although the tilt is generally ignored (Wielandt and Forbriger, 1999). A colocated recording of tilt by a rotational seismograph allows correction of the horizontal records for tilt and thus reconstruction of the true horizontal translation. Many horizontal broadband seismographs, particularly on the ocean bottom, are extremely noisy at long periods (periods < 50 sec) due to very small tilts that induce noise in the horizontal component. Independent recording of tilt will allow the removal of this noise, providing a much better signal-to-noise ratio on the horizontal component (Crawford and Webb, 2000). In addition, near-source horizontal recordings of some phenomena, such as strombolian eruptions of volcanoes, naturally contain both translational and tilt components. Tilt records allow the disentangling of these records, facilitating a much more straightforward interpretation (Aster *et al.*, 2003, Chouet *et al.*, 2003).

Design Concept and Description of the New Instrument

In essence, the instrument consists of a torsion balance having a natural period of oscillation significantly longer than the longest period of the rotational seismic motions of interest. The angular position of the torsion balance is recorded at regular intervals by means of a high-resolution optical lever of large dynamic range. As noted in the introduction, such a torsion balance will couple negligibly to the rotational oscillations of the Earth, and an optical lever that is firmly coupled to the Earth will faithfully record the seismic rotational oscillations. Here the challenge faced by the experimentalist is twofold: (1) to develop a mechanical torsional oscillator with a low enough natural frequency ($\sim 10^{-3}$ Hz) and the fabrication of an optical lever with an angular resolution $\Delta\psi$ better than $\sim 10^{-6}\,\mathrm{rad\,Hz}^{-1/2}$; (2) to make the instrument robust and field worthy for the study of near-field strong motions at frequencies higher than $\sim 10^{-2}$ Hz. There are two basic subsystems of such an instrument: the torsional oscillator and the optical lever for the angular measurements. The designs that meet the requirements of a robust rotational seismometer are described subsequently.

Design of the Torsion Balance

The balance is designed to have its natural frequency of oscillation substantially below that of the lowest frequencies of interest in near-field and engineering studies, say below 10^{-2} Hz. Furthermore, it is to be made insensitive to ground tilts and several other unwanted background torques. These requirements are met in the following way: the balance bob consists of a circular mirror of diameter $\sim$40 mm, with its normal in the horizontal plane as shown in Figure 3. The mirror is mounted within an aluminum framework whose moment of inertia may be adjusted as required. Typically, the whole configuration weighs $\sim$50 g and has a moment of inertia of $\sim$400 $\mathrm{g\,cm}^2$ about the suspension axis. The suspension fiber is made of, say, SS-304 alloy with a rigidity modulus of $n = 7.8 \times 10^{11}$ dyne/cm^2, which for reducing the restoring torque has a rectangular cross section. For example, in the fabrication of a simple apparatus to prove the concept, we have chosen $(x \times y) = 7\ \mu\mathrm{m} \times 110\ \mu\mathrm{m}$ and the fiber length $l = 3$ cm. Following Champion and Davy (1936), the torsional rigidity of the fiber is given by

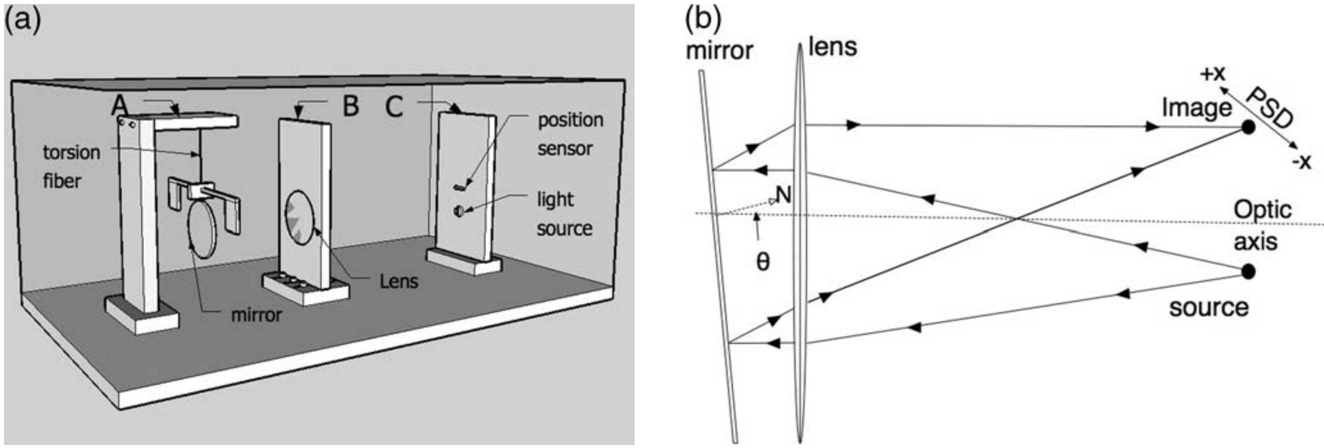

Figure 3. (a) simple cut-out drawing of the prototype torsional oscillator is shown. Plate A is used to suspend the balance bob (mirror and extension arms), plate B holds the 200 mm focal length lens, and plate C holds the light source (an LED) and the position sensing detector (a PSD). This simplified drawing does not show the magnetic damper. (b) The schematic diagram of the optics of the system is shown.

$$k_f = \frac{x^3 yn}{3l} = 0.3 \text{ dyne cm/rad.} \tag{2}$$

The angular frequency of natural oscillations is given by

$$\omega_0 = \left(\frac{k_f}{I}\right)^{1/2} \approx 2.8 \times 10^{-2} \text{ rad/sec,} \tag{3}$$

which corresponds to a period of ~200 sec or a frequency of ~5×10^{-3} Hz, that is, significantly smaller than the frequencies of interest.

The oscillations of the balance as a simple pendulum are damped by attaching the top of the fiber of the torsion balance to a copper disk suspended by a torsionally stiff wire of circular cross section, thus, constituting a small simple pendulum. A set of permanent magnets generate fields that thread through the disk. Eddy currents generated due to pendular swings are dissipated in the disk, thereby damping the pendular motions. Because a suspension fiber of rectangular cross section will, in principle, generate a finite coupling between tilt and torsion of the fiber, it is important to isolate the balance from ground tilts. However, because we have chosen the torsionally stiff wire above the eddy-current damper to have a circular cross section, it will just bend following the tilt but with no torsion. The stainless steel fiber attached to it will not bend, and thus the balance will be isolated from coupling to tilts. In a simpler system, we may just choose a torsion fiber of circular cross section.

A robust, yet sensitive, optical lever has been developed for the present application. It consists of a slit illuminated by a high-intensity light emitting diode (LED; 50,000 mcd) emitting in a forward cone of angle ~7°. The slit is located at the focal plane of a lens of aperture $f \sim 200$ mm (see Fig. 3). This optical design is generally referred to as the autocollimating configuration, and it ensures that the image quality and the angular displacement of the image due to motions of the mirror are sensibly independent of changes in the temperature of the surroundings. The optical image falls on a position sensitive diode (PSD; Hamamatsu C9068), and the position is capable of being read out with a maximum frequency of ~100 Hz. The best positional accuracy that is achievable with this system is $\Delta x \sim 3 \times 10^{-5}$ mm Hz$^{-1/2}$, which corresponds to an angular displacement of the mirror by $\Delta\psi = \Delta x/2f \approx 7.5 \times 10^{-8}$ rad Hz$^{-1/2}$. This sensitivity of the optical lever is sufficient for the engineering applications and near-field studies.

The whole instrument should be enclosed within an airtight container, insulated thermally to avoid convection currents, and shielded from variations of the magnetic fields by a μ-metal sheath. The sketch of the simplified instrument built for proving the design concepts is shown in Figure 3.

Response Function of the Instrument to Periodic Signals

We begin this section with an analysis of the response of the instrument to torsional seismic oscillations and show that the signal put out by it has a unity gain at frequencies significantly above the natural frequency of the torsion balance, with a phase lag of 180°. We then discuss the root mean square (rms) amplitudes induced by the thermal fluctuations and show that these essentially define the detection threshold. The analysis treatment here closely follows that described by Cowsik (2007).

Response Function of the Balance. In deriving the response function, we first note that only the point of suspension of the balance and the autocollimating optical lever will faithfully follow the seismic angular oscillations defined in equation (1) by $R(\omega, r, \theta, \phi)$. Considering one of the Fourier components $R(\omega)$, the torque acting on the balance due to the angular twist of the fiber is given by

$$S(\omega) = k_f R(\omega) \cos\omega t = I\omega_0^2 R(\omega) \cos\omega t. \tag{4}$$

The response of the balance to this component may be written as (Marion, 1996)

$$\ddot{\psi} + 2\beta\dot{\psi} + \omega_0^2\psi = a(\omega)\cos(\omega t), \tag{5}$$

where the angular displacement of the balance is represented by ψ, the damping parameter by β, the resonant frequency by ω_0, and the driving term by $a(\omega) = \omega_0^2 R(\omega)$. The signal recorded by the autocollimator may then be shown to be (Cowsik, 2007)

$$\begin{aligned}\alpha &= \frac{a(\omega)}{[(\omega_0^2-\omega^2)^2+4\omega^2\beta^2]^{1/2}}\cos(\omega t-\delta) - \frac{a(\omega)}{\omega_0^2}\cos(\omega t)\\ &= a(\omega)\left\{\frac{\cos(\delta)}{[(\omega_0^2-\omega^2)^2+4\omega^2\beta^2]^{1/2}} - \frac{1}{\omega_0^2}\right\}\cos(\omega t)\\ &\quad + a(\omega)\left\{\frac{\sin(\delta)}{[(\omega_0^2-\omega^2)^2+4\omega^2\beta^2]^{1/2}}\right\}\sin(\omega t)\\ &\approx -\frac{a(\omega)}{\omega_0^2}\cos(\omega t)\\ &= R(\omega)\cos(\omega t), \qquad \text{for } \omega \gg \omega_0,\end{aligned} \tag{6}$$

where α is the angle measured by the optical lever and δ is the phase lag with respect to the ground motion. We show in Figure 4 the amplitude and phase of the response function given by equation (6) for various values of (ω/ω_0) in units of $a(\omega)$, for $\beta = \omega_0/200$ corresponding to $Q \approx 100$.

Several interesting features emerge from inspection of Figure 4. The response of the instrument at frequencies above the natural frequency ω_0 rapidly reaches unity, even though it is exactly π out of phase with the seismic oscillations. At frequencies below ω_0, the response declines, initially rapidly close to ω_0 and subsequently as $(\omega/\omega_0)^2$. In the quadrature phase the response is weaker and decreases as $(\omega_0/\omega)^2$ at higher frequencies. Because ω_0 is significantly below the lowest frequencies of interest, the instrument will be capable of recording the seismic motions with unit response.

Sensitivity of the Instrument. The thermal fluctuations of the torsion balance determine the threshold sensitivity of the balance. These are easily calculated by noting the interconnection that the fluctuating thermal torques bear with the damping term or the quality factor of the balance, as first explained by Einstein (1905) and worked out with progressively increasing detail by Chandrasekhar (1943) and Kubo (1966) who emphasized that the autocorrelation function of the thermal torques is a Dirac-delta function. These were estimated (Cowsik, 2007) to be

$$\langle\alpha_{\mathrm{Th}}^2(\omega)\rangle^{1/2} = \frac{\langle a_T^2\rangle^{1/2}}{[(\omega^2-\omega_0^2)^2+4\beta^2\omega^2]^{1/2}} \approx \langle a_T^2\rangle^{1/2}/\omega^2, \quad \text{for } \omega \gg \omega_0, \tag{7}$$

where $\langle a_T^2(\omega)\rangle^{1/2} = (kT/\pi IQ)^{1/2}$, Q being the quality factor of the balance. Now consider a seismic signal that peaks at some frequency ω_s and has a quality factor Q_s. The thermal amplitude within the bandwidth sets the typical threshold for the observations

$$\alpha_{\mathrm{sensitivity}}(\omega_s) = \frac{\langle a_{\mathrm{Th}}^2(\omega_s)\rangle^{1/2}}{\omega_s^2}\left(\frac{\omega_s}{Q_s}\right)^{1/2} = \left(\frac{kT\omega_0}{\pi I\omega_s^3 QQ_s}\right)^{1/2}. \tag{8}$$

Inserting typical values $T = 300$ K, $\omega_0 = 3.7 \times 10^{-2}$ rad/sec, $I = 150$ g cm^2, and $\omega_s \approx 10^{-1}$ rad/sec, corresponding to the typical frequencies of near-field motions, $Q = 100$ and $Q_s = 30$, we find $\alpha_{\mathrm{sensitivity}} \approx 3 \times 10^{-9}$ rad. Thus, thermal backgrounds do not pose any significant restrictions in observing strong ground motions. However,

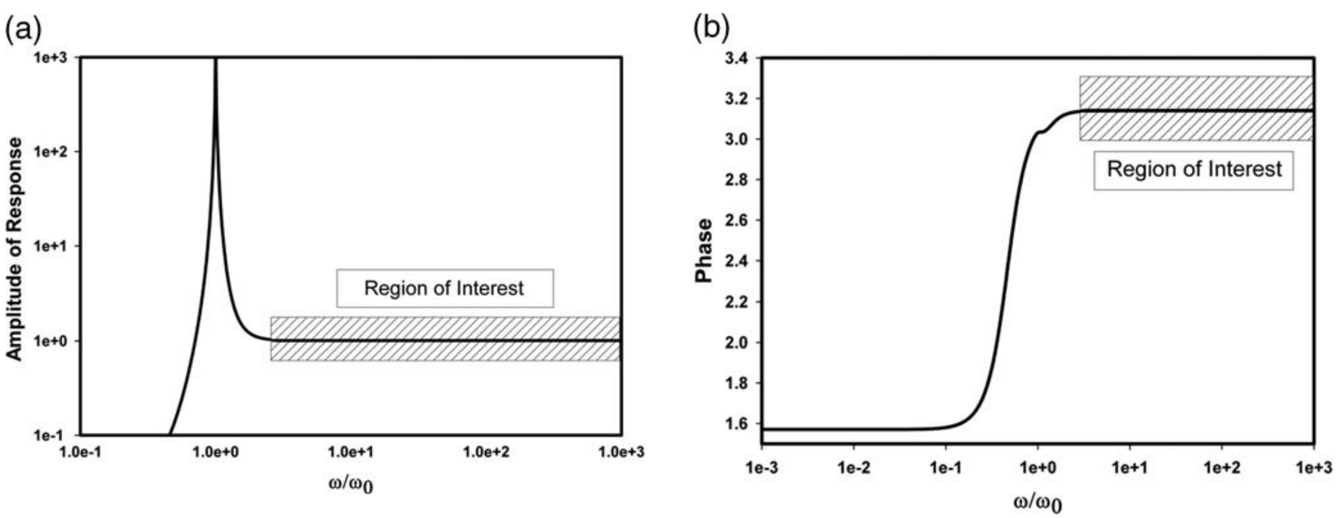

Figure 4. Amplitude of the response function given by equation (6) in units of $a(\omega)$. Notice that in the relevant region, the balance exhibits unit response.

one does have to isolate the instrument from other sources of noise.

Isolation of the Responses to Torsion from Tilts. Let us consider a small tilt of the ground,

$$\Delta = \Delta_0 \cos \omega_h t, \tag{9}$$

about some axis in the horizontal plane. The frequency of the tilt oscillations ω_h is expected to be much smaller than the oscillations of the torsion balance as a whole as a simple pendulum, which is

$$\omega_p = \sqrt{\frac{g}{l}} \approx 10 \text{ rad/sec}\,. \tag{10}$$

Accordingly, the suspension fiber of the balance will continue to be aligned with the local gravity field. The body of the apparatus would, however, always be aligned with the ground. Because the cross section of the fiber of the eddy-current damper that attaches the balance to the vacuum chamber has a circular cross section (as described earlier in the section titled Design of the Torsion Balance), the bending of the fiber due to tilts does not cause any twists. Thus, we may conclude that the normal to the mirror attached to the pendulum bob will always point in the horizontal plane, that is, perpendicular to local gravity.

In contrast, the axis of the autocollimator will follow the tilt faithfully. Accordingly, the measured rotation angle will have a change in the absolute normalization by a factor of $\cos \Delta \approx 1 - \Delta^2/2$. Because ground tilt Δ is expected to be small, this normalization change may be neglected. Thus, in the present design the cross coupling between different motions such as translations and rotations around the three axes is extremely small, in contrast with the experience with conventional seismometers measuring translations.

Design of Seismometers for Measuring Tilts. The design principle described in previous sections can be carried over for building seismometers to measure tilts as well, which may be thought of as rotations about an axis that lies in the horizontal plane. For the discussion of tilts, it would be useful to formally define the axes about which angular displacement occur. The rotational seismometer discussed previously pertains to rotations about the local zenith; this is defined as the $\hat{z}$ direction. The $\hat{x}$ and $\hat{y}$ axes are chosen in the horizontal plane, with say, $\hat{x}$ pointing towards the east and $\hat{y}$ pointing towards the north. Now let us consider a cube suspended by a torsional fiber; optical levers reflecting light beams off each face of the cube can measure two angular coordinates toward which the normal to the face is pointing. Because three angles are needed to specify the orientation of the cube, we need to observe at least two faces that are perpendicular to each other. When angular displacements about the $\hat{z}$ axis, $\alpha(z, t)$, occur, these will induce torques about the $\hat{z}$ axis proportional to the torsion constant of the suspension fiber, and the response of the balance will be as described earlier, which can be recorded by an optical lever measuring angular displacements about the $\hat{z}$ axis.

Now consider the effects of the ground tilts on the instrument, say about the $\hat{x}$ axis, specified by $\alpha(x, t)$ or equivalently $\tilde{\alpha}(x, \nu_t)$, where $\tilde{\alpha}$ is the Fourier amplitude. Let ν_t be much smaller than the pendular frequency of the balance, $\nu_0 \approx \frac{1}{2\pi}\sqrt{\frac{g}{l}}$, where l is the length of the suspension fiber and g is the acceleration due to gravity, that is, $\nu_t \ll \nu_0$. Then the suspension fiber would be aligned with the $\hat{z}$ direction, but the optical lever observing the $\hat{y}\,\hat{z}$ face of the cube will record $\tilde{\alpha}(x, t)$ faithfully. If, on the other hand, $\nu_t \gg \nu_0$, the response of the balance will be small and proportional to $(\nu_0/\nu_t)^2$. In contrast, the local gravity field $\vec{g}$ might be thought of as a very low-frequency field, with $\nu_g \approx 0$, which will induce unit response on the balance, and the pendulum will act as a plumb line so that we will have a good measurement of $\alpha(x, t)$. However, if $\nu_t \sim \nu_0$, then the suspended bob will respond strongly to the tilts and reliable measurements cannot be made.

Let us now discuss translations, that is, linear accelerations along the $\hat{x}$ and $\hat{y}$ axes, and consider accelerations $a(x, t)$ or equivalently $\tilde{a}(x, \nu_H)$ to be specific. This will correspond to a torque on the instrument $\tau_H \approx \tilde{a}(x, \nu_H)lm$ in the $\hat{y}$ direction, where m is the mass of the suspended bob. This leads to an apparent tilt, $\alpha_{tH} \approx \tau_H/gm = \tilde{a}/g$. In the simple design discussed here, this cannot be avoided. On the other hand, the ideal design for a tiltmeter will be to suspend a symmetric mass with a two-axis flex bearing located at its center of gravity. Such an instrument will execute flexural oscillations about the $\hat{x}$ and $\hat{y}$ axes, say with a frequency ν_f. Linear accelerations along the $\hat{x}$ and $\hat{y}$ axes, or indeed along any horizontal axis, will not lead to any torque, as the inertial forces act through the center of gravity. For $\nu_H \gg \nu_f$, we will reliably measure the tilt without significant coupling to horizontal accelerations. It is difficult to design such flexural oscillators with frequencies below $\sim 10^{-2}$ Hz, and tilt observations will be limited to $\nu \gtrsim 3 \times 10^{-2}$ Hz. For lower frequencies, the simple pendulum discussed in the earlier paragraphs of this section may provide better possibilities.

Test Procedure for the Robust Instrument

It will be useful to begin the characterization of the robust instrument by mounting it in close proximity to arrays of translation seismometers deployed around earthquake prone zones for measurement of the near-field characteristics. Then the analysis proceeds along the lines used by Cochard *et al.* (2006) and Igel *et al.* (2005, 2007) in their pioneering effort with the ring laser angular motion sensors developed by Schreiber *et al.* (2006). It is expected that this new robust instrument will be compact enough to be mounted on test facilities being developed by the USGS scientists for absolute calibrations. Such studies will be very useful in assessing possible cross talk between translational, rotational, and tilt modes in the response of the instrument.

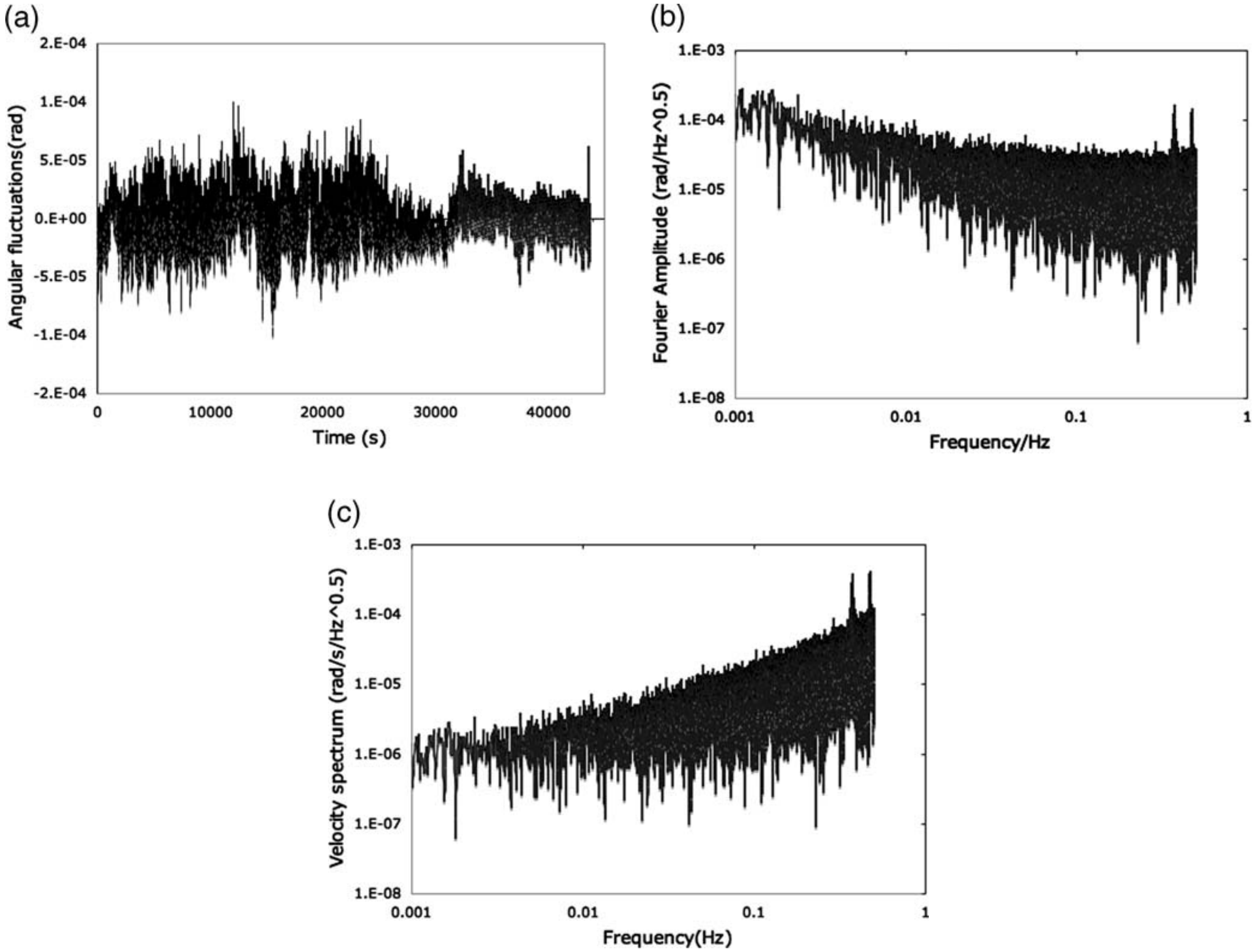

Figure 5. (a) The mean angular position of the balance (α in radians) acquired at 1 sec intervals, after removing the general long term drift. (b) The digital Fourier transform $\tilde{\alpha}$, normalized such that $\sum \tilde{\alpha}^2 = \sum \alpha^2$. (c) The velocity spectrum, $\dot{\tilde{\alpha}} = 2\pi\nu\tilde{\alpha}$.

Proof of Concept—Prototype Toy Model

A very simplified prototype new instrument described in this article has been constructed without any engineering sophistication and is set up on a tabletop. Efforts are underway to ensure proper insulation from environmental disturbances. This is for the purposes of proving the design concepts discussed previoulsy. Before briefly reporting the results from this setup, we note that this simple instrument already displays the sensitivity needed for engineering and near-field studies as assessed by the USGS panel chaired by Evans *et al.* (2007). The preliminary results are reported in Figure 5, and these indicate that the response noise is about 10^{-4} rad sec^{-1} Hz$^{-1/2}$ at ~0.5 Hz and decreases at lower frequencies reaching about 10^{-6} rad sec^{-1} Hz$^{-1/2}$ at 0.001 Hz. Even at this stage it shows promise to probe the rotational motions at levels prescribed by the USGS panel, summarized in Figure 2. There are two discernible peaks at 0.37 and 0.47 Hz whose origin we cannot be sure of. We are hopeful that a considerable improvement in the noise level should be possible with better engineering of the design.

Conclusion

We conclude this article by noting that the basic design concept of using a torsion balance as a filter to detect rotational seismic motions is validated and may be implemented for the construction of rotational seismometers, even though much work lies ahead to make them more sensitive and reliable for deployment in the field.

Data and Resources

All data used in this article came from the prototype instrument we have described and from published sources listed in the references.

References

Adelberger, E. G., B. R. Heckel, and A. E. Nelson (2003). Tests of the gravitational inverse-square law, *Ann. Rev. Nucl. Part. Sci.* **53,** 77–121.

Aki, K., and P. G. Richards (2002). *Quantitative Seismology, Theory and Methods*, Freeman, San Francisco, 608 pp.

Aster, R., S. Mah, P. Kyle, W. McIntosh, N. Dunbar, J. Johnson, M. Ruiz, and S. McNamara (2003). Very long period oscillations of Mount Erebus Volcano, *J. Geophys. Res.* **108,** no. B11, 2522, doi 10.1029/2002JB002101.

Bouchon, M., and K. Aki (1982). Strain, tilt, and rotation associated with strong ground motion in the vicinity of earthquake faults, *Bull. Seismol. Soc. Am.* **72,** 1717–1738.

Boynton, P. E., D. Crosby, P. Ekstrom, and A. Szumilo (1987). Search for an intermediate-range composition-dependent force, *Phys. Rev. Lett.* **59,** 1385–1389.

Braginsky, V., and V. Panov (1972). Verification of equivalence of inertial and gravitational masses, *JETP Lett.* **34,** 463–466.

Castellani, A., and Z. Zembaty (1996). Comparison between earthquake rotation spectra obtained through different experimental sources, *Eng. Struct.* **18,** 597–603.

Champion, F. C., and N. Davy (1936). *Properties of Matter* (Blackie & Son, Ltd., London, Bombay, Glasgow).

Chandrasekhar, S. (1943). Stochastic problems in physics and astronomy, *Rev. Mod. Phys.* **15,** 1–89.

Chouet, B. A., P. Dawson, T. Ohminato, M. Martini, G. Saccorotti, F. Giudicepietro, G. De Luca, G. Milana, and R. Scarpa (2003). Source mechanisms of explosions at Stromboli Volcano, Italy, determined from moment-tensor inversion of very-long-period data, *J. Geophys. Res.* **108,** no. B1, 2019, doi 10.1029/2002JB001919.

Cochard, A., H. Igel, B. Schuberth, J. Wassermann, and W. Suryanto (2006). Rotational motions in seismology: theory, observation, simulation, *Earthquake Source Asymmetry, Structural Media and Rotation Effects*, R. Teisseyre, M. Takeo, and E. Majewski (Editors), Springer, Berlin, 391–412.

Cowsik, R. (1981). A new torsion balance for studies in gravitation and cosmology, *Indian J. Phys., Part B* **55B,** 497–512.

Cowsik, R. (2007). An instrument for direct observations of seismic and normal-mode rotational oscillations of the Earth, *Proc. Natl. Acad. Sci.* **104,** no. 17, 6893–6898.

Cowsik, R., N. Krishnan, P. Saraswat, S. N. Tandon, S. Unnikrishnan, U. D. Vaishnav, C. Viswanadham, and G. P. Puthran (1989). Torsion balance experiments for the measurement of weak forces in nature, *Indian J. Pure Appl. Phys.* **27,** 691–709.

Cowsik, R., N. Krishnan, S. N. Tandon, and C. S. Unnikrishnan (1988). Strength of intermediate-range forces coupling to isospin, *Phys. Rev. Lett.* **61,** 2179–2181.

Cowsik, R., N. Krishnan, S. N. Tandon, and C. S. Unnikrishnan (1990). Strength of intermediate-range forces coupling to isospin, *Phys. Rev. Lett.* **64,** 336–339.

Cowsik, R., R. Srinivasan, S. Kasturirangan, A. Senthil Kumar, and K. Wagoner (2007). Design and performance of a sub-nanoradian resolution autocollimating optical lever, *Review Sci. Instrum.* **78,** 035105-1–035105-7.

Crawford, W. C., and S. C. Webb (2000). Identifying and removing tilt noise from low-frequency (<0.1 Hz) seafloor vertical seismic data, *Bull. Seismol. Soc. Am.* **90,** 952–963.

Deuss, A., and J. H. Woodhouse (2001). *Geophys. J. Int.* **146,** 833–842.

Dvorak, J. J., and D. Dzurisin (1997). Volcano geodesy: the search for magma reservoirs and the formation of eruptive vents, *Rev. Geophys.* **35,** 343–384.

Dziewonski, A. M., and D. L. Anderson (1981). Preliminary reference Earth model (PREM), *Phys. Earth Planet. Interiors* **25,** 297–356.

Einstein, A. (1905). Investigations on the theory of Brownian movement, *Ann. Phys. Lpz.* **17,** 549–560.

Evans, J. R., H. I. Igel, L. Knopoff, T. L. Teng, and M. D. Trifunac (2007). Rotational seismology and engineering—online proceedings for the first international workshop, *U.S. Geol. Surv. Open-File Rept. 2007-1144*, version 2.0., Appendix 2.5, 36–37.

Huang, B.-S. (2003). Ground rotational motions of the 1999 Chi-Chi, Taiwan earthquake as inferred from dense array observations, *Geophys. Res. Lett.* **30,** no. 6, 1307, doi 10.1029/2002GL015157.

Igel, H., A. Cochard, J. Wassermann, A. Flaws, U. Screiber, A. Velikoseltsev, and N. Pham Dinh (2007). Broad-band observations of earthquake-induced rotational ground motions, *Geophys. J. Int.* **168,** 182–196.

Igel, H., H. K. Lee, and M. I. Todorovska (2006). Inauguration of the International Working Group on Rotational Seismology (IWGoRS), *AGU Fall Meeting 2006*, Rotational Seismology Sessions: S22A, S23B.

Igel, H., U. Schreiber, A. Flaws, B. Schuberth, A. Velicoseltsev, and A. Cochard (2005). *Geophys. Res. Lett.* **3L,** L08309.

Jones, R. V., and J. C. S. Richards (1959). Recording optical lever, *J. Sci. Instrum.* **36,** 90–94.

Kanamori, H. (1994). *Annu. Rev. Earth Planet. Sci.* **22,** 207–307.

Kubo, R. (1966). The fluctuation-dissipation theorem, *Rep. Prog. Phys.* **29,** 255–284.

Lee, H. K., M. Çelebi, M. I. Todorovska, and M. Diggles (Editors), (2007). Rotational seismology and engineering—online proceedings for the first international workshop, in *U.S. Geol. Surv. Open-File Rept. 2007-1144*, version 2.0.

Marion, J. B. (1996). *Classical Dynamics of Particles and Systems*, Academic Press, New York.

Newman, R., P. G. Nelson, and D. M. Graham (1990). Search for an intermediate-range composition-dependent force coupling to N-Z, *Phys. Rev. D* **42,** 963–976.

Oliveira, C. S., and B. A. Bolt (1989). Rotational components of surface strong ground motion, *Earthq. Eng. Struct. Dyn.* **18,** 517–526.

Pancha, A., T. H. Webb, G. E. Stedman, D. P. McLeod, and U. Schreiber (2000). Ring laser detection of rotations from teleseismic waves, *Geophys. Res. Lett.* **27,** 3553–3556.

Rhie, J., and B. Romanowicz (2004). Excitation of Earth's incessant free oscillations by atmosphere-ocean-seafloor coupling, *Nature* **431,** 552–556.

Ritter, R. C., C. E. Goldblum, W. T. Ni, C. T. Gillies, and C. Speake (1990). Experimental test of equivalence principle with polarized masses, *Phys. Rev. D* **42,** 977–991.

Roll, P. G., R. Krotkov, and R. H. Dicke (1964). The equivalence of inertial and passive gravitational mass, *Ann. Phys.* **26,** 442–517.

Schreiber, K. U., G. E. Stedman, H. Igel, and A. Flaws (2006). Ring laser gyroscopes as rotation sensors for seismic wave studies, in *Earthquake Source Asymmetry, Structural Media and Rotation Effects*, R. Teisseyre, M. Takeo, and E. Majewski (Editors), Springer, Berlin, ISBN 978-3-540-31336-6, 377–390.

Stein, S., and M. Wysession (2003). *An Introduction to Seismology, Earthquakes, and Earth Structure*, Blackwell, Oxford.

Suda, N., K. Nawa, and Y. Fukao (1998). Earth's background free oscillations *Science* **279,** 2089–2091.

Suryanto, W., H. Igel, J. Wassermann, A. Cochard, B. Schuberth, D. Vollmer, F. Scherbaum, U. Schreiber, and A. Velikoseltsev (2006). First comparison of array-derived rotational ground motions with direct ring laser measurements, *Bull. Seismol. Soc. Am.* **96,** 2059–2071.

Takeo, M. (1998). Ground rotational motions recorded in near-source region of earthquakes, *Geophys. Res. Lett.* **25,** 789–792.

Weilandt, E., and T. Forbriger (1999). Near-field seismic displacement and tilt associated with the explosive activity of Stromboli, *Ann. Geofis.* **42,** 407–416.

Wiens, D. A., P. J. Shore, S. H. Pozgay, A. W. Sauter, and R. A. White (2005). Tilt recorded by a portable broadband seismograph: the 2003 eruption of Anatahan Volcano, Mariana Islands, *Geophys. Res. Lett.* **32,** L18305, doi 10.1029/2005GL023369.

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